ECE 4880 RF Systems Fall 2016
Homework 1 Solution

Reading before homework:

- Chap. 1: Review of EM theory and resonator circuits
- ECE 3030 lecture notes: lecture1, lecture2 and lecture13
- Lee's *The Design of CMOS Radio Frequency Integrated Circuits*, *2nd Ed.*, Chap. 1 and Chap. 2.1.

1. (**Radio as Traveling Waves**) If we define radio as a device that uses "electromagnetic radiation of traveling waves into space without wires or waveguides", answer and briefly explain the following:

(a) Is a microwave oven a radio? (**4 pts**)

Even though there is no communication purposes for the microwave oven (2.45GHz radiation without modulation), according to the definition, it is actually a radio. I think this is a good reminder that any strong radio wave can cause some heating concerns, especially when you are near a broadcasting station. Your body parts that do not have strong blood circulation (like the lens and vitreous humor of your eyes) are particularly vulnerable.

(b) Is the cable TV a radio? (**4 pts**)

Cable TV has EM waves propagating in the cable as a waveguide, and hence all waves are confined to the media within the cable. This will not be a radio. However, if the shielding of the cable is not sufficient, unintended radio transmission can leak out.

(c) Is the portable GPS device a radio? (**4 pts**)

GPS is a very sensitive radio receiver to listen to the signals from 4 – 6 satellites.

(d) Are two inductor coils separated in space with nonzero mutual inductance "radio"? This is basically a smart card with near-field communication (NFC). (**4 pts**)

Inductive coupling is most often magneto-quasi-static, and no EM traveling wave. So, this does not fit our radio definition, but NFC does transmit information wirelessly within a distance of about 10cm. The silver lining of the short distance is that there is really no need to consider multiple access, as we can safely assume there would be only one NFC transceiver within the communication range.

(e) Is the Easy Pass on cars for highway tolling a radio? (**4 pts**)

Easy pass is a UHF (around 900MHz) backscattering radio system. It must be a traveling wave as the antenna (less than 15 cm) is much smaller than the reading distance (about 10 m).

(f) Does playing Pokemon-Go involve a radio? (**4 pts**)

Your smart phone, and most of its applications that depend on voice or data transmission, is a radio.

2. (**Maxwell Equations**) From the Maxwell equations,

$$\nabla . \varepsilon_o \vec{E} = \rho$$

$$\nabla . \mu_o \vec{H} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \mu_o \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \varepsilon_o \vec{E}}{\partial t}$$

Use the simplest steps and assumptions to derive a plane-wave propagation equation in free space (no net charge or polarization). You can assume a plane wave traveling in z direction with electric field ONLY in the x direction and magnetic field ONLY in the y direction. A wave equation is a second-order differentiation in both space and time. (**8 pts**) What is the propagation speed of the plane wave? (**4 pts**) Write down the wave solution for the plane-wave electric fields and magnetic fields. (**4 pts**)

For any wave propagation, we will look for the ratio of the second derivatives in time and space to be proportional with a constant v^2, where v is the propagation time constant. For example, for a 1D wave (3D can have more degrees of freedom) propagating in x with velocity v, the equation will look like:

$$\frac{\partial^2 a(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 a(x,t)}{\partial t^2}$$

In free space, we have: $\rho = \vec{J} = 0$

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\nabla \times \left(\frac{\partial \mu_o \vec{H}}{\partial t}\right) = -\frac{\partial \mu_o \nabla \times \vec{H}}{\partial t} = -\mu_o \varepsilon_o \frac{\partial^2 \vec{E}}{\partial t^2},$$

Defining: $c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \approx 3 \times 10^8$ m/s .

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

By using the vector identity: $\nabla \times \left(\nabla \times \vec{F}\right) = \nabla \left(\nabla . \vec{F}\right) - \nabla^2 \vec{F}$, we can have the general 3D EM wave propagation equation as

$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$, or in scalar form of the electric field in the Cartesian coordinates:

$$(1)\quad \frac{\partial^2 E_x}{\partial x^2}+\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2}$$

$$(2)\quad \frac{\partial^2 E_y}{\partial x^2}+\frac{\partial^2 E_y}{\partial y^2}+\frac{\partial^2 E_y}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}$$

$$(3)\quad \frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\frac{\partial^2 E_z}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}$$

If we now invoke the plane wave assumption, i.e., $E_y = E_z = 0$ and

$\nabla . \varepsilon_o \vec{E} = 0 \qquad \Rightarrow \qquad \frac{\partial E_x}{\partial x} = 0$, i.e., $E_x = E_x(z, t)$.

