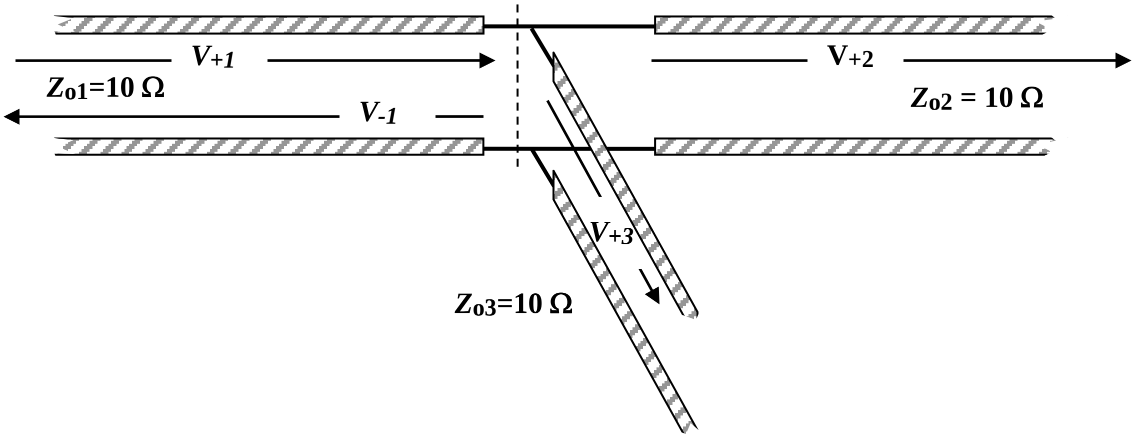
ECE 4880 RF Systems Fall 2016
Homework 2 Solution

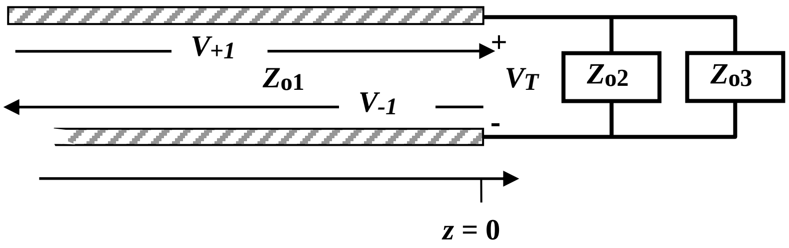
Reading before homework:

- Lecture note Chap. 1
- ECE 3030 lecture notes: Lecture02 and Lecture20
- Lee's *The Design of CMOS Radio Frequency Integrated Circuits*, *2nd Ed.*, Chap. 3.

1. (**Transmission line as power splitter**) Power splitters are often used in RF signal chains into two or more output directions. A schematic of a 1×2 microwave splitter is shown below. You need to figure out what fraction of the input power is reflected, and what fraction of the input power is transmitted into each of the output transmission lines. Before you can do that you need to find the amplitudes V_{-1}, V_{+2} , and V_{+3} of the voltage waves in terms of the input wave amplitude V_{+1}. The three transmission lines can be viewed as infinitely long in this simple case



(a) Looking to the right of the dashed line, the two output transmission lines can be represented as lumped impedances so the equivalent circuit becomes as shown below. Find the amplitude V_{-1} of the reflected wave in terms of the input wave amplitude V_{+1} (**5 pts**) and find the fraction of the input power that is reflected. (**5 pts**)



As all transmission lines here are considered with infinitely long, the impedances looking into lines 2 and 3 are just Z_{o2} and Z_{o3}.

$$\Gamma_L = \frac{\frac{Z_{o2} \| Z_{o3}}{Z_{o1}} - 1}{\frac{Z_{o2} \| Z_{o3}}{Z_{o1}} + 1} = \frac{\frac{5}{10} - 1}{\frac{5}{10} + 1} = -\frac{1}{3} . \; V_{-1} = -\frac{1}{3} V_{+1} .$$

The power reflected is: $\dfrac{\frac{|V_{-1}|^2}{2Z_{o1}}}{\frac{|V_{+1}|^2}{2Z_{o1}}} = \dfrac{1}{9}$.

