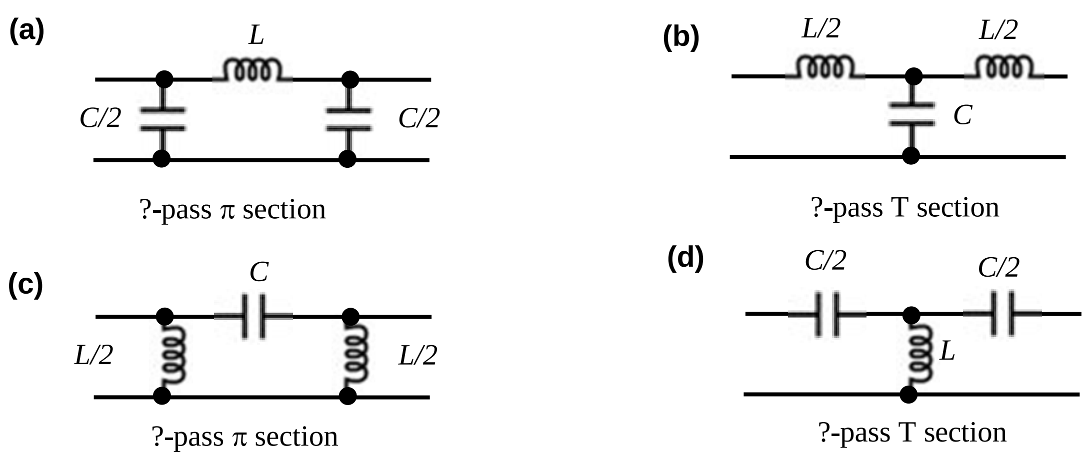
ECE 4880 RF Systems Fall 2016
Homework 3 Solution

Reading before homework:

- Lecture note Chap. 2
- Lee's *The Design of CMOS Radio Frequency Integrated Circuits*, *2nd Ed*., Chap. 3.

1. (**Filter by LC Banks**) Filters are used extensively in RF systems and the passive filters are often accomplished by LC resonators. For the following four filters, denote whether they are low-pass or high-pass. Note that the π and T network construction is often popular due to its symmetry (see Prob. 2). (**12 pts**)



(a) Low-pass π section

(b) Low-pass T section.

(c) High-pass π section.

(d) High-pass T section.

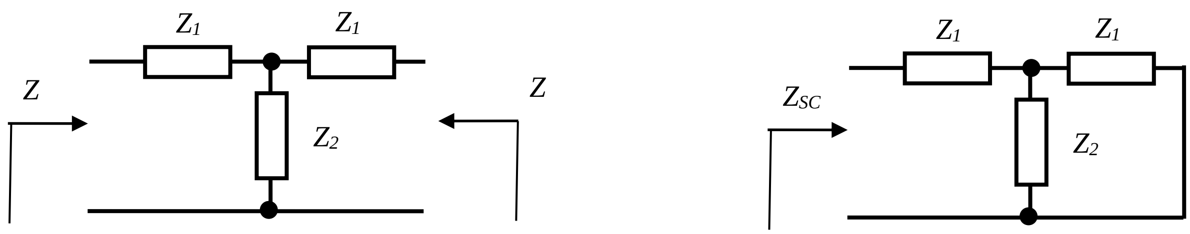
LC bank filter analyses are rather complicated for the Butterworth (flat pass band but less sharp transition to stop band) and Chebyshev (sharp transition to stop band with ripples in either pass band or stop band gains) filters. If you have tried to write down the transfer function of the above network, you will notice that not only additional poles/zeroes exist (and resonance as well), but also there is a strong depedence on the source and load termination. However, LC bank filters are still preferred in the RF band to RC or LC based filters due to power consumption, especially around power amplifiers. Remember that whatever power consumption will be eventually heat, which will not only hurt efficiency but need heat sink. A 3dB insertion loss in the pass band means half of the signal power will become heat, and any power in the stop band will be reflected or heat here!

When LC tank filters are used (T, π, m-derived, ladder, etc.), as the transfer function is termination dependent, it is important to **match the input and output impedance** to obtain the spectral transfer function. This is often for the pass band of the filter.

