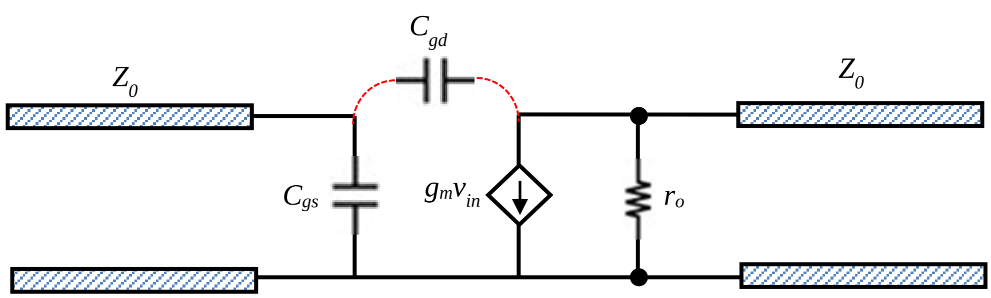
ECE 4880 RF Systems Fall 2016
Homework 4 Solution
(Please **box** your quantitative final answers)

1. (**Antenna dBi**) You are given an antenna below with a 10dBi rating in 2.4GHz with a bandwidth of 0.5GHz. From its size, you know that this is not a multi-antenna array. Ignoring the polarization effects and beam steering, which of the following is true? (**10 pts**)

(a) The antenna can cover all of the space in the front hemisphere.
(b) When you walk a receiver from the left to the right side, you would not observe much change in the received power strength.
(c) The antenna can at most cover a solid angle of 72°.
(d) The antenna will not be able to cover the front-left-down corner at all.
(e) The antenna can be used to enhance your 1.9GHz cell phone for better reception (if you can plug it in).

Answer (c). 10 dBi means the solid angle will be 720°/10 = 72° even when the emission is very efficient. What you see as the picture may be deceiving to determine the angles of the focused beam, as the surface is just the anti-reflective coating, and the real metal antenna structure is underneath. Therefore, we do not know about which corner it may cover by just knowing 10 dBi and the outer shape. The bandwidth given is centered around 2.4GHz, so you can expect efficient T/R only for 2.15GHz – 2.65GHz.

2. (**Matching network for MOSFET**) An inexperienced engineer put a MOSFET directly to implement an RF amplifier. Assume that the MOSFET has been properly DC-biased to be in the saturation region, and within the voltage range of operation can be approximated by the π network shown below. The input and output have transmission line of impedance of Z_0.



(a) Write down the Y matrix for the π network, first ignoring C_{gd}. (**5 pts**)

From the definition of the Y parameters, we can put down by inspection:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} j\omega C_{gs} & 0 \\ g_m & \dfrac{1}{r_o} \end{bmatrix}$$

(b) Write down the S matrix using the Y network in (a), assuming both input and output are connected by the transmission line of Z_0 impedance. (**5 pts**) Observe how gm appears in the four S parameters, and rationalize the expression. (**5 pts**)

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{(1+Z_0Y_{11})(1+Z_0Y_{22})-Z_0^2Y_{12}Y_{21}} \begin{bmatrix} (1-Z_0Y_{11})(1+Z_0Y_{22})+Z_0^2Y_{12}Y_{21} & -2Z_0Y_{12} \\ -2Z_0Y_{21} & (1+Z_0Y_{11})(1-Z_0Y_{22})+Z_0^2Y_{12}Y_{21} \end{bmatrix}$$

$$= \frac{1}{(1+j\omega C_{gs}Z_0)\left(1+\frac{Z_0}{r_o}\right)} \begin{bmatrix} (1-j\omega C_{gs}Z_0)\left(1+j\frac{Z_0}{r_o}\right) & 0 \\ -2Z_0g_m & (1+j\omega C_{gs}Z_0)\left(1-\frac{Z_0}{r_o}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1-j\omega C_{gs}Z_0}{1+j\omega C_{gs}Z_0} & 0 \\ \dfrac{-2Z_0g_m}{(1+j\omega C_{gs}Z_0)\left(1+\frac{Z_0}{r_o}\right)} & \dfrac{1-\frac{Z_0}{r_0}}{1+\frac{Z_0}{r_0}} \end{bmatrix}$$

It is intuitive to observe S_{11} and S_{22}. From the circuit diagram, you can see that S_{11} is entirely decoupled from the right side, while S_{22} is entirely decoupled from the left side. The form of S_{11} and S_{22} is simply how the reflection coefficient is defined.