(b) Find the total voltage V_T at the point z=0 in the figure above in terms of the input wave amplitude V_{+1}. (**5 pts**)

$V_T = V_{+1} + V_{-1} = \dfrac{2}{3} V_{+1}$.

(c) Find the fraction of the input power transmitted in the each of the two output transmission lines. Do all your fractions (reflected and transmitted) add up to unity? They should. (**5 pts**)

Power transmitted into line 2: $P_2 = \dfrac{|V_T|^2}{2Z_{o2}}$. Fraction of P_2 to P_1 = $\dfrac{\frac{|V_T|^2}{2Z_{o2}}}{\frac{|V_{+1}|^2}{2Z_{o1}}} = \dfrac{4}{9}$.

Check: $\dfrac{1}{9} + 2 \times \dfrac{4}{9} = 1$.

(d) Suppose you could choose the impedances Z_{02} and Z_{03} of the output transmission lines to be whatever you wanted. Choose these values such that you simultaneously satisfy the following two conditions: (i) No fraction of the input power is reflected ; (ii) The output transmission line with impedance Z_{02} has twice as much power going into it as the transmission line with impedance Z_{03}. (**5 pts**)

No reflection: $Z_{o2} \| Z_{o3} = 10\Omega$; $P_2 = 2P_3$ or $Z_{o3} = 2Z_{o2}$.

Solve and we obtain: $Z_{o2} = 15\Omega$ and $Z_{o3} = 30\Omega$.

2. (**Historical aspects**) From the radio development history,
   (a) What was the experimental difficulty to demonstrate a working radio receiver for Marconi and Bose? (**5 pts**)

The received signal is too weak (on the order of several mV) to be transduced to a signal that human can read easily by eyes or ears. There is no effective electronic amplifier except the transformer, which is very difficult to be impedance matched to the antenna (this will be shown later).

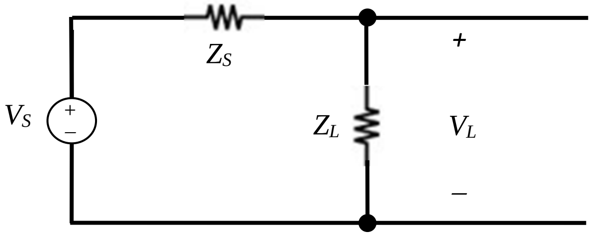
(b) Can Eiffel Tower be erected as a TV broadcast antenna? (**5 pts**)

The experimental radio demonstration (no earlier than 1890) is later than the erection, so it is not possible to be the initial purpose of Eiffel Tower.

(c) Delivering electric power wirelessly with a near DC source can be similar to lightning. Why is it so difficult even for genius like Nikola Tesla (although he is also the inventor of induction motors, he is mainly thinking about DC power delivery through thin air as a conductor)? (**5 pts**)

The voltage required will be VERY high, even for very thin air. Lightning indeed has very high voltage up to 10kV during the rain cloud formation. You can appreciate the plot in the “Back to Future”, where the mad scientist can only get the high voltage from lightning to power his time machine.

3. (**Conjugate Matching by Derivation**) For a AC Thevenin source $V_S\cos(\omega t)$ to deliver RF power to the load Z_L, assume that both source and load impedances Z_S and Z_L are complex, i.e., $Z_S = R_S + jX_S$ and $Z_L = R_L + jX_L$, where $R_S, R_L > 0$.



