ECE 4880: RF Systems

Fall 2016

Chapter 9: Phase Noises in the Signal Chain

Reading Assignments:

1. T. H. Lee, *The Design of CMOS Radio Frequency Integrated Circuits*, 2^{nd} *Ed*, Cambridge, 2004. Sec. 18.1 – 18.4.
2. W. F. Eagan, *Practical RF System Design*, Wiley, 2003, Chap. 9.

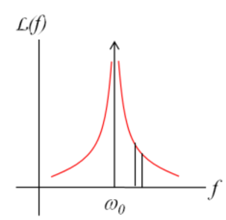
Game Plan for Chap. 8: Frequency strategy

1. Effects of amplitude and phase noises in RF transceivers
2. Intuitive modeling of amplitude and phase noises by LC resonators
3. Empirical models and parameters

Let's take a review on the noises. A noise-free single-tone signal can be expressed as: $v(t) = A\cos(\omega_0 t)$. We can see two kinds of "added noises" in this signal:

1. Amplitude noise (AM): stochastic process by : $v(t) = A(t)\cos(\omega_0 t)$
2. Phase noise (PM): stochastic process by: $A\cos(\omega_0 t + \varphi(t))$

The phase noise is often represented as one-sided spectral density of a signal's phase deviation or phase instability $S_\varphi(f)$. In other words, the phase noise is often defined by $\mathcal{L}(f)$ (pronounced as script *L* of *f*), which represents the noise power relative to the carrier contained in a 1Hz bandwidth centered at the carrier frequency, which has a unit of dBc/Hz. The "c" in dBc reminds you that the noise power is normalized by the carrier. As an example, For a *1/f* phase noise, $\mathcal{L}(f)$ can be –80dBc/Hz at 10kHz offset or –90dBc/Hz at 100kHz.



9.1 Effects of phase noises in RF transceivers

Continuing with our treatment of linear time-varying systems, we will now treat the effect of phase noises in the overall architecture before we put everything together for the present RF applications. When we discuss about noises in Chap. 4, we have listed where the noise power originates from, and how the "additive noise" propagates the signal-to-noise ratio along the signal chain. Phase noise is not a new noise source. Its physical origin is the SAME as the additive noise in the gain module before, including thermal, shot, and Flicker noises. However, as the mixer is a special element that multiplies two signals to achieve the purpose of frequency conversion (aka heterodyning) and channel selection, the phase noise with its multiplicative nature needs special treatment. As shown in Fig. 9.1,

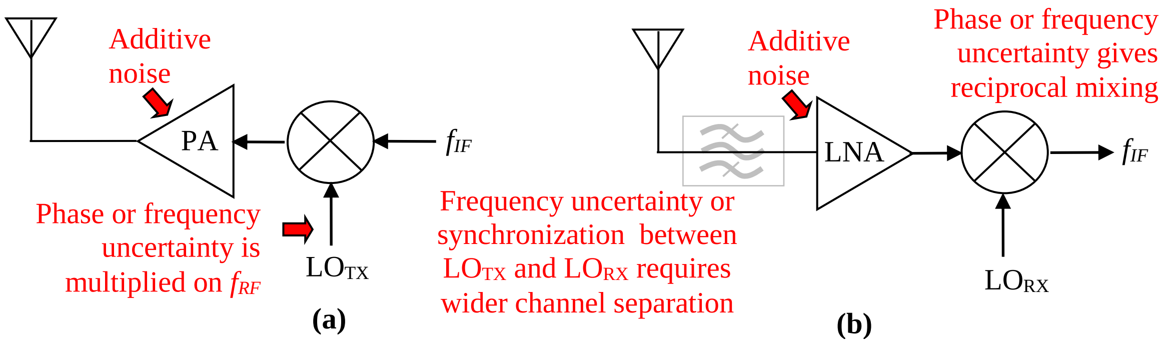


Fig. 9.1. Phase noises enter the mixers in TX and RX. (a) In TX, phase noises are multiplied (convoluted) on the intended signal; (b) In RX, phase noises (or spread LO_{RX} spectrum) will enlarge the influence of the interferer. The uncertainty in LO_{TX} and LO_{RX} will demand wider channel separation. LO_{TX} phase shift at the receiver however carries the time-of-flight information.