(c) Let's look at S_{21}, which is the transfer voltage gain. In the quasi-static limit, we know that the transfer voltage gain should be close to $-g_m(r_o \parallel Z_0)$. Under what condition does the above S_{21} approach $-2g_m(r_o \parallel Z_0)$? (**5 pts**) Explain the additional factor of 2 here. (**5 pts**)

Observe from the equation above, we know that:

$$S_{21} = \frac{-2Z_0g_m}{(1+j\omega C_{gs}Z_0)\left(1+j\frac{Z_0}{r_o}\right)}$$

To approach $-2g_m(r_o \parallel Z_0)$, we will need $\omega C_{gs}Z_0 << 1$ and $Z_0 << r_o$.

The additional factor of 2 comes from the nearly open reflection at the capacitive input, which makes the voltage across C_{gs} appear two times bigger by the superposition of V_+ and V_-.

(d) Obtain the numerical values of the S matrix (complex numbers) with C_{gs} = 100fF; g_m = 50mS; r_o = 2kΩ; Z_0 = 50Ω. First at 10MHz and then at 10GHz. Just keep two significant digits to simplify your calculation, i.e., 1.0 + 0.04 ≅ 1.0. (**10 pts**)

At 10MHz,

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} (1 - j\omega C_{gs} Z_0)\left(1 + j\frac{Z_0}{r_o}\right) & 0 \\ -2Z_0 g_m & (1 + j\omega C_{gs} Z_0)\left(1 - j\frac{Z_0}{r_o}\right) \end{bmatrix} \cong \begin{bmatrix} 1 & 0 \\ -5 & 0.95 \end{bmatrix}$$

You can see that most of the energy injected will be reflected as an open circuit at either side (C_{gs} or r_o), which makes sense from the impedance point of view. The transistor has a large self gain ($g_m r_o$ = 100) so that some of the transconductance gain is still preserved.

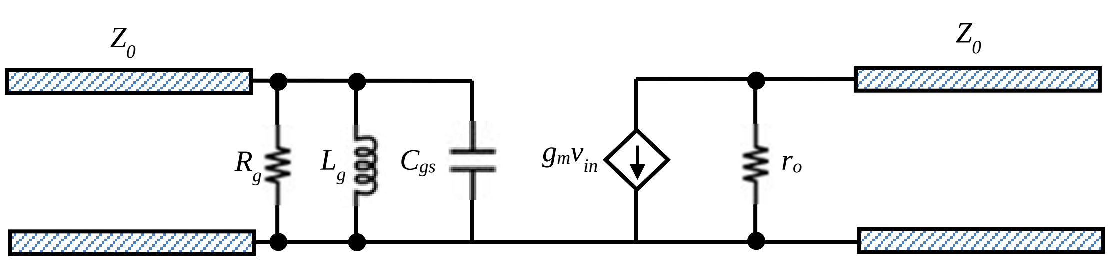
At 10GHz,

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} (1 - j\omega C_{gs} Z_0)\left(1 + j\frac{Z_0}{r_o}\right) & 0 \\ -2Z_0 g_m & (1 + j\omega C_{gs} Z_0)\left(1 - j\frac{Z_0}{r_o}\right) \end{bmatrix} \cong \begin{bmatrix} 0.82 - j0.57 & 0 \\ -4.4 + j1.4 & 0.95 \end{bmatrix}$$

We can see that the first main deviation is caused by the deviation of unmatched ωC_{gs} at input. The reflection at the source becomes slightly smaller and the drain side is changed. Transconductance is also smaller with an imaginary part. When the real part of the transconductance magnitude is smaller than 1, we call it the cutoff frequency.

In RF power amplifier design, impedance matching is critical. Although for relatively lower power output like 0dBm in Bluetooth, large transconductance gain can still give some amplification even in high frequency. This is important for very small packages where inductance becomes too expensive to implement.