We will not have time to treat the filter circuits (in ECE 5790), but will deal with the nonideal issues in the signal level later. We will introduce the ABCD matrix for more rigorous derivation later as well.

Above are first-order constant-k type filters. Other popular LC bank filters include ladders and m-derived (for better impedance matched and more degrees of freedom in design, which is related to the image impedance method in Prob. 2).

2. (**The image impedance method in filter design**) To find the impedance for the π and T networks above, the image impedance method offers a convenient view, which is often employed in the filter design for easier impedance characterization. Consider the T network below. Due to symmetry, we can conclude that both sides have the same Z.



(a) Derive $Z^2 = Z_1^2 + 2Z_1Z_2$ (**5 pts**). This will be the impedance when you have a large number of these modules in cascade.

$Z = Z_1 + \dfrac{1}{\dfrac{1}{Z_2} + \dfrac{1}{Z + Z_1}}$; Move all terms with Z to left, and all other terms to the right. We will obtain $Z^2 = Z_1^2 + 2Z_1Z_2$. This Z corresponds to the impedance when infinite number of identical stages is connected, and will surely change if a load is added. You can see that when there are sufficient stages, the load will become less and less important.

(b) We define the short-circuit impedance Z_{SC} at the left end when the right end is short circuited (shown as an example in the right), and the open-circuited impedance Z_{OC} at the left end when the right end is open circuited. Derive $Z = \sqrt{Z_{OC}Z_{SC}}$. (**5 pts**) This relation offers an easy method for measuring the impedance of the T network.

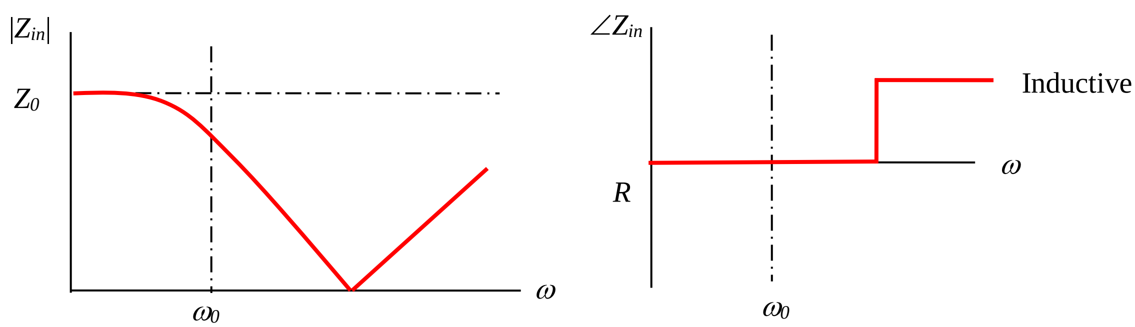
$Z_{OC} = Z_1 + Z_2$; $Z_{SC} = Z_1 + \dfrac{Z_1Z_2}{Z_1 + Z_2}$. We get $Z = \sqrt{Z_{OC}Z_{SC}}$. The formula is very useful in practical measurements, as cascading many stages may not be practical unless that is what we intend to build in the first place.

(c) If $Z_1 = j\omega L/2$ and $Z_2 = 1/j\omega C$, what is the impedance Z of this T network in Prob. 1b as a function of frequency? (**5 pts**) Draw Z vs. ω in the Bode plots of $|Z|$ and $\angle Z$. (**10 pts**)

$$Z^2 = \left(\frac{j\omega L}{2}\right)^2 + 2\left(\frac{j\omega L}{2}\right)\frac{1}{j\omega C} = \frac{L}{C} - \frac{\omega^2 L^2}{4}.$$

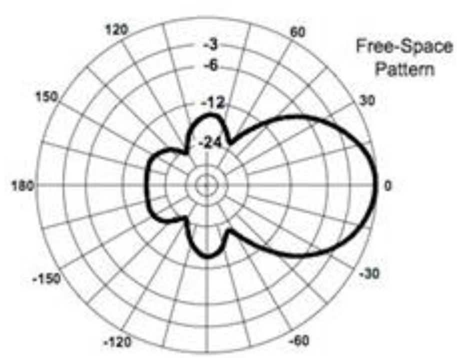
Notice that in this lossless ideal case Z^2 is always real ($\mathrm{Im}(Z^2) = 0$). This is rather peculiar, as Z has to be pure real or imaginary. In general, when X is complex, $\sqrt{X} = \sqrt{|X|}\exp(\angle X/2)$, and only when $\angle X/2$ is multiple of $\pi/2$, we can have $\mathrm{Im}(X) = 0$. Whenever there is some resistive loss, this abrupt change will be smeared out.

