ECE 4880 RF Systems Fall 2016
Homework 7 Solution

Reading before homework:

- Lecture summary on Nonlinearity and Interplay Between Noise and Nonlinearity
- Egan's book, Chaps. 4 and 5.

1. (**Single Side Band Nonlinear and Noise Interplay**) A single-side band (SSB) signal can be approximated by the positive and negative frequency domain as below after the mixer and filter. This signal is fed into a power amplifier where only IM2 nonlinearity is important. Assume $a_1 > a_2 v_{in}$, give the shape and rough size estimation of the output voltage with noise and nonlinearity interplay. Hint, for the positive frequency, use the interaction between any two frequencies between $(f_c - BW, f_c)$. (**10 pts**)

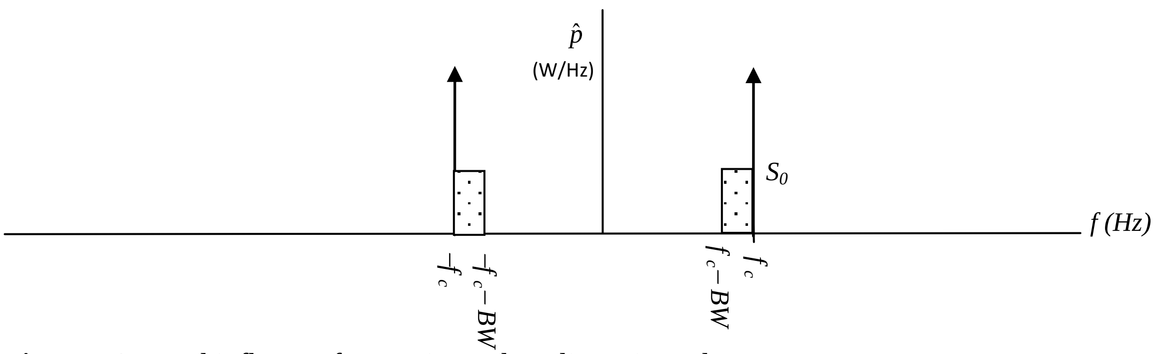

Fig. P.1**.** Spectral influence from noise and nonlinear interplay.

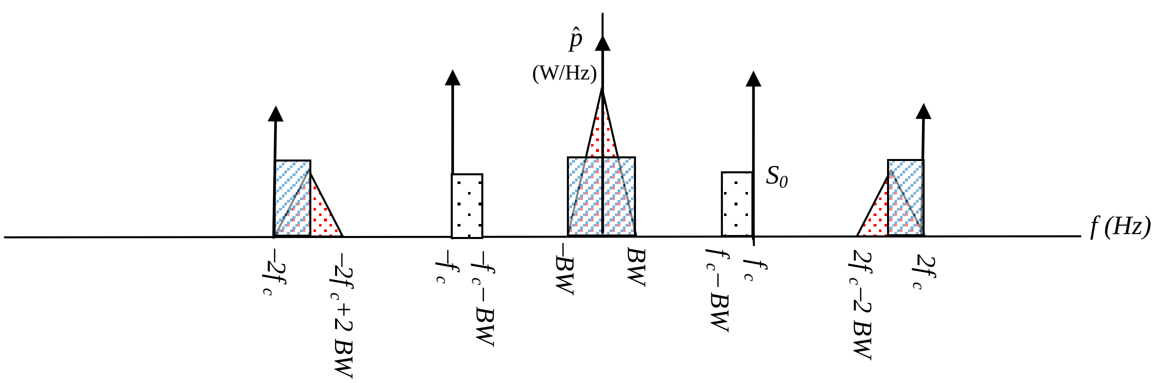

Fig. Sol.1**.** Black: from linear term. Red: IM2 of the Gaussian block only. Blue: convolution of the carrier and the Gaussian block.

2. (**1-dB compression**) For a power amplifier with 20dB gain, IIP_{IM3} = 30 dBm and IIP_{H2} = 20dBm (same PA from Homework 6) for the band 800MHz – 1.2GHz,
   (a) Assume the VTC is only valid for the given bandwidth and outside the band the gain is much smaller, which or both 2nd and 3rd-order nonlinearity is important for obtaining 1-dB compression point? (**5 pts**)

   Within a small bandwidth, the only important nonlinearity is 3rd order. Two 3rd-order terms are important:

- IM3at $(2f_a - f_b)$ and $(2f_b - f_a)$
- Signal distortion at f_a and f_b.