If LO can be approximated with a delta function in spectrum (monotone sinusoidal waves from a perfect oscillator), then the multiplication will not result in any further distortion of the original data waveform, but ONLY a frequency translation by f_{LO} (up or down conversion). The finite precision to the delta function can be treated as a combination of amplitude (envelope function) and phase (zero crossing) noises, generally termed as "*reciprocal mixing*" of amplitude modulation (AM) and frequency modulation (FM). The amplitude of LO will affect the mixer gain and nonlinearity, which can be treated just like in any other discrete-element amplifiers. On the other hand, the phase noise can be viewed as additional frequency components or uncertainty in zero-crossing points. The two views have similar influence on the multiplication if finite number of cycles is used in eventual evaluation. We will not distinguish frequency modulation or phase modulation, although in general they can be different especially in long-time integration. The slight spreading in LO will translate into the mixer product in term of symbol distortion, spread spectrum (similar to "pseudo-noise" injection in CDMA) and timing uncertainties, as shown in Fig. 9.2.

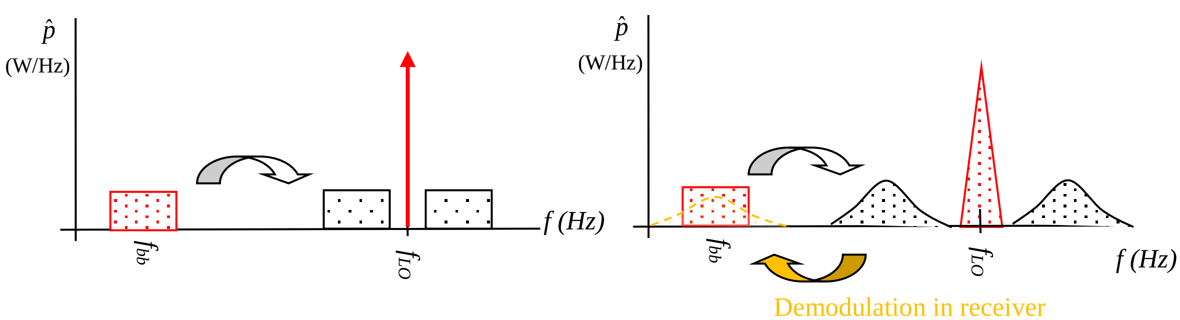


Fig. 9.2. Mixing of ideal LO and LO with realistic phase noises.

Notice that the spreading can be significant enough to overshadow the adjacent band, if both the phase noise and the interference signal are strong.

The nonlinearity in the mixer, or combined with the nonlinearity of the LNA is particularly troublesome, as the spur analysis in the last chapter. We will now focus on the intuitive understanding of phase noise, and leave the noise and nonlinearity interplay details still only to numerical tools. The phase noise concerns pre-date the radio, because for sure it is important when people intended to build accurate clocks out of pendulum and springs.

9.1.1 Effects of the LO phase noises

There are three main detrimental effects by the LO phase noises. Remember that in the heterodyne scheme (frequency conversion to fit the FCC and antenna requirements, including homodyne), the LO mixing happens twice. Once at the transmitter and once at the receiver. The purpose of the radio link is to retrieve the information before the transmitter modulation by the receiver after demodulation, which can be polluted by the (at least two) mixers, the nonlinearity of the amplifiers (in particular PA and LNA), and any interference received by the receiver antenna including self jamming.

1. Receiver desensitization by the transmitter and receiver LO phase noises: A strong interferer can have a longer-than-expected tail due to its own LO phase noise, and desensitize a neighboring channel. This is more serious especially when the receiver has a high sensitivity but insufficient dynamic range, as the large interferer can saturate the base band when the digital signal processing cannot have any information left to work on.

2. Jitter is the uncertainty in synchronization (or zero-crossing points) of the source. The phase noise in LO can make the oscillator that generates LO look jittery. Any stable oscillation is constrained by two conditions: loop gain = 1 and loop phase delay = $2n\pi$. The loop gain fluctuation around 1 (long-term integration to be 1) is the amplitude noise or AM; the loop delay fluctuation around $2n\pi$ is the phase noise or PM. If the oscillation is generated by transistors, both fluctuations can be traced back to the transistor current fluctuation due to thermal, shot and Flicker noises. Therefore, phase noise has the same physical origin of noise power, but is manifested differently from the LO feeding into the mixer.