At low frequency ($\omega << \omega_0$), $Z = \sqrt{\dfrac{L}{C}} = Z_0$, which is of no surprise. At $\omega_0 = \dfrac{1}{\sqrt{LC}}$, $Z = \sqrt{\dfrac{3L}{4C}} = \dfrac{\sqrt{3}}{2}Z_0$. At $2\omega_0$, Z = 0! At very high frequency, as C is a short circuit, $Z = \dfrac{j\omega L}{2}$.



3. **(Practice of dB and dBm**) Translate to or back from dB, dBm or dBi for the following (**3 pts** each)
   (a) An RF amplifier with power gain of 100 and voltage gain of 10. Answer the amplifier gain in dB.
   (b) An antenna with 8 dBi in reference with an imaginary isotropic lossless antenna. Answer the ratio of peak power in this antenna and the imaginary isotropic antenna.
   (c) A differential amplifier with voltage gain of 40. Answer the amplifier gain in dB.
   (d) A filter with pass-band power loss of –3dB and stop-band power loss of at least –60dB. What are the percentage of transmitted power in the pass-band and stop-band?
   (e) A power amplifier with 36dBm maximum output power. Answer in Watt.
   (f) A power amplifier with 0dBm input and 30dB gain. What is the output power in dBm?

a) Power gain: unitless, and should be denoted in dB = 10 $\log_{10}$. Power gain of 100 = 20 dB.
b) 8 dBi: unitless. In comparison with the imaginary isotropic antenna, the power emitted is $10^{0.8}$ = 6.3 times more. This can be accomplished in a main lobe of emission of about 720^o/9 = 114^o (solid angle) as shown in the figure below.



c) Voltage gain: unitless, and should be denoted in dB = 20 $\log_{10}$. Voltage gain of 100 = 32dB.

d) Filter is defined on power gain, and should be denoted in dB = 10 log10. In-band loss of –3dB is 0.50 power transmitted inside the filter pass-band, and stop-band loss of –60dB is 10^{-6} smaller power transmitted outside the band (for sure, it will become heat, or reflected, as the filter is designed for impedance match only in stop band).

e) Power amplifier with 36dBm output power is $10^{3.6} \times$ 1mW = 4W (limit for ISM bands with circular polarization).

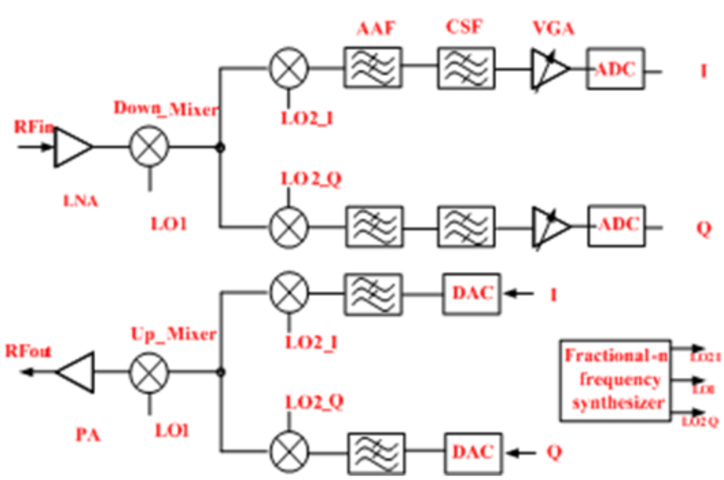
f) 0dBm + 30dB = 30dBm, which is $10^{30/10} \times$ 1mW = 1W.

