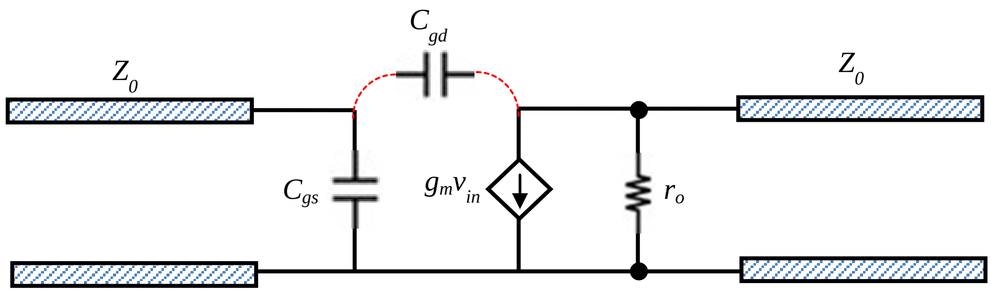
ECE 4880 RF Systems Fall 2016
Homework 4
Due at 5pm of 9/23 in the Phillips Hall Dropbox
(Please **box** your quantitative final answers)

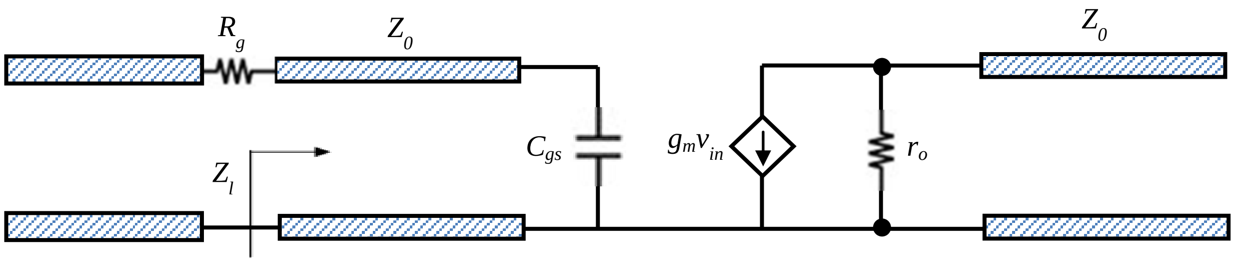
1. (**Antenna dBi**) You are given an antenna below with a 10dBi rating in 2.4GHz with a bandwidth of 0.5GHz. From its size, you know that this is not a multi-antenna array. Ignoring the polarization effects and beam steering, which of the following is true? (**10 pts**)

   (a) The antenna can cover all of the space in the front hemisphere.
   (b) When you walk a receiver from the left to the right side, you would not observe much change in the received power strength.
   (c) The antenna can at most cover a solid angle of 72^o.
   (d) The antenna will not be able to cover the front-left-down corner at all.
   (e) The antenna can be used to enhance your 1.9GHz cell phone for better reception (if you can plug it in).

2. (**Matching network for MOSFET**) An inexperienced engineer put a MOSFET directly to implement an RF amplifier. Assume that the MOSFET has been properly DC-biased to be in the saturation region, and within the voltage range of operation can be approximated by the π network shown below. The input and output have transmission line of impedance of Z_0.



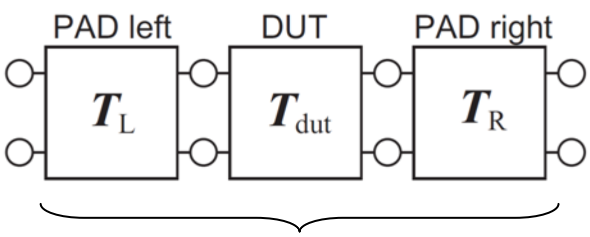
   (a) Write down the Y matrix for the π network, first ignoring C_{gd}. (**5 pts**)
   (b) Write down the S matrix using the Y network in (a), assuming both input and output are connected by the transmission line of Z_0 impedance. (**5 pts**) Observe how gm appears in the four S parameters, and rationalize the expression. (**5 pts**)
   (c) Let's look at S_{21}, which is the transfer voltage gain. In the quasi-static limit, we know that the transfer voltage gain should be close to $-g_m(r_o \parallel Z_0)$. Under what condition does the above S_{21} approach $-2g_m(r_o \parallel Z_0)$? (**5 pts**) Explain the additional factor of 2 here. (**5 pts**)
   (d) Obtain the numerical values of the S matrix (complex numbers) with C_{gs} = 100fF; g_m = 50mS; r_o = 2kΩ; Z_0 = 50Ω. First at 10MHz and then at 10GHz. Just keep two significant digits to simplify your calculation, e.x., $1.0 + 0.04 \cong 1.0$. (**10 pts**)
   (e) Suggest an input impedance matching network just outside the π network at 10GHz so that the S_{11} is minimized. (**5 pts**) Give numerical values of your matching network circuit element parameters (**5 pts**)
   (f) Use the input impedance matching network below with an input transmission line length to cancel the reactance and a series element of lumped $R_g = Z_0$ = 50Ω. What is the input transmission line length at 10GHz? (**5 pts**)



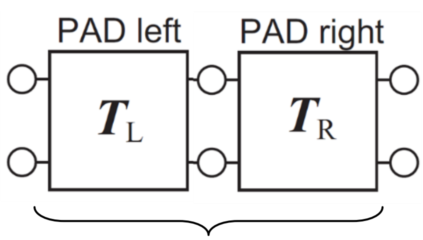
(g) Express T_{11} explicitly by the circuit elements in the π network in Part (c). Give your answer in the low frequency and high output resistance limit (i.e., $Z_0 << r_o$.) Is T_{11} positive or negative? (**5 pts**)

(h) Now we will add in the effect of C_{gd}. Assume C_{gd} is only big enough to affect Y_{12}, which is originally zero. Find Y_{12} by the definition of $Y_{12} = i_1/v_2$ when $v_1 = 0$ (**5 pts**). Find the approximate expression for S_{21}. What is the modification of S_{21} at high frequency? (**5 pts**)

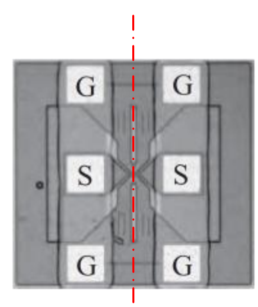
3. (**TL de-embedded network**) In the through-only de-embedding scheme, we can observe the mirror symmetry of the left and right L and R network of the GSG pads for the through network on a chip or PCB board.



Total Measurement = T_{meas}



Through-Only Measurement = $T_{through}$



Mirror symmetry in Through-Only Measurement

(a) If the left pad can be expressed as: $[S]_L = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, write down $[S]_R$ by the mirror symmetry (**5 pts**).

(b) Write down $[T]_L$, $[T]_R$ and $[T]_{through}$ in terms of $[S]_L$. (**5 pts**)

(c) What is the symmetry that can be observed in $[T]_{through}$? (**5 pts**) If at a given frequency $[T]_{through}$ is the only available measurements, can we derive $[S]_L$ and $[S]_R$? Justify your answer from the number of variables in each case (**5 pts**).

(d) By using the definition of the Y matrix and the mirror symmetry between $[Y]_L$ and $[Y]_R$, write down the relations between the elements of $[Y]_L$ and $[Y]_R$. (**5 pts**)