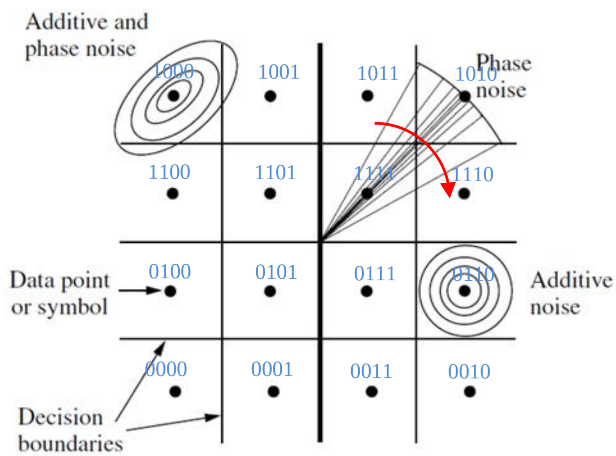


Fig. 9.3. Phase errors in 4-bit Q-ary modulation can cause errors from two nonadjacent decision boundaries.

3. Larger bit error rate in Q-ary modulation that can be harder to correct. For example, as shown in Fig. 9.3, the 4-bit Q-ary modulation (2 bits from FM and 2 bits from AM) can make errors across two non-adjacent domains. Adjacent domains have only one bit in error, and are often easier to correct in error correction code (ECC).

9.2 Intuitive modeling of phase noises by LC oscillators

We will show an intuitive phase noise model in the LC resonators as shown in Fig. 9.4. This can be either indeed the electrical oscillator, such as the Colpitts oscillator or Clapp oscillator or cross-coupled oscillator (where electrostatic and magnetostatic energies balanced each other), or just a model for mechanical or piezoelectric oscillators (where two energies balance each other such as potential and kinetic energy in pendulum, or strain and electrostatic energy in quartz crystal oscillators).

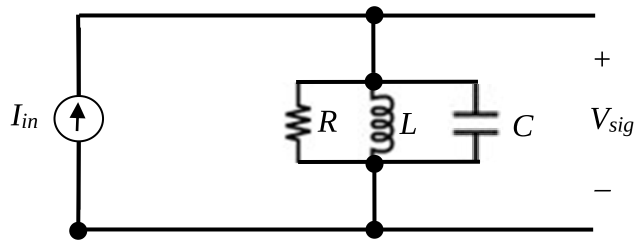


Fig. 9.4. A RLC resonator with a source current injection, compensating the energy loss in R to maintain stable oscillation.

As the loop gain has to be 1 for stable oscillation, $I_{in}V_{out}$ has to be exactly V_{out}^2/R in the long-term integration. This equality can just be valid in small signals, where I_{in} can be replaced with a transistor or any device with a negative differencial resistance (NDR) which cancels R.

9.2.1 AM and PM separation

Let's first assume that the resonator has achieved the loss cancellation by I_{in}, and is oscillating indefinitely when there is no further noise perturbation. Notice that the stable solution requires the form of $V_A\cos(\omega_0 t)$, which is DC balance and has an average stored power of $CV_A^2\omega_0/2$. Now an instantaneous perturbation ΔI_{in} happens in I_{in}, and injects a small amount of charge into the LC resonating system. There are three interesting cases:

1. Injection at the peak voltage: The LC system will have an additional energy of $V_A\Delta I_{in}$ and will follow the solution of a higher amplitude (all amplitudes are allowed in the resonator), as shown in Fig. 9.5(a). In this case, the zero crossing point will NOT change, and we only have amplitude modulation, no phase modulation[1]!!!

2. Injection at the zero-crossing point: As the voltage is zero, the injected current will not add any energy, so it has to stay at the same V_A magnitude and frequency ω_0. The additional charge will however help the voltage reach its maximum and the next zero-crossing point faster, as shown in Fig. 9.5(b). This is a phase modulation without amplitude modulation!

3. Injection at random time with random sign of I_{in}: True noise has unpredictable timing and sign (so that the DC level is zero, and no synchronization possible). If I_{in} has true noise, then it would be shown as amplitude and phase modulation with no special preference to either.

[1] The effect to compensate oscillation loss (such as friction or resistance) with least influence to the phase/frequency inaccuracy is known for more than 400 years in making mechanical clocks and watches, where the spring "kick" only happens at the pendulum point with maximum velocity.

