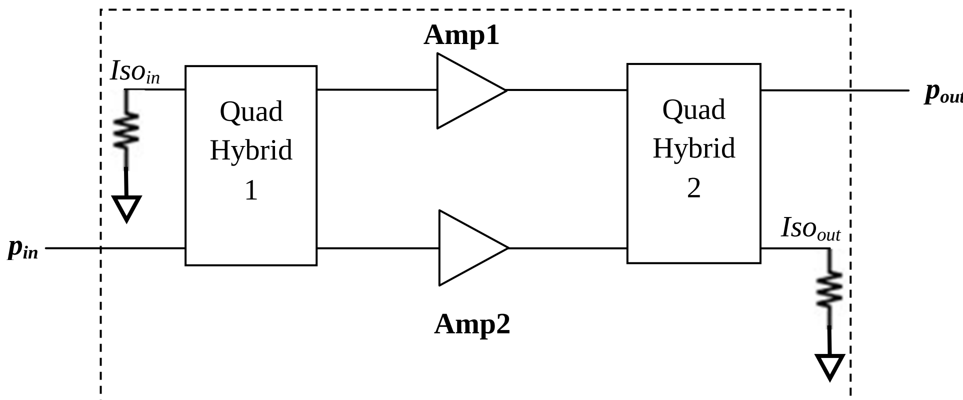
ECE 4880 RF Systems Fall 2016
Homework 8 Solution

Reading before homework:

- Lecture summary on Signal Splitting and Combining; Frequency Strategy
- Egan's book, Chaps 6 and 7
- Simulink tutorial

1. (**Quadrature hybrid amplifier**) To improve the linearity and signal reflection of LNA, a quadrature hybrid architecture is used, as shown below. You measure the S parameters for Amp1 with 50Ω cables and obtain:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{amp1} = \begin{bmatrix} 0.1 & 0 \\ 8 & 0.05 \end{bmatrix}$$



(a) What are the input and output impedance of Amp1 in Ω? What is the amplifier gain in dB? Notice that S parameters are voltage ratios. (**6 pts**)

$S_{11} = \Gamma_{Lin} = \frac{V_-}{V_+} = \frac{Z_{in}/Z_o - 1}{Z_{in}/Z_o + 1} = 0.1$. We can solve to get Z_L = 61Ω.

$S_{22} = \Gamma_{Lout} = \frac{V_-}{V_+} = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1} = 0.05$. We can solve to get Z_L = 55Ω.

The linear gain is $20\log_{10}(S_{21})$ = 18.1dB.

(b) First assume that Amp1 and Amp2 are identical in every aspect. What are the S parameters for the block in the dash-line box? Remember that S parameters are complex numbers. (**6 pts**)

As all reflections will be cancelled out at p_{in} and p_{out} due to the –180° phase shift of the two paths. The voltage gain magnitude will remain the same as Amp1 and Amp2 have –3dB input, and then are combined. However, there is a –90° phase shift.

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{hybrid} = \begin{bmatrix} 0 & 0 \\ -8j & 0 \end{bmatrix}$$

(c) If you measure the S parameters for Amp2 and find that they are slightly different as below.

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{amp2} = \begin{bmatrix} 0.1 & 0 \\ 7 & 0.1 \end{bmatrix}$$

Estimate the S parameters for the block in the dash-line box now? Assume the quadrature hybrid is still ideal. (**8 pts**) For p_{in} = 0dBm, estimate the power dissipated in Iso_{in} and Iso_{out}. (**8 pts**) **Hint**: when you calculate the voltage through the hybrid, the path without phase shift is $1/\sqrt{2}$, and the one with 90° shift is $-j/\sqrt{2}$.

For S_{11}, it is still matched, so we will have S_{11} = 0.

