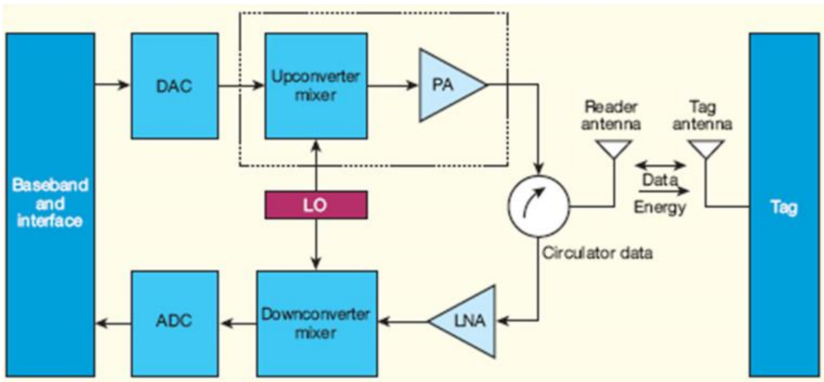
ECE 4880 RF Systems Fall 2016
Homework 5 Solution

Reading before homework:

- Lecture summary on Gain Modules and Noises
- Egan's book, Chaps 2 and 3.

1. (**Ideal design of an RFID reader**) A typical RFID reader transceiver block diagram is shown below. The reader listens to the echo modulated by the tag. You can consider that the reader is composed of two radio links: the reader-to-tag downlink and the tag-to-reader uplink. This is a device to be operated around 900MHz with a line-of-sight reading range of 10m. FCC regulation dictates no more than 4W emission in this ISM band.



(a) First assume that both reader and tag antennas are 0 dBi for the best angle coverage. At the 10m distance, how much power in dBm is impinging on the tag? How much power in dBm is impinging on the reader receiver? What is the link budget (power ratio from TX to RX)? (**5 pts**) If the tag has a RF-to-DC converter of 20% efficiency, how much power in μW can the tag recover for use to execute the air protocol and backscattering modulation? (**5 pts**)

The wavelength at 900MHz is: 0.33m. The free space loss of 10m distance for the down link is:

$$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi r}\right)^2 \Psi_T \Psi_R = \left(\frac{0.33}{4\pi \times 10}\right)^2 = 0.69 \times 10^{-5} = -51 \text{ dB.}$$

36 dBm – 51 dBm = –15 dBm. This is the power impinging on the tag.
–15 dBm – 51 dBm = –66 dBm. This is the power impinging on the reader receiver. Both are quite large in the typical RFID setup.

–15 dBm = 32 μW. 20% of this power will be 6.3 μW. This is relatively large for a passive tag to operate. Present commercial RFID tags have a sensitivity close to –20 dBm.

(b) At the 10m distance, if the circulator provides 60dB rejection and the transmitter can be considered as the main interference for the receiver, what is the signal-to-interference (SIR) ratio in dB? (**5 pts**)

36 dBm – 60dB = –24 dBm. The receiver signal is at –66 dBm, and hence SIR = –42dB. This is an SIR that can be reasonably solved with code modulation at the backscattered signal.

(c) At the 10m distance, what is the gain in dB needed in the receiving chain so the input to ADC has a minimal voltage amplitude of 10mV? Assume the tag antenna has an impedance of 50Ω. (**5 pts**)

The receiver has –66dBm lowest power at the antenna,

$-66\text{dBm} = 0.25\times10^{-6}$ mW = $V^2/(2\times50)$. $V = 0.16$mV.

The gain (both for voltage and power) in the receiver chain just needs

10mV/0.15mV = 62.5 V/V = 36 dB.

This is rather small gain in the receiver, as the impinging power is large. Notice that because ADC is at the baseband with no need for impedance matching to avoid reflection, calculation in voltage is a better way.