One of the possible traveling wave solution can be expressed as:

$$\vec{E} = \hat{x} E_x(z-ct) = \hat{x} E_o \cos\left(\frac{2\pi}{\lambda}(z-ct)\right) = \hat{x} E_o \cos(\omega t - k z)$$

where we define the angular frequency $\omega = \frac{2\pi c}{\lambda} = 2\pi f$ and wavevector $k = \frac{2\pi}{\lambda}$.

We can derive the magnetic field as:

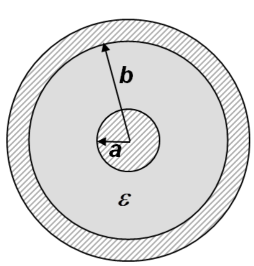
$$\frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu_o}\nabla \times \vec{E} = -\hat{y}\frac{k}{\mu_o}E_o \sin(\omega t - k z)$$

$$\Rightarrow \quad \vec{H} = \hat{y}\frac{k}{\mu_o \omega}E_o \cos(\omega t - k z)$$

$$\Rightarrow \quad \vec{H} = \hat{y}\frac{E_o}{\eta_o}\cos(\omega t - k z)$$

The result can be generalized for a 3D plane wave travelling in the $\vec{k}$ direction as:
$\vec{E}(\vec{r}, t) = \hat{n} E_o \cos(\omega t - \vec{k}.\vec{r})$ where $\vec{k}.\hat{n} = 0$.

3. (**Circuits and Waveguides**) The RF signal is embedded in a 1GHz carrier wave in a coaxial cable to be connected to an antenna. The permittivity of the media in the coaxial cable is $4\varepsilon_0$.



Solution: $C = \frac{2\pi \varepsilon}{\log\left(\frac{b}{a}\right)}$ log(): natural log

$L = \frac{\mu_o}{2\pi}\log\left(\frac{b}{a}\right)$ $LC = \mu_o \varepsilon$

(a) What is the quarter wavelength in the air and in the coaxial cable? (**5 pts**)

The plane wave velocity in the media is the speed of light divided by the square root of the relative permittivity. Therefore, the quarter wavelength is:

In air: $\frac{\lambda}{4} = \frac{c}{4f} = 7.5cm$

In coaxial cable: $\frac{\lambda}{4} = \frac{c/\sqrt{4}}{4f} = 3.8cm$.

(b) In your radio system of 1GHz when you use a 100cm coaxial cable between the power amplifier and the antenna, should this cable be modeled as a resistor, a RC circuit, a RLC circuit, or a transmission line? Briefly explain. (**5 pts**)

100cm is much longer than the wavelength in the coaxial cable, so it is best treated as a transmission line.

(c) If you are requested to describe the 100cm cable as a discrete LC line, what is the constraint on the length you can use for each *L* and *C* segment? (Hint: use the Bragg frequency). (**5 pts**)

A discrete transmission line description needs to have f_{bragg} >> 1GHz.

$$f_{bragg} = \frac{1}{\pi}\frac{v}{\Delta z}.$$

$\Delta z << 4.8cm$.

(d) If the characteristic impedance of the coaxial cable is 50Ω, what is the unit-length inductance and capacitance in nH/cm and pF/cm, respectively? (**5 pts**) If you need to imitate the behavior of this coaxial cable on a PCB board, how many stages of discrete LC you have to use? (**5 pts**)

$Z_o = \sqrt{\frac{L}{C}}$ =50Ω; $v = \frac{1}{\sqrt{LC}}$ =1.5×10^{10}cm/s. We got: *C* = 1.3pF/cm; *L* = 3.3nH/cm.

By the definition of $f_{bragg} = \frac{1}{\pi}\frac{v}{\Delta z}$, we will then have *N* >> 21. To be safe, we need to use more than 25 segments to imitate the distributive 100cm transmission line in 1GHz so that the operation is far away from the f_{bragg} cutoff.

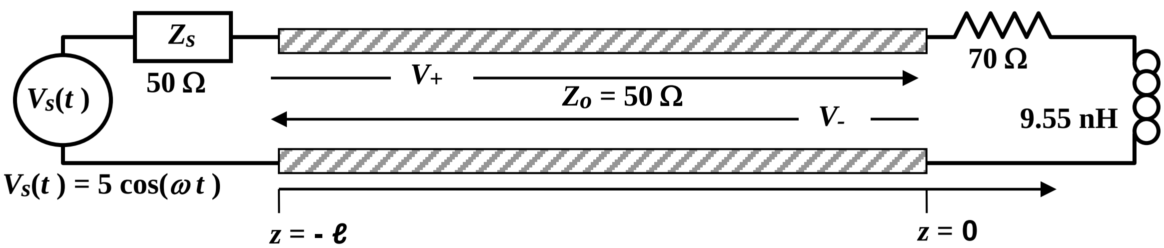
Sometime you need to create phase shift or effects from the Smith chart for your transceiver design within your small radio PCB. However, not-fully-shielded waveguides in PCB is subject to the PCB material and any close-by conductor. This is the reason that a coaxial cable with full outer shielding is a more controllable and known transmission line than coplanar waveguides and slot lines printed on PCB. However, it is more difficult and expensive to have a coaxial cable on PCB. Therefore, discrete LC lines are your next choice. You will need to consider the Bragg frequency and the carrier frequency as shown above though.