(a) What is the power $P_L \equiv \frac{1}{2}\mathrm{Re}\left(V_L I_L^*\right)$ delivered to Z_L? Express P_L in terms of V_S, R_S, X_S, R_L and X_L. (**5 pts**)

$$P_L = \frac{1}{2}\mathrm{Re}\left(V_L I_L^*\right) = \frac{1}{2}\mathrm{Re}\left(\left(V_S \frac{Z_L}{Z_S + Z_L}\right)\left(\frac{V_S}{Z_S + Z_L}\right)^*\right)$$

$$= \frac{1}{2}\left|V_S\right|^2 \frac{\mathrm{Re}\left(Z_L\right)}{\left|Z_S + Z_L\right|^2} = \frac{1}{2}\left|V_S\right|^2 \frac{R_L}{\left|Z_S + Z_L\right|^2} = \frac{1}{2}\left|V_S\right|^2 \frac{R_L}{\left(R_L + R_S\right)^2 + \left(X_L + X_S\right)^2}$$

(b) When P_L is at its maximum, we will have $\frac{\partial P_L}{\partial R_L} = 0$ and $\frac{\partial P_L}{\partial X_L} = 0$. Use the relation to derive the conjugate matching criteria: $R_L = R_S$; $X_L = -X_S$. Or $Z_L = Z_S^*$. (**5 pts**)

$$\frac{\partial P_L}{\partial R_L}=0 \quad \Rightarrow \quad (R_L+R_S)^2+(X_L+X_S)^2-2R_L(R_L+R_S)=R_S^2-R_L^2+(X_L+X_S)^2=0$$

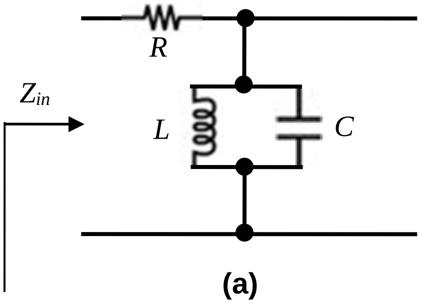
$$\frac{\partial P_L}{\partial X_L}=0 \quad \Rightarrow \quad -2(X_L+X_S)=0$$

As R_S, $R_L > 0$, we have $R_L = R_S$; $X_L = -X_S$ for the conjugate matching criteria.

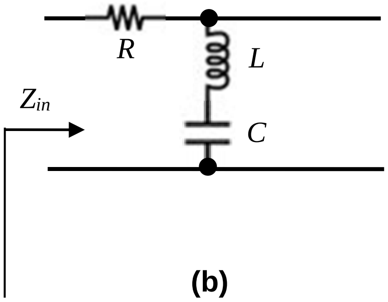
(c) What is the maximum power delivered to Z_L at the conjugate matching condition? (**5 pts**)

$$P_{L\max}=\frac{1}{2}|V_S|^2\frac{R_L}{|Z_S+Z_L|^2}=\frac{1}{2}|V_S|^2\frac{R_L}{(2R_L)^2}=\frac{|V_S|^2}{8R_L}$$

4. (**LC Resonator Circuits**) For the series and parallel LC circuits below, express the impedance Z_{in} in the frequency space (**5 pts**), and then draw $|Z_{in}|$ (**5 pts**) and $\angle Z_{in}$ (**5 pts**) for both circuits as a function of frequency. From $\angle Z_{in}$, denote the capacitive and inductive nature of Z_{in}. (**5 pts**).



(a)



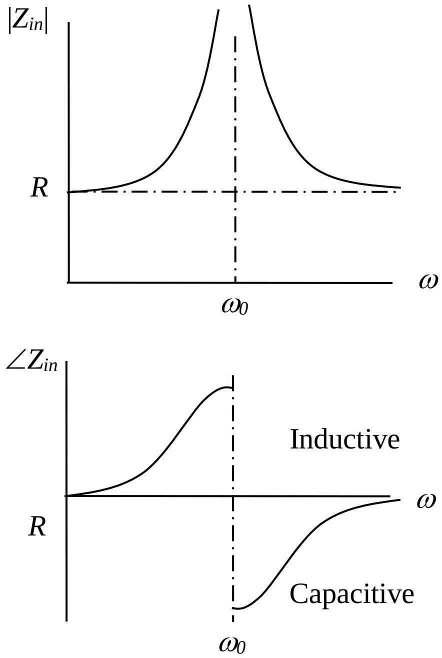
(b)