We will need to trace the two different paths for S_{21} and S_{22}.

$$S_{21} = \underbrace{\frac{1}{\sqrt{2}} \cdot (-j) \cdot 8 \cdot \frac{1}{\sqrt{2}}}_{path1} + \underbrace{\frac{1}{\sqrt{2}} \cdot 7 \cdot \frac{1}{\sqrt{2}} \cdot (-j)}_{path2} = -7.5j$$

$$S_{22} = \underbrace{\frac{1}{\sqrt{2}} \cdot 0.05 \cdot \frac{1}{\sqrt{2}}}_{path1} + \underbrace{\frac{-j}{\sqrt{2}} \cdot 0.1 \cdot \frac{-j}{\sqrt{2}}}_{path2} = -0.025$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{hybrid} = \begin{bmatrix} 0 & 0 \\ -7.5j & -0.025 \end{bmatrix}$$

V_+ at pin will be 0.32V for 0dBm p_{in}.

At Iso_{in}, we have the voltage as: $V_{-isoin} = \underbrace{0.32\frac{-j}{\sqrt{2}} \cdot 0.1 \cdot \frac{1}{\sqrt{2}}}_{path1} + \underbrace{0.32\frac{1}{\sqrt{2}} \cdot 0.1 \cdot \frac{-j}{\sqrt{2}}}_{path2} = -0.032j$

The power dissipation at the Iso_{in} resistor is then: $p_{isoin} = \frac{(V_{-isoin})^2}{2Z_0} = 10.0\mu W$ = –20dBm.

At Iso_{out}, we have the voltage as: $V_{isoout} = \underbrace{0.32\frac{-j}{\sqrt{2}} \cdot 8 \cdot \frac{-j}{\sqrt{2}}}_{path1} + \underbrace{0.32\frac{1}{\sqrt{2}} \cdot 7 \cdot \frac{1}{\sqrt{2}}}_{path2} = -0.16$

The power dissipation at the Iso_{out} resistor is then: $p_{isoout} = \frac{(V_{isoout})^2}{2Z_0} = 0.256mW$ = –5.92dBm.

(d) If both Amp1 and Amp2 have IIP_{H2} = 20dBm (but their S parameters are slightly different like in part (c)), for p_{in} = 0dBm, estimate the 2nd harmonic power at p_{out}. Assume that H2 will make the total output voltage at Amp1 and Amp2 smaller (i.e., a_2 is negative). (**8 pts**) Compare your answer of the H2 power if p_{in} = 0dBm is just fed into Amp1. (**8 pts**)

The S parameters are defined with small signals in the linear system, but we can still calculate the 2nd harmonic voltage in the path separately.

Given IIP_{H2} = 20dBm, we know A_{IIPH2} = 3.2V and can calculate a_2 at IP_{H2} by the following equation:

$$A_{OIPIM\,2} = |a_1| A_{IIPIM\,2} = |a_2| A_{IIPIM\,2}^2 .$$

For Amp1, $8 \cdot 3.2 = |a_2| (3.2)^2$; $a_2 = -2.56$ (V^{-1})

For Amp2, $7 \cdot 3.2 = |a_2| (3.2)^2$; $a_2 = -2.24$ (V^{-1})

$$V_{H2} = \underbrace{-2.56 \cdot \left(0.32 \cdot \frac{-j}{\sqrt{2}}\right)^2 \cdot \frac{1}{\sqrt{2}}}_{path1} + \underbrace{(-2.24)\left(0.32 \frac{1}{\sqrt{2}}\right)^2 \cdot \frac{-j}{\sqrt{2}}}_{path2} = 0.093 - 0.081j$$

$$p_{outH2} = \frac{|0.093 - 0.081j|^2}{100} = 0.15mW = -8.2\text{dBm}.$$

If only Amp1 is used, we know that the linear gain is $20\log_{10}(8)$ = 18.1dB, and thus,