4. (**Superheterodyne RF transceiver block diagram**) We have not covered this in the lecture, so the problem is simple and self contained. Just as a warm-up for the lecture on "Idealized transceiver design". Assume the local oscillator 1 (LO1) implements the carrier frequency of 900MHz, and LO2 implements the intermediate frequency (IF) of 10.7MHz. This is called the superheterodynae scheme. The quadrature elements *I* and *Q* are for phase/magnitude handling/extraction and improved linearity (reasons will be clear in the long run). *I* and *Q* are sometime the same signal with 90° phase shift. The 10-bit ADC and DAC have the sampling data at 200kHz. Assume that the ADC and DAC will take voltages roughly from 1mV to 1V without severe distortion/underflow/overflow (for example 1mV at ADC will give 0000000001_2 and 1V will roughly give 1111111111_2). Follow through the ideal signal chain in the following questions. The mixer is implementing the simplified function for frequency conversion:

$$v_{in} = A(t)\sin(\omega_{in}t); \qquad v_{LO} = C\sin(\omega_{LO}t;);$$
$$v_{out} = \frac{CA(t)}{2}\left(\cos((\omega_{LO}+\omega_{in})t)+\cos((\omega_{LO}-\omega_{in})t)\right)'$$

where *A(t)* is a slow-varying function in time.

We will temporarily ignore the frequency synthesizer, and assume all desired LO frequencies are available.



- LNA: low-noise amplifier
- I, Q: quadrature signals
- Down_mixer: down-conversion mixer
- LO: local oscillator
- AAF: anti-aliasing filter (low-pass)
- CSF: channel select filter (band-pass)
- VGA: variable gain amplifier
- ADC: analog-to-digital converter
- DAC: digital-to-analog converter
- Up_mixer: up-conversion mixer
- PA: power amplifier

(a) What frequency should the low-pass filter to the right of the LO2 mixer be? Give a rough estimate. What are the operating frequency range of the LNA and PA? (**5 pts**)

It should be slightly higher than the base band of the ADC/DAC sampling frequency, say at 400kHz. For the receiver, all high-frequency components around LO1 and LO2 are filtered out. For the transmitter, the digital output (broadband) will contain only the fundamental baseband frequency when it enters the LO2 IF mixer. Both LNA and PA are in the carrier frequency, or 900MHz here.

(b) Assume RF_{in} in the receiver is in the range of 1pW (–90dBm) to 0.1mW (–10dBm), as we do not know how close the other radio is, and the dynamic range here is 10^8 or 80dB, a common number. Further assume that the receiving antenna as well as the input impedance of LNA is 50Ω, what is the voltage level at RF_{in}? (**5 pts**)

For a given average power, the relation to the peak voltage amplitude is $P = V^2/2R$ because:

$$\int \frac{V^2 \cos^2(\omega t)}{R} dt = \frac{V^2}{2R}$$

The voltage swing at RF_{in} will be: 10μV at 1pW and 100mV at 0.1mW, spanning 10^4 times.

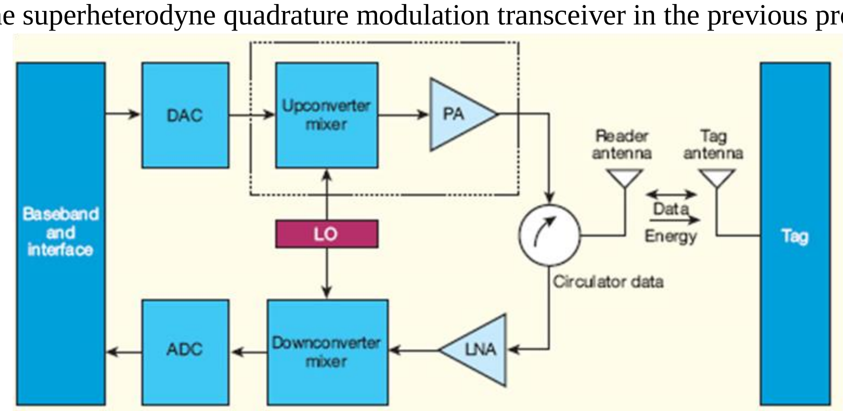
(c) For the minimum signal at RF_{in}, how much gain is needed for the ADC to recognize the signal? (**5 pts**)

1mV/10μV = 100, or $20\log_{10}100$ = 40dB. This is a relatively modest gain as –90dBm sensitivity is quite high in common radios. Wi-Fi receivers are usually around –110dBm, and GPS around –130dBm.