(d) At the 10cm distance, repeat (a) – (c). Assuming the system is still in the far-field range as 10 cm > $\lambda/2\pi$ (**5 pts**). If this is the minimal distance for the tag to be read, what is the required dynamic range in dB for the reader receiver? (**5 pts**)

$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi r}\right)^2 \Psi_T \Psi_R = \left(\frac{0.33}{4\pi\times0.1}\right)^2 = 0.069$. The link budget for the downlink = $\frac{P_T}{P_R}$ = 12 dB. The total link budget now is 24 dB.

36 dBm – 12 dB = 24 dBm. This is the power impinging on the tag. Very strong and attenuator may be needed.

24 dBm – 12 dB = 12 dBm. This is the power impinging on the reader receiver. Very large still.

24 dBm = 251 mW. 20% of this power will be 59mW. This is enough power to run a decent microcontroller! Many UHF tags can be run in full microcontroller mode when they are close to the reader with 5cm – 10cm distance.

The circulator leaking remains the same –24 dBm. The receiver signal is at 12 dBm, and hence SIR = 36dB, which is very large.

The dynamic range of the receiver is 12 dBm – (–54 dBm) = 66 dB.

In most designs, the PA will be tuned down when the receiver has found strong backscattering signal. This will not only relieve the dynamic range requirement in the receiver chain, but more importantly save power (as many RFID readers are mobile and use battery).

(e) What is the voltage amplitude out of the 50Ω reader transmitting antenna with 36dBm? If the DAC in the transmitter outputs signals with 0.5V voltage amplitude and the PA has a gain of 16 dB, what is the gain needed in the transmitter mixer? (**5 pts**)

The voltage amplitude at the antenna is: 20V!

The power before PA is at: 20dBm. As PA often has matched impedance to avoid reflection (after the mixer, it is at the carrier frequency), this still has 3.3V amplitude. Therefore, the mixer needs to provide 3.3/0.5 = 6.6 or in voltage gain this is 16.4 dB. Although this is not large, 3.3V often has serious nonlinearity already for most active mixers.

(f) If you are given a reader antenna of 8dBi while the tag antenna still at 0 dBi, assume the range is limited by the power the tag can collect (called tag sensitivity limited or downlink limited), what is the operation range now? At the new maximal range, what is the power in dBm at the reader receiver? (**5 pts**)

The downlink scales with r^{-2}, so with 8dB more in the link budget, the range will be extended for $10^{8/20} = 2.5$ times, which is at 25 m.

The downlink will now be 54dB loss, and so is the uplink. However, as the antenna gain is used to increase the range, both the tag and the receiver have the same power impinging at –15 dBm and –66 dBm still.

(g) Through some great designs by engineers, the tag sensitivity is improved to –20dBm, with the antenna set in (f), what is the operation range now? At the new maximal range, what is the power in dBm at the receiver? For the same minimal distance at 10cm, what is the present required dynamic range in dB for the reader receiver? (**5 pts**)

$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi r}\right)^2 \Psi_T \Psi_R = \left(\frac{0.33}{4\pi \times r}\right)^2 = -56dB - 8dB = -64dB = 7.9 \times 10^{-7}$. The range r is now at 42m.

Check: we have 5dB more in the downlink budget, and the distance should be $10^{5/20}$ more than 25m from Part (f).

The power impinging at the tag is: –20 dBm.
The power impinging at the reader receiver is: –76 dBm.

At 10cm with the 8dBi antenna still with 12dB path loss,

The power at the tag: 36 dBm +8 dB – 12 dB = 32 dBm. This is the power impinging on the tag. Very strong and attenuator may be needed.

32 dBm – 12 dB + 8 dBi = 28 dBm. This is the power impinging on the reader receiver.

The receiver dynamic range now will need to be: 28 dBm – (–76dBm) = 104 dB. This is very large, and detailed variable gain amplifier will be needed.