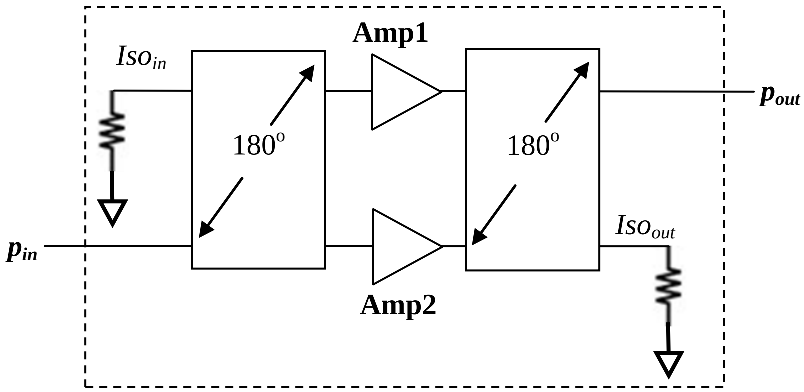
$p_{outH2} = 2p_{out1} - OIP_{H2}$ = 2·18.1dBm – (20dBm+18.1dB) = –1.9dBm = 0.65mW.

We can use the voltage equation as well,

$V_{H2} = -2.56 \cdot 0.32^2 = 0.256$ V. p_{outH2} = 0.65mW = –1.9dBm.

We can see that p_{outH2} in the quad hybrid is about 6dB lower indeed. The difference comes from the slight mismatch in Amp1 and Amp2. This is mainly from the 3dB lower input power to Amp1 and Amp2 when we use the quadrature hybrid.

2. (**180° hybrid amplifier**) For the 180° hybrid amplifier shown below, we will use the same amplifier in Prob. 1. Basically repeat part (b), (c) and (d) of Prob. 1 in the three sub-problems below.



(a) First assume that Amp1 and Amp2 are identical in every aspect as described below.

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{amp1} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{amp2} = \begin{bmatrix} 0.1 & 0 \\ 8 & 0.05 \end{bmatrix}$$

What are the S parameters for the block in the dash-line box? Remember that S parameters are complex numbers. (**6 pts**) **Hint**: when you calculate the voltage through the hybrid, the path without phase shift is $1/\sqrt{2}$, and the one with 180° shift is $-1/\sqrt{2}$.

$$S_{11} = \frac{-1}{\sqrt{2}} \cdot 0.1 \cdot \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot 0.1 \cdot \frac{1}{\sqrt{2}} = 0.1$$

$$S_{21} = \frac{-1}{\sqrt{2}} \cdot 8 \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot 8 \cdot \frac{-1}{\sqrt{2}} = -8$$

$$S_{22} = \frac{1}{\sqrt{2}} \cdot 0.05 \cdot \frac{1}{\sqrt{2}} + \frac{-1}{\sqrt{2}} \cdot 0.05 \cdot \frac{-1}{\sqrt{2}} = 0.05$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{hybrid} = \begin{bmatrix} 0.1 & 0 \\ -8 & 0.05 \end{bmatrix}$$

We can see that the 180° hybrid does not improve on signal reflection.

(b) Now the S parameters for Amp2 is slightly different as shown below. Estimate the S parameters for the block in the dash-line box now? Assume the 180° hybrid is still ideal. (**6 pts**) For p_{in} = 0dBm, estimate the power dissipated in Iso_{in} and Iso_{out}. (**6 pts**)

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{amp2} = \begin{bmatrix} 0.1 & 0 \\ 7 & 0.1 \end{bmatrix}$$

$$S_{11} = \frac{-1}{\sqrt{2}} \cdot 0.1 \cdot \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot 0.1 \cdot \frac{1}{\sqrt{2}} = 0.1$$

$$S_{21} = \frac{-1}{\sqrt{2}} \cdot 8 \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot 7 \cdot \frac{-1}{\sqrt{2}} = -7.5$$

$$S_{22} = \frac{1}{\sqrt{2}} \cdot 0.05 \cdot \frac{1}{\sqrt{2}} + \frac{-1}{\sqrt{2}} \cdot 0.05 \cdot \frac{-1}{\sqrt{2}} = 0.05$$

$$S_{22} = \underbrace{\frac{1}{\sqrt{2}} \cdot 0.05 \cdot \frac{1}{\sqrt{2}}}_{path1} + \underbrace{\frac{-1}{\sqrt{2}} \cdot 0.1 \cdot \frac{-1}{\sqrt{2}}}_{path2} = 0.075$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{hybrid} = \begin{bmatrix} 0.1 & 0 \\ -7.5 & 0.075 \end{bmatrix}$$

V_+ at p_{in} will be 0.32V for 0dBm p_{in}.