For the circuit in Fig. (a),

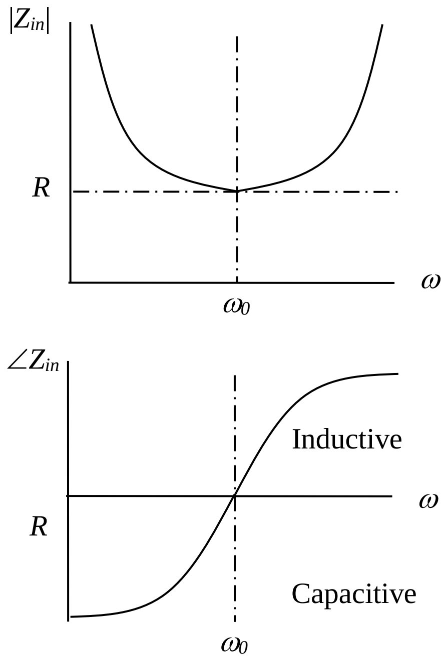
$$Z_{in}=R+\frac{1}{\frac{1}{j\omega L}+j\omega C}=R+j\left(\frac{\omega L}{1-\omega^2 LC}\right)=R+j\left(\frac{\omega L}{1-\frac{\omega^2}{\omega_o^2}}\right);\text{ where }\omega_0=\frac{1}{\sqrt{LC}}$$

For the circuit in Fig. (b),

$$Z_{in}=R+j\omega L+\frac{1}{j\omega C}=R+j\omega L\left(1-\frac{1}{\omega^2 LC}\right);$$

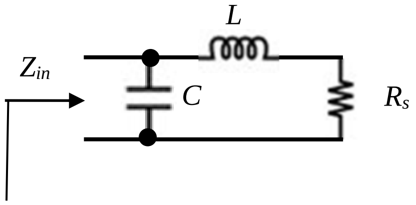


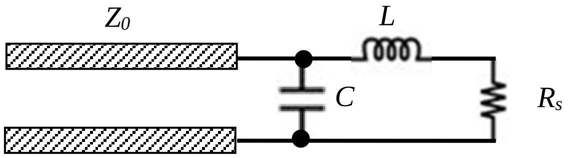
(**a**)



(**b**)

5. (**Resonator as impedance transfer**) The resonator can be used to transfer resistance to make it seem larger or smaller at the resonant frequency. An example for "**impedance up conversion**" is shown below, and is called the "L-section". This is another way to match the impedance to or with the transmission line, as shown in the right figure.





(a) Derive Z_{in} in the complex phasor expression. (**5 pts**)

$$Z_{in} = \frac{1}{j\omega C + \dfrac{1}{R_s + j\omega L}} = \frac{R_s + j\omega L}{\left(1-\omega^2 LC\right) + j\omega R_s C}.$$

(b) Close to $\omega_0 = \dfrac{1}{\sqrt{LC}}$, what is the real part of the impedance? If we define the quality factor as $Q \equiv \dfrac{\omega L}{R_s}$, express Z_{in} in terms of R_S and Q. (**5 pts**)

$Z_{in} \cong \dfrac{L}{CR_s} + \dfrac{1}{j\omega_0 C} = Q^2 R_s + \dfrac{1}{j\omega_0 C}$, where $Q \equiv \dfrac{\omega L}{R_s}$. We can see that R_s seems to be Q^2 times larger!!!

(c) For $R_s = 1\Omega$, give the L and C values that can give an up conversion of Re(Z_{in}) to 50Ω at 1GHz for matching with a transmission line. What is the reactance of Z_{in} at such condition? (**5 pts**)

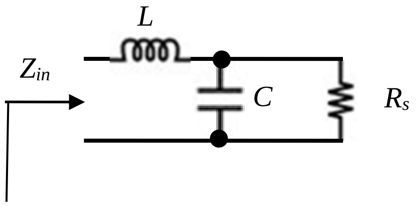
At $\omega_0 = \dfrac{1}{\sqrt{LC}}, Z_{in} \cong \dfrac{L}{CR_s} + \dfrac{1}{j\omega_0 C}$.