(h) FCC allows operations from 902MHz to 928MHz for this application. If each channel has a 500kHz bandwidth, how many channel can be used? If you are given a 5MHz quartz oscillator for the frequency reference, give an example of the needed N and M in your rational frequency synthesizer to cover the entire band with a changing LO frequency. (**5 pts**)

There would be 52 channels to use. As 0.5MHz resolution is needed, one of the easy choice is to set $N = 10$, and $M = 9020, 9025, \ldots 9280$. However, very large N and M will cause even more delay in switching frequency. Therefore, we can explore the rational number to be: $N = 2$, $M = 1804, 1805, \ldots 1856$. This is still too large, and multiple stages of frequency conversion may be necessary. Notice that this is a rational frequency synthesizer, and N and M have to be integers.

(i) The reader uses a "noncoherent amplitude modulation" where the bit rate can just be half of the bandwidth. To leave sufficient channel isolation, you will use a data bandwidth of 200kHz with 150kHz side skirt for better channel isolation. If each data packet has 256 bits, what is the duration of each packet delivered? What is the number of cycles the carrier has gone through for each bit of transmission? (**5 pts**)

There would be 10^5 bits per second, so the data packet of 256 bits will take 2.56ms. For each bit, there are in total 900MHz/100KHz = 9000 carrier cycles. We can see that there can be a lot of averages to be taken for accurate magnitude and phase modulation measurements.

(j) With the extended tag sensitivity in (g), if there is a wall with additional 15dB attenuation, , what is the operating range now assuming still the same tag sensitivity limitation at –20dBm? What is the power impinging on the tag and the reader receiver now? (**5 pts**).

The downlink has 15dB more loss, which will make the range to be $10^{15/20}$ = 5.6 times smaller at 62.8m/5.6 = 11.2m. This is still an impressive range going through walls.

At the maximal range, the power impinging on the tag and reader receiver will remain the same at –20 dBm and –76dBm, respectively.

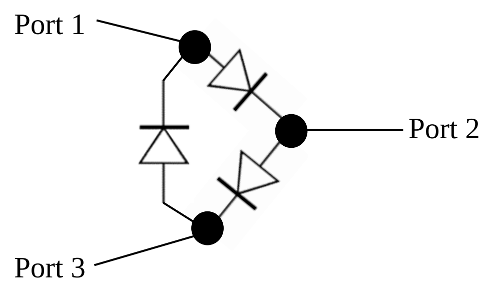
(k) The DAC output at the transmitter has many additional frequency components that are up-converted by the mixer. As the PA impedance is around 50 Ω only for the designed bandwidth between 850MHz to 1050MHz, insert a filter after the DAC or after the mixer in the transmitter signal chain to alleviate this problem. Denote the type and give a rough pass/stop band design to the filter. (**5 pts**)

If we add the filter after the DAC, we would use a low-pass filter with a passband below 300kHz. The high frequency component will not enter the mixer.

If we add the filter after the mixer, we would use a band-pass filter with a passband between 850MHz to 1050MHz. Notice that we give some room for the passband so that the insertion loss can be small.

We will hope for a minimal passband loss (say –1dB). As the circulator provides –60dB isolation, if we have additional –40 dB isolation from the filter stop band, we will only have at most 36 dBm – 40dB – 60dB = –64dBm leakage to the receiver in the undesirable band, assuming the PA is broadband.

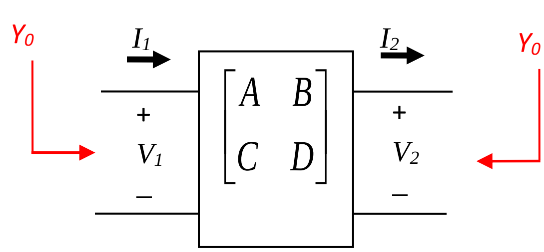
2. (**Circulator considerations**) A student suggests a diode implementation of the circulator for broadband applications as shown above. Choose all correct answers below (can be more than one) (**5 pts**)



(a) The circulator will have a clockwise path only, as the counterclockwise path is always blocked by a reverse leakage diode.
(b) The reverse biased diode from Port 1 to Port 3 may not work well in high frequency due to capacitive coupling.
(c) The circulator will not work well as there is a DC leaky path through Port 1 to Port 3.
(d) Although the circulator will have serious nonlinearity, it will be acceptable in duplex radio transceivers as the signals will always be very small at Port 1 and Port 3.
(e) The impedance at Port 1 and Port 3 will be a strong function of the voltage magnitude, which makes impedance matching difficult.