At Iso_{in}, we have the voltage as: $V_{-isoin} = \underbrace{0.32 \frac{-1}{\sqrt{2}} \cdot 0.1 \cdot \frac{1}{\sqrt{2}}}_{path1} + \underbrace{0.32 \frac{1}{\sqrt{2}} \cdot 0.1 \cdot \frac{1}{\sqrt{2}}}_{path2} = 0$

The power dissipation at the resistor is 0. All reflected power was in p_{in} (which is NOT a good thing)!

At Iso_{out}, we have the voltage as: $V_{isoout} = \underbrace{0.32\frac{-1}{\sqrt{2}}\cdot 8\cdot\frac{1}{\sqrt{2}}}_{path1} + \underbrace{0.32\frac{1}{\sqrt{2}}\cdot 7\cdot\frac{1}{\sqrt{2}}}_{path2} = -0.16$

The power dissipation at the resistor is then: $p_{isoout} = \frac{(V_{isoout})^2}{2Z_0} = 0.256mW$ = –5.92dBm.

(c) If both Amp1 and Amp2 have IIP_{H2} = 20dBm (but their S parameters are slightly different), for p_{in} = 0dBm, estimate the 2nd harmonic power at p_{out}. Assume that H2 will make the total output voltage at Amp1 and Amp2 smaller (i.e., a_2 is negative). (**8 pts**)

The S parameters are defined with small signals in the linear system, but we can still calculate the 2nd harmonic voltage in the path separately.

Given IIP_{H2} = 20dBm, we know A_{IIPH2} = 3.2V and can calculate a_2 at IP_{H2} by the following equation:

The individual amplifier nonlinearity is the same as in Prob. 1.

For Amp1, $8\cdot 3.2 = |a_2|(3.2)^2$; $a_2 = -2.56$ (V^{-1})

For Amp2, $7\cdot 3.2 = |a_2|(3.2)^2$; $a_2 = -2.24$ (V^{-1})

$$V_{H2} = \underbrace{-2.56\cdot\left(0.32\cdot\frac{-1}{\sqrt{2}}\right)^2\cdot\frac{1}{\sqrt{2}}}_{path1} + \underbrace{(-2.24)\left(0.32\frac{1}{\sqrt{2}}\right)^2\cdot\frac{-1}{\sqrt{2}}}_{path2} = -0.0116$$

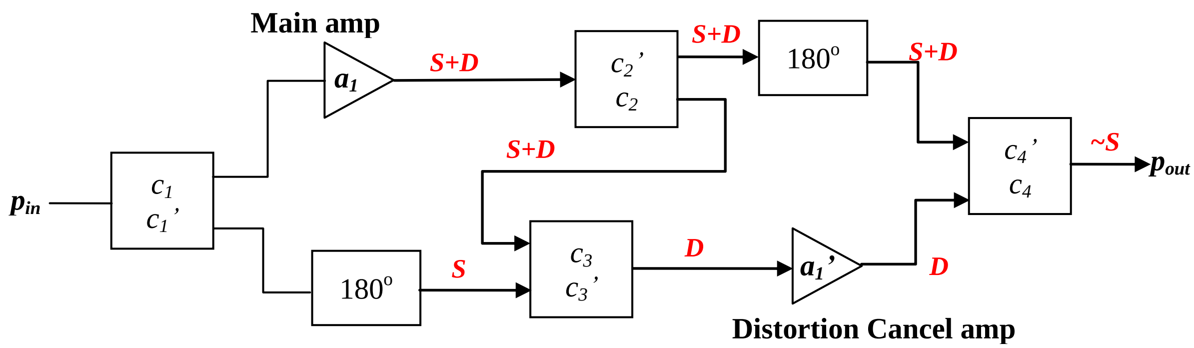
$$p_{outH2} = \frac{|0.0116|^2}{100} = 1.34\mu W = -28.7\text{dBm}.$$

Notice that this is a lot lower, and only result from the Amp1 and Amp2 mismatch. If Amp1 and Amp2 match well, p_{outH2} will be 0!!! This is the unique advantage of 180° hybrid over quadrature hybrid.