Therefore, if VTC only represents the bandwidth of A and B signals, 1-dB compression will only be affected by IM3 and signal distortion. H2, H3, and IM2 are all outside the usual bandwidth.

(b) Give $I_{1dBcomp}$ and $O_{1dBcomp}$. (**10 pts**)

With only IM3, $I_{1dBcomp} = IIP_{IM3}$ – 9.64dB = 30dBm – 9.64dB = 20.36dBm.

$O_{1dBcomp} = OIP_{IM3}$ – 10.64dB = 30dBm + 20dB – 10.64dB = 39.36dBm.

(c) Calculate the magnitudes of Taylor coefficients a_1, a_2 and a_3 for the VTC. Use the standard impedance of 50Ω. The Taylor expansion of v_{out} is expressed in volts. (**15 pts**)

Notice that a_1, a_2 and a_3 have different units for the Taylor expansion of v_{out} expressed in volts. The Taylor coefficient a_1 is unitless, a_2 is V^{-1} and a_3 is V^{-2}.

a_1 is the linear gain, and hence $a_1 = 10^{20/20}$ (voltage dB) = 10.

$A_{IIPIM2} = \left|\frac{a_1}{a_2}\right|$, where A_{IIPIM2} is the voltage amplitude of v_{in} at the intercept point IP_{IM2}.

$IIP_{IM2} = IIP_{H2}$ – 6dB = 14dBm = 25mW.

$$A_{IIPIM2} = \sqrt{2 \cdot Z_0 \cdot IIP_{IM2}} = 1.58V.$$

$$|a_2| = \frac{|a_1|}{A_{IIPIM2}} = 6.33 \ (V^{-1}).$$

$A^2_{IIPIM3} = \frac{4}{3}\left|\frac{a_1}{a_3}\right|$, where A_{IIPIM3} is the voltage amplitude of v_{in} at the intercept point IP_{IM3}.

IIP_{IM3} = 30 dBm = 1W.

$$A_{IIPIM3} = \sqrt{2 \cdot Z_0 \cdot IIP_{IM3}} = 10V.$$

$$|a_3| = \frac{4}{3}\frac{|a_1|}{A^2_{IIPIM3}} = 0.133 \ (V^{-2}).$$

3. (**Transfer function to IP**) For a power differential amplifier whose VTC can be described by: $v_{out} = 20 \times v_{in} + 0.1 \times v_{in}^3$ with v_{in} and v_{out} in V and I/O impedance Z_0 of 50Ω in all frequency of interest,

(a) What is the linear voltage and power gain in dB? (**5 pts**)

The linear gain is $20\times\log_{10}(20)$ = 26 dB. This is for both voltage and power.

(b) What is the unit of 0.1 in the VTC description? If f_a = 1GHz and f_b = 1.1GHz, what are the IM3 frequencies? (**5 pts**)

The cubic coefficient must have a unit of V^{-2}.

The IM3 frequencies are $2f_a - f_b$ = 0.9 GHz and $2f_b - f_a$ = 1.2GHz.

(c) Find IIP_{IM3}, OIP_{IM3}, IIP_{H3}, OIP_{H3} and $I_{1dBcomp}$. (**10 pts**)

$A_{IIPIM3}^2 = \frac{4}{3}\left|\frac{a_1}{a_3}\right|$, where A_{IIPIM3} is the voltage amplitude of v_{in} at the intercept point IP_{IM3}.

$$A_{IIPIM3} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} = 16.3 \text{ (V).}$$

$$IIP_{IM3} = \frac{A_{IIPIM3}^2}{2Z_0} = 2.67W = 34.2 \text{ dBm.}$$

$OIP_{IM3} = IIP_{IM3}$ + 26dB = 60.2dBm.

$IIP_{H3} = IIP_{IM3}$ + 4.77dB = 39.0dBm.

$OIP_{H3} = IIP_{H3}$ + 26dB = 65.0dBm.

$I_{1dBcomp} = IIP_{IM3}$ – 9.64dB = 24.6dBm.

4. (**Signal desensitization and jamming**) An LNA as the first stage in the receiver has 15dB gain and noise figure of 3dB. The LNA designer did not anticipate large input signal, so the nonlinearity was not carefully designed with a low IIP_{IM3} = 10 dBm. We will still use $Z_0 = 50\Omega$ in all frequency of interest,

(a) In the two-tone signal, if a smaller signal $A\cos(\varphi_a)$ is received at –60dBm, will the IM3 term be an important distortion consideration when no other signal is present? (**5 pts**)

OIP_{IM3} = 10dBm + 15dB = 25dBm.