(e) If the coaxial cable has an inner conductor diameter of 1mm, what will be the diameter of the outer metal shield? (**5 pts**)

$C = \dfrac{2\pi\,\varepsilon}{\log\left(\dfrac{b}{a}\right)} = 1.3\,pF/cm$. Here, $\varepsilon = 4\times 8.85\times 10^{-14}$F/cm. $b = 5.5a$ = 5.5mm.

4. (**Transmission line review**)

Consider the following transmission line where the transmission velocity is 1.5×10^{10}cm/s:



The circuit is operating at a frequency of 1.0 GHz. At the load end of the circuit there is a 70 Ω resistor in series with a 9.55 nH = 9.55x10^{-9} H inductor. The length of the transmission line is such that: $\ell = \dfrac{21}{16}\lambda$, where λ is the wavelength of waves in the transmission line at a frequency of 1.0 GHz.

(Do not worry about the precision reading off the Smith Chart if you choose to use one. For homework and learning, the Smith Chart is actually preferred, as the visualization will give you more intuition than pure algebra.) However, you can also use the equations directly for computation.

(a) Find the load reflection coefficient Γ_L (give a numerical value). (**5 pts**)

$\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{20 + j60}{120 + j60} = \dfrac{1+j}{3}$. Check: $|\Gamma_L| = \dfrac{\sqrt{2}}{9} < 1$.

(b) Find the impedance $Z(z = -\ell)$ looking into the transmission line at $z = -\ell$ (feel free to use the Smith Chart or calculation). Give a numerical value for your answer. (**5 pts**)

Remember the impedance repeats itself for every half wavelength, so if you use the Smith Chart, you will rotate for just 5λ/16.

We can calculate as well:

$$Z(Z = -\ell) = Z_0 \frac{1 + \Gamma_L e^{-j2k\ell}}{1 - \Gamma_L e^{-j2k\ell}} \quad \text{where } \exp(-j2kl) = \exp\left(-j\frac{5\pi}{4}\right) = \frac{-1+j}{\sqrt{2}}.$$

It happens that after the rotation in Smith Chart or the complex calculation above:

$Z(Z=-\ell)=18\Omega$, with the reactance part at 0.

(c) Find the average power dissipated in the impedance $Z(z = -\ell)$ in the above circuit (give a numerical value). Notice that $V_s(t) = 5\cos(\omega t)$ in volt. (**5 pts**)

$$P(z=-\ell)=\frac{1}{2}\mathrm{Re}\left(\frac{|V(z=-\ell)|^2}{Z(z=-\ell)}\right) \text{ where } V(z=-\ell)=V_S\frac{Z(z=-\ell)}{Z(z=-\ell)+Z_S}$$

We have then $P(z=-\ell)=$ 0.049W = 17dBm.

(d) What is the impedance at $z = 0$ looking towards the source? (**5 pts**)

We will short any source voltage when finding impedance. As Z_S matches with Z_0, there is no reflection, which corresponds to the center of the Smith Chart. The impedance at the load end will always be $Z_0 = 50\Omega$ regardless of the transmission line length.

(e) In the following, we will change the length of the transmission line for impedance purposes. If we need the real part of the $Z(z = -\ell)$ to be 50Ω, what is the smallest ℓ we can use? What is the complex impedance $Z(z = -\ell)$? (**5 pts**)

The impedance transform is easiest from the Smith Chart. Precision is not an issue, as the Smith Chart is mostly used in an ideal design phase. Detailed optimization is often done later with full simulation.

$Z_n(z=-\ell)=1+jR$. From the Smith chart we find the first intersection with the $\mathrm{Re}(Z_n) = 1$ circle is at $\ell=(0.34-0.19)\lambda=0.15\lambda$ with $Z_n = 1 - j0.96$. Remember that all impedance is normalized on the Smith Chart.

$Z = 50\times(1 - j0.96) = 50 - j48.$

(f) A impedance match to Z_s is needed. Give a series stub runner design with the stub line length (**5 pts**). Give a parallel stub runner design with the stub line length. (**5 pts**). Assume the stub runner is open terminated.

For the series stub runner design to cancel –j0.96, we will use here an open-terminated stub runner to with length = 0.13λ.

For the parallel shunt stub runner design, we will have an open-terminated stub runner at 0.12λ.