(d) Assume that we are implementing amplifiers in monolithic microwave integrated circuits (MMIC) where device matching can be done almost perfectly and good passive elements can be constructed. Answer the following design needs with "quad", "180°" or "both" for the scenarios given. (**6 pts**)

- Need to improve the overall linearity. (Both)
- Need to have minimal IM2 and H2. (180°)
- Need to relax the input impedance match without the penalty of signal reflection (quad)

3. (**Feedthrough distortion cancellation**) For the feedthrough distortion compensation, assume the main amplifier a_1 has a gain of 20dB and $I_{1dBcomp}$ = 20dBm. All couplers have $c_i + c_i' = 1$ with no added noise or nonlinearity (passive lossless). We will assume $c_1 = c_2' = c_4' = 0.9$.



(a) What is the final gain of the amplifier in dB with respect to p_{in}? Assume only IM3 is important for distortion, what is IIP_{IM3} for a_1 in dBm? (**6 pts**)

0.9 = –0.45dB.
0.1 = –10.0dB

The main path gain is: $c_1a_1c_2'c_4'$ = 20dB – 0.45dB×3 = 18.65dB. The other path does not contain signal power (only distortion is left before a_1'.)

$IIP_{IM3} = I_{1dBcomp}$ + 9.64dB = 29.64dBm for a_1.

(b) What are the required values for c_3 ,c_3', and a_1' to cancel the distortion? (**6 pts**)

$c_1a_1c_2c_3 = c_1'c_3'$. $0.9\times100\times0.1c_3 = 0.1\times c_3'$. Therefore $c_3'/c_3 = 90$. We have c_3 = *0.011* or *–19.6dB*, and c_3' = *0.989* or *–0.048dB*.

$c_2'c_4' = c_2c_3a_1'c_4$. $0.9\times0.9 = 0.1\times0.011\times a_1'\times0.1$. Thus, a_1' = *7,364 or 38.7dB*. The large gain usually requires two amplifiers in cascade.

(c) As the coupler design is to minimize the degradation of the signal gain, you may have obtained a VERY large a_1' required to cancel the distortion. Let's see if that is achievable. Assume you have p_{in} at 10dBm (which is lower than $I_{1dBcomp}$ so you know distortion is still small), and the major distortion is from IM3. Estimate the input power level at a_1 and a_1'. (**6 pts**) You should observe that because the input at a_1' is so small, the large gain will not saturate or cause severe distortion.

Input at a_1 is 9.55dBm.

To estimate distortion, we have p_{out1} at 29.55dBm. IIP_{IM3} = 29.64dBm and OIP_{IM3} = 49.64dBm. The distortion power out of a_1 is about 3×29.55 – 2×49.64 = –10.63dBm.

Input at a_1' is about –10.63dBm – 10dB (by c_2) – 19.6dB (by c_3) = –40.23dBm! This means the large gain is probably achievable with two cascading gain stages and the output of a_1' will be around –1.53dBm and still not saturate.

(d) Assume that the compensation a_1' has a very low $I_{1dBcomp}$ = –20dBm due to the large gain (as an amplifier with large gain can easily get saturated, and hence a very low $I_{1dBcomp}$). Estimate the reduction (in dB) for the signal distortion power. (**6 pts**)

$IIP_{IM3,a1'}$ = –10.36dBm and $OIP_{IM3,a1'}$ = 28.34dBm.

As the distortion of a_1 is mostly cancelled, the IM3 distortion from a_1' is now the main distortion component at the final output, which can be estimated at:

3×(–10.63) – 2×28.34 – 10 = –98.57dBm.

The improvement we have achieved for distortion reduction is: –10.63dBm – (–98.57dBm) = 87.9dB!!! A total success!!!