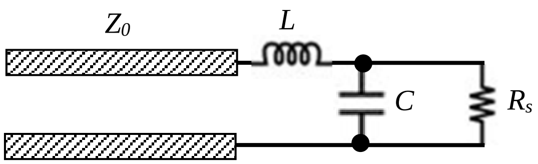
We then have $\dfrac{L}{CR_s} = 50\Omega$ and $\sqrt{LC} = \dfrac{1}{2\pi \cdot 1GHz}$.

This gives us $L = 1.12\text{nH}$, $C = 22.5\text{pF}$ and $Z_{in} = 50\Omega - j\,7.1\Omega$.

We can easily put one more inductor in series to make Z_{in} to be exactly 50Ω.

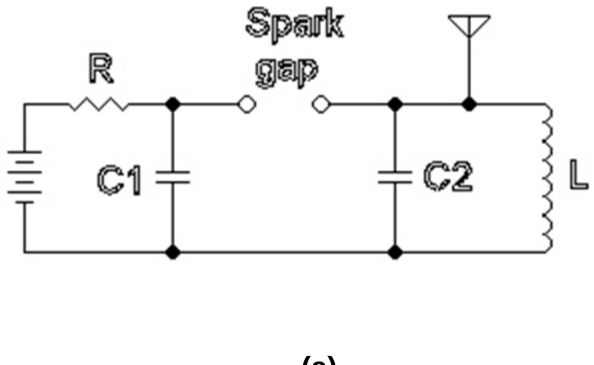
To complete the story, the "**impedance down conversion L**" section will be:



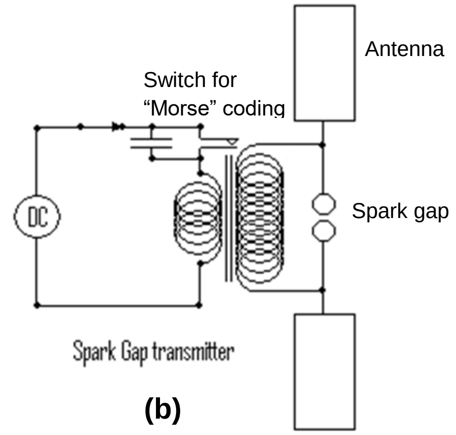


This circuit works when $R_s > Z_{in}$.

6. (**Spark Gap Transmitter**) In the earliest days of radio history, the traveling RF waves were often generated by the "spark gap transmitter". The gap is very small (< 0.5mm), and can be caged in sealed glass chambers to control the gas content. When high voltage is built up across the gap, an arc will be generated as a fast transient short circuit to feed energy to the resonator and antenna. The frequency component of the spark is very broad, but the resonator and antenna are designed to radiate at specific frequency with reasonable bandwidth. Message is typically coded by a mechanical switch with on-off-keying amplitude modulation. Although the spark gap is a simple and practical production, the bandwidth is not efficiently used in consideration of the power consumption.



(a)



(b)

(a) In Fig. (a), what is the approximate frequency of emission (hint: the resonant frequency when the spark is generated)? (**5 pts**)

It would be the resonant frequency set by $L(C_1 + C_2)$, which is $1/\left(2\pi\sqrt{L(C_1 + C_2)}\right)$. The spark generated by ionizing the air has a fast random waveform, and hence a very broad frequency spectrum. However, only the RF power close to the resonant frequency will be emitted from the antenna. Unintended broad spectrum emission is not only inefficient in power but also cause interference for other channels.

(b) In Fig. (a), what is the role of the RC_1 time constant in the spark gap transmitter? (**5 pts**)

The time constant RC1 set a bound for the startup time of each spark formation, and can be thought to set the upper bound of the baseband data. If the baseband is too fast, human ear cannot distinguish the beeps by on-off keying.

(c) A hundred years ago, it is difficult to reliably create a thin gap (which had also postponed the realization of bipolar transistors). Explain how Fig. (b) is an improved version of Fig. (a). (**5 pts**)

When the gap cannot be thin, we will need a high voltage to generate a spark, which is again difficult to get. The transformer, similar to the spark plug we use today in combustion engine, boosts the voltage effectively. The mechanical switch for coding the baseband is also shown.