Answer: (b), (c), (e). Not only there is reverse-biased capacitive leakage, the two forward passes from 1-2 and 2-3 will combine to become a path for 1-3. The forward biased diode has a nonlinear IV curve, and hence nonconstant resistance with respect to voltage, which makes the impedance match very hard.

3. (**ABCD matrix and S parameters**) When we change the view from the traveling waves to static voltages and currents, we have defined the relations of S parameters and the Y matrix as shown below.



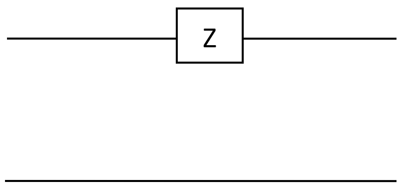
$$I_1 = Y_0(v_{o1} - v_{i1})$$
$$V_1 = v_{o1} + v_{i1}$$
$$I_2 = Y_0(v_{o2} - v_{i2})$$
$$V_2 = v_{o2} + v_{i2}$$

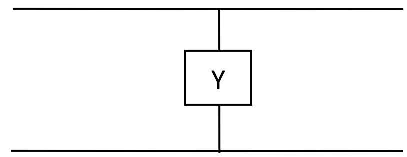
We will still assume that the ports 1 and 2 have admittance of Y_0, and we define the ABCD matrix as:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Please be careful that I_2 is defined in the opposite direction with the Y and Z matrix to enable cascading.

(a) The ABCD matrix is convenient in module cascade just like the T matrix (but T is in the traveling wave view). Write down the ABCD matrix for the two simple cases below. (**5 pts**)





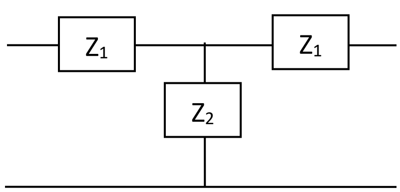
By definition, for the Z in series, we have $I_1 = I_2$ and $ZI_1 = (V_1 - V_2)$ or $V_1 = V_2 + ZI_1$. The ABCD matrix is then

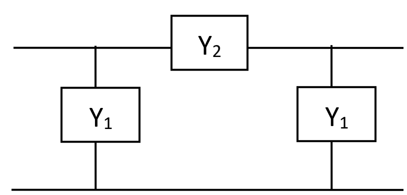
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}.$$

For Y in parallel, we have $V_1 = V_2$ and $I_1 = YV_2 + I_2$. The ABCD matrix is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

(b) Use the matrix multiplication to find the ABCD matrix for the following two reciprocal T and π networks (**5 pts**)





For the T network, we have

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{Z_1}{Z_2} & 2Z_1 + \dfrac{Z_1^2}{Z_2} \\ \dfrac{1}{Z_2} & 1 + \dfrac{Z_1}{Z_2} \end{bmatrix}$$

For the π network, we have

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{Y_2} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} = \begin{bmatrix} 1+\frac{Y_1}{Y_2} & \frac{1}{Y_2} \\ 2Y_1+\frac{Y_1^2}{Y_2} & 1+\frac{Y_1}{Y_2} \end{bmatrix}$$

(c) Express the S matrix with the ABCD parameters. (**5 pts**)

Remember that the definition of the S parameters is:

$$\begin{bmatrix} v_{i1} \\ v_{o2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}\begin{bmatrix} v_{o1} \\ v_{i2} \end{bmatrix}$$