(e) Suggest an input impedance matching network just outside the π network at 10GHz so that the S_{11} is minimized. (**5 pts**) Give numerical values of your matching network circuit element parameters (**5 pts**)



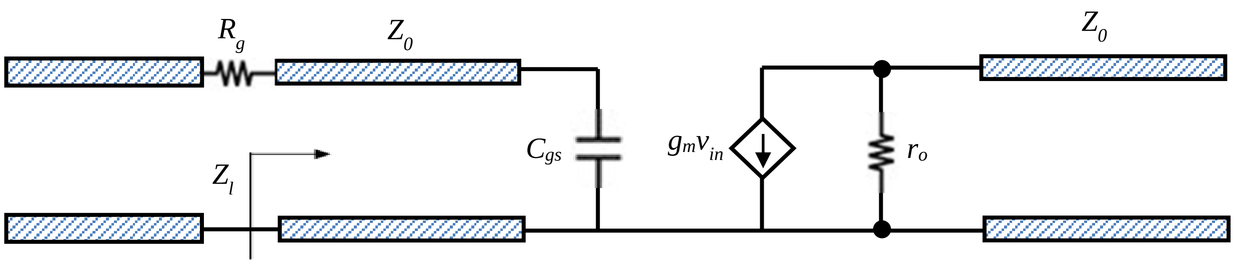
We will add the RL network above, where the complex input impedance is now:

$$Z_{in} = \frac{1}{\frac{1}{R_g} + \frac{1}{j\omega L_g} + j\omega C_{gs}}$$

To match Z_{in} with Z_0 at 10 GHz, by inspection we have $R_g = 50\Omega$ and

$$L_g = \frac{1}{\omega^2 C_{gs}} = 2.5\text{nH}.$$

(f) Use the input impedance matching network below with an input transmission line length to cancel the reactance and a series element of lumped $R_g = Z_0 = 50\Omega$. What is the input transmission line length at 10GHz? (**5 pts**)



The matching network is shown above. The reactance of $C_{gs} = -j159\Omega$. $C_{gs}/Z_0 = -j3.2$. On the Smith Chart to transfer to $Z_l = 0$, we have $l = (0.50 - 0.29)\lambda = 0.63\text{cm}$.

(g) Express T_{11} explicitly by the circuit elements in the π network in Part (c). Give your answer in the low frequency and high output resistance limit (i.e., $Z_0 << r_o$.) Is T_{11} positive or negative? (**5 pts**)

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{S_{21}} \begin{bmatrix} 1 & -S_{22} \\ S_{11} & -S_{11}S_{22} \end{bmatrix}.$$

$$T_{11} = \frac{1}{S_{21}} = -\frac{1}{2Z_0 g_m}.$$

Notice that in this simple model, T_{11} is always real and negative regardless of the input and output matching network.

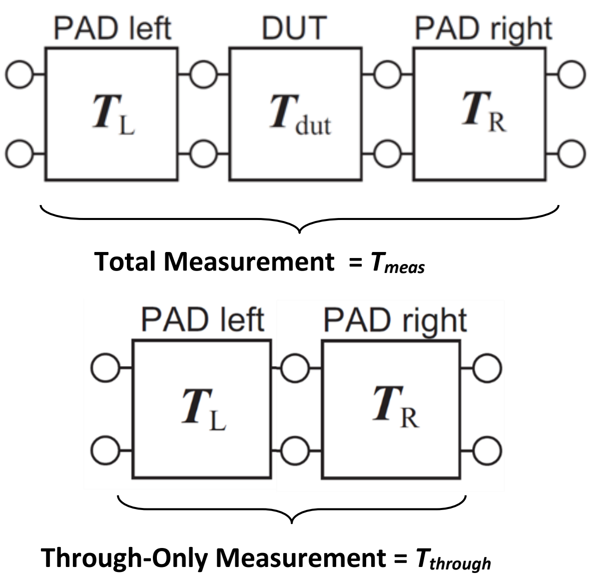
(h) Now we will add in the effect of C_{gd}. Assume C_{gd} is only big enough to affect Y_{12}, which is originally zero. Find Y_{12} by the definition of $Y_{12} = i_1/v_2$ when $v_1 = 0$ (**5 pts**). Find the approximate expression for S_{21}. What is the modification of S_{21} at high frequency? (**5 pts**)