We know p_{outIM3} = –60dBm – {2×[25 – (–60)]dB) = –230dBm, which is not important here. Therefore, for the signal itself, the nonlinearity in LNA is NOT important at all.

(b) What will be the large signal voltage $B\cos(\varphi_b)$ that will desensitize this small signal? The large signal B may be from an in-band jammer, although $\varphi_a \neq \varphi_b$ and B does not cause a direct interference as it could be rejected by the mixer without the LNA nonlinearity. The desensitized point is defined when the interfering voltage generates a distortion magnitude at the signal frequency that is half of the originally intended signal voltage. Does the desensitized point depend on p_{in}? (**10 pts**)

For this LNA, we have $a_1 = 10^{20/15} = 5.6$.

$$A_{IIPIM3} = \sqrt{2 \cdot Z_0 \cdot IIP_{IM3}} = 1V.$$

$$|a_3| = \frac{4}{3}\frac{|a_1|}{A_{IIPIM3}^2} = 7.47$$ (V^{-2}). (This is a rather large a_3!)

The distortion is half of the intended output signal at the desensitization point:

$$Distortion \cong \frac{3a_3}{2}AB^2 = \frac{a_1 A}{2}.$$

$B = \sqrt{\frac{1}{3}\left|\frac{a_1}{a_3}\right|} = \frac{1}{2}A_{IIPM3}$ = 0.5V. The jamming signal at the receiver just needs to be: 2.5mW or 4.0dBm (yes, 6dB lower than IIP_{IM3}). This receiver can be easily jammed!

We can also see from the above equation, A (which gives p_{in}) will not affect the desensitized point.

You can see that for this LNA, as long as there is a jamming $B\cos\varphi_b$ of 0.5V magnitude at the receiver within the band, the LNA 3^{rd}-order nonlinearity will cause all signals within the band to be desensitized.

(c) Recalculate the jamming desensitization voltage B if LNA has a better design with IIP_{IM3} = 40dBm. (**5 pts**)

$$A_{IIPIM3} = \sqrt{2 \cdot Z_0 \cdot IIP_{IM3}} = 32V.$$

$B = \sqrt{\frac{1}{3}\left|\frac{a_1}{a_3}\right|} = \frac{1}{2}A_{IIPM3}$ = 16V. The jamming signal at the receiver now needs to be: 2.56W or 34dBm. Therefore, this receiver is now much more robust.

We can see that IIP_{IM3} is still an important parameter for LNA!!!

5. (**Cascade with gain, noise figure and OIP3**) Module 1 is an amplifier with g_1 = 20dB; F_1 = 10dB; $OIP3_1$ = 38 dBm. Module 2 is a filter with pass band g_2 = –0.5dB; F_2 = 3dB (more noise than that of attenuator); $OIP3_2$ = 30 dBm as ferromagnetic inductor is used. Assume all IM terms add up coherently.
(a) Calculate the cascade gain, noise figure and OIP3 for Module 1 – 2 and Module 2 – 1. (**10 pts**)

Cascade module 1 – 2 has g_{cas12} = 19.5dB.

The cascade 1-2 noise factor is $f_{cas12} = 10^1 + (10^{0.3} - 1)/100 = 10.01$ or the noise figure F_{cas12} = 10dB;

As all IM3 terms add coherently, we have $\frac{1}{OIP_{IM3,cas}} = \frac{1}{g_2 \cdot OIP_{IM3,1}} + \frac{1}{OIP_{IM3,2}}$.

Therefore, $OIP3_{cas12}$ = 667mW = 28dBm.

Cascade module 2 – 1 has g_{cas21} = 19.5dB.

The cascade 2-1 noise factor is $f_{cas21} = 10^{0.3} + (10^1 - 1)/10^{-0.5} = 30.1$ or the noise figure F_{cas21} = 14.8dB;

As all IM3 terms add coherently, we have $OIP3_{cas21}$ = 5.94W = 37.7dBm.

(b) If we hope for the least nonlinearity, which cascade should be chosen? If we hope for the least noise, which cascade should be chosen? (**10 pts**)

For lower noise, we will have the amplifier before filter (cascade 1-2), which uses the gain in amplifier to mitigate the noise in filter.

For better linearity, we will have the filter before the amplifier (cascade 2-1), where the filter will NOT see large v_{in} for its nonlinearity.