We will use S_{21} as an example for solution here.

$$S_{21} \equiv \left.\frac{v_{out2}}{v_{in1}}\right|_{v_{in2}=0} \equiv \left.\frac{v_{o2}}{v_{o1}}\right|_{v_{i2}=0}$$, when $v_{i2} = 0$, we get:

$$\begin{aligned} I_1 &= Y_0\left(v_{o1} - v_{i1}\right) \\ V_1 &= v_{o1} + v_{i1} \\ I_2 &= Y_0 v_{o2} \\ V_2 &= v_{o2} \end{aligned}$$, we can then have:

$$\begin{aligned} v_{o1} + v_{i1} &= \left(A + BY_0\right)v_{o2} \\ v_{o1} - v_{i1} &= \left(C/Y_0 + D\right)v_{o2} \end{aligned}.$$

Add the two equations together:

$$S_{21} \equiv \left.\frac{v_{o2}}{v_{o1}}\right|_{v_{i2}=0} = \frac{2}{A + BY_0 + C/Y_0 + D}.$$

Following the same procedure, we have

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{A + BY_0 + C/Y_0 + D}\begin{bmatrix} A + BY_0 - C/Y_0 - D & 2\left(AD - BC\right) \\ 2 & -A + BY_0 - C/Y_0 + D \end{bmatrix}$$

(d) Express the Y matrix with ABCD parameters, and then the ABCD matrix with the Y parameters. (**5 pts**)

Following similar procedure as above, we will find:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{B}\begin{bmatrix} D & BC - AD \\ -1 & A \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{Y_{21}}\begin{bmatrix} -Y_{22} & -1 \\ Y_{12}Y_{21} - Y_{11}Y_{22} & -Y_{11} \end{bmatrix}$$

(e) The S and T matrices are used for traveling waves, and Y, Z and ABCD (with different definition of I_2 direction) are used for static voltages and currents. They all have their own advantages for the module properties. For a reciprocal network (i.e., the network has symmetry looking from the two ports), we know $Y_{12} = Y_{21}$. What is the requirement on the S and ABCD parameters? Confirm the ABCD parameter property in the T and π networks in (b). (**5 pts**)

If $Y_{12} = Y_{21}$, we know that $S_{12} = S_{21}$, and $AD - BC = 1$, which is the determinant of the ABCD matrix.

To check if the T and π networks are reciprocal, we can observe:

For the T network, we have

$$\left(1+\frac{Z_1}{Z_2}\right)\left(1+\frac{Z_1}{Z_2}\right)-\left(\frac{1}{Z_2}\right)\left(2Z_1+\frac{Z_1^2}{Z_2}\right)=1$$

For the π network, we have

$$\left(1+\frac{Y_1}{Y_2}\right)\left(1+\frac{Y_1}{Y_2}\right)-\left(\frac{1}{Y_2}\right)\left(2Y_1+\frac{Y_1^2}{Y_2}\right)=1$$

(f) For passive lossless network (i.e., no resistive loss or source), we know that all Y parameters are imaginary, i.e., $\mathrm{Re}(Y_{11}) = \mathrm{Re}(Y_{12}) = \mathrm{Re}(Y_{21}) = \mathrm{Re}(Y_{22}) = 0$. Also, all Z parameters are imaginary too. What is the requirement on the ABCD parameters? Confirm the ABCD parameter property in the simple series and parallel element of (a), when $Y = 1/Z = j\omega C$. (**5 pts**)

A and *D* will be real; *B* and *C* will be imaginary. Or $\mathrm{Im}(A) = \mathrm{Im}(D) = \mathrm{Re}(B) = \mathrm{Re}(C) = 0$.

We can see this is true for:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{1}{j\omega C} \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix}.$$

Notice that these properties are hard to observe directly on the S and ABCD parameters, although the S matrix is easiest in microwave measurements, and the ABCD matrix is the easiest in cascade of modules expressed in static currents and voltages.