(d) The range of the minimum and maximum voltage of RF_{in} is too large for the 10-bit ADC. This is where the variable gain amplifier (at the low-frequency baseband, so not hard to implement). What is the minimal range of gain the VGA needs to provide? (**5 pts**)

$10^4/2^{10} \cong 10$, or 20dB in voltage gain. Often the range is even larger, as the gain is easy to control at the baseband. LNA will provide reasonable gain and minimal noise, as all noise at this stage will be further amplified in the following stages.

5. (**Backscattering RF transceiver considerations**) A typical RFID reader transceiver block diagram is shown below. Notice that this is a "homodyne" amplitude-modulation transceiver, and much simpler than the superheterodyne quadrature modulation transceiver in the previous problem.



The operation is that the reader sends a modulated RF signal and listens to the echo from the RFID tag. The passive tag needs to collect power from the RF carrier, and run a small logic signal to put modulation on the echo. The RFID system is most often limited by the tag sensitivity, i.e., the tag needs to convert enough RF power from the RF carrier sent by the reader. Therefore, many other parts of the radio are adapted for that purpose. The transceiver in the tag is even more simplified, because if the carrier is powerful enough for run the logic circuits, the radio signal is very strong and easy to design.

(a) What is the different requirement in the local oscillator here in comparison with that of the quadrature superheterodyne transceiver in problem 4, in terms of number of LO generators and frequency accuracy? (**5 pts**)

The quadrature superheterodyne transceiver needs two LO1, LO2_I and LO2_Q, while the homodyne of RFID receiver here just needs one. In two-way radio, as the carrier frequency in the transmitter chain and receiver chain may not be identical (coming from two different frequency synthesizers that may have slight manufacturing variations, or in full duplex scenario transmitter and receivers are on two different channels), the LO in the transmitter and receiver chain needs to be "slightly modified" individually to be "synchronized". In RFID, as we are listening to the echo, no such synchronization complication exists, so one LO is sufficient, and accuracy requirement between two units.

(b) The transmitter and the receiver share one antenna through a component called "circulator". The circulator has low loss from PA to antenna as well as from antenna to LNA, but high "isolation" from PA to LNA directly. A duplex transceiver can use an RF switch to time share the antenna, but here as the tag can only be on with the transmitter on, time sharing is not possible and a circulator is necessary. Assume the circulator pass-path has –1dB loss, and –60dB loss for stop-path, when the PA outputs 36dBm power, what is the power of the self jamming signal at the LNA? (**5 pts**) If the receiver has a –70dBm sensitivity (i.e., the lowest echo to be heard clearly has power of –70dBm), what is the power ratio between the self jamming and the lowest echo? (**5 pts**) (very simple, just for your information)

36 dBm – 60dB = –24 dBm. Notice that dB is in ratio, so it can operate on dBm as such. Corresponding to the –60dBm receiver sensitivity, the self jamming signal is –24dBm – (–70dBm) = 46dB larger, or 40,000 times larger. The self jamming will need to be further resolved from distinction between the transmitter modulation and tag modulation.

(c) At the –70dBm receiver sensitivity (i.e., the lowest echo to be heard has power of –60dBm), what is the peak voltage for 50Ω impedance? (**5 pts**) As we do not use a variable-gain amplifier here, for a 12-bit ADC (analog-to-digital converter), what is the largest power (in mW and dBm) and peak voltage at the LNA input without causing overflow? What is the dynamic range of the receiver in dB? (**5 pts**)

$P = 10^{-10} W = \frac{V_{peak}^2}{2R}$, and we have $V_{peak,min}$ = 0.1mV, which is pretty large, and not much gain is needed.

A 12-bit ADC can give 4,096 levels, which corresponds to the maximum and minimum voltage ratio at the LNA, i.e., $V_{peak,max}$ = 0.4V. $P_{receiving,max}$ = 1.6mW = 2.0dBm. The dynamic range is: 2.0 dBm – (–70dMm) = 72 dB (not great, but acceptable).