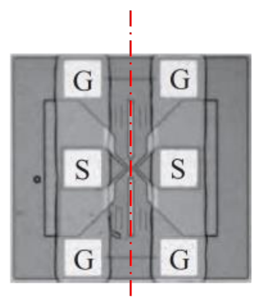
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} j\omega C_{gs} & 0 \\ g_m & \dfrac{1}{r_o} \end{bmatrix} + \begin{bmatrix} 0 & -j\omega C_{gd} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} j\omega C_{gs} & -j\omega C_{gd} \\ g_m & \dfrac{1}{r_o} \end{bmatrix}$$

$$S_{21} = \frac{-2Z_0Y_{21}}{(1+Z_0Y_{11})(1+Z_0Y_{22}) - Z_0^2Y_{12}Y_{21}} = \frac{-2Z_0 g_m}{(1+j\omega Z_0C_{gs})\left(1+\dfrac{Z_0}{r_o}\right) + j\omega Z_0^2 C_{gd} g_m}$$

We can see that C_{gd} will decrease the effective gain when it becomes important. In very high frequency for a large g_mZ_0, S_{21} is then entirely dominated by C_{gd} as the term of C_{gd} is magnified by g_mZ_0 (Miller effect) in comparison with the prefactor of C_{gs}. g_mZ_0 is basically the gain at the low frequency. This is true even if we add the matching network.

3. (**TL de-embedded network**) In the through-only de-embedding scheme, we can observe the mirror symmetry of the left and right L and R network of the GSG pads for the through network on a chip or PCB board.





Mirror symmetry in Through-Only Measurement

(a) If the left pad can be expressed as: $[S]_L = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, write down *[S]$_R$* from the mirror symmetry (**5 pts**)

$[S]_R$ is the mirror image of $[S]_L$, and its S matrix will just switch the terminal indices of 1 and 2.

When $[S]_L = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$; $[S]_R = \begin{bmatrix} S_{22} & S_{21} \\ S_{12} & S_{11} \end{bmatrix}$

(b) Write down $[T]_L$, $[T]_R$ and $[T]_{through}$ in terms of $[S]_L$. (**5 pts**)

$$[T]_L = \frac{1}{S_{21}}\begin{bmatrix} 1 & -S_{22} \\ S_{11} & -S_{11}S_{22} \end{bmatrix}; \quad [T]_R = \frac{1}{S_{12}}\begin{bmatrix} 1 & -S_{11} \\ S_{22} & -S_{11}S_{22} \end{bmatrix};$$

$$[T]_{through} = [T]_L[T]_R = \frac{1}{S_{12}S_{21}}\begin{bmatrix} 1-S_{22}^2 & -S_{11}+S_{11}S_{22}^2 \\ S_{11}-S_{11}S_{22}^2 & -S_{11}^2+S_{11}^2S_{22}^2 \end{bmatrix};$$

(c) What is the symmetry that can be observed in $[T]_{through}$? (**5 pts**) If at a given frequency $[T]_{through}$ is the only available measurements, can we derive $[S]_L$ and $[S]_R$? Justify your answer from the number of variables in each case (**5 pts**).

$[T]_{through}$ has symmetry on the off diagonal as $T_{21} = -T_{12}$. As $[T]_{through}$ has only 3 useful complex numbers instead of 4 as in $[S]_L$, we cannot derive $[S]_L$ without further assumptions. One additional de-embedding structures are needed for exact representation of $[S]_L$ and $[S]_R$. Similar to SOLT, a "reflect" R structure (open) or a "line" L structure (a known transmission line) can be used. When both of R and L are used together with "through", we are almost back to the conventional SOLT, and it was called "TRL de-embedding".

(d) By using the definition of the Y matrix and the mirror symmetry between $[Y]_L$ and $[Y]_R$, write down the relations between the elements of $[Y]_L$ and $[Y]_R$. (**5 pts**)

$y_{11L} = y_{22R}$; $y_{22L} = y_{11R}$.

$y_{12L} = y_{21R}$; $y_{21L} = y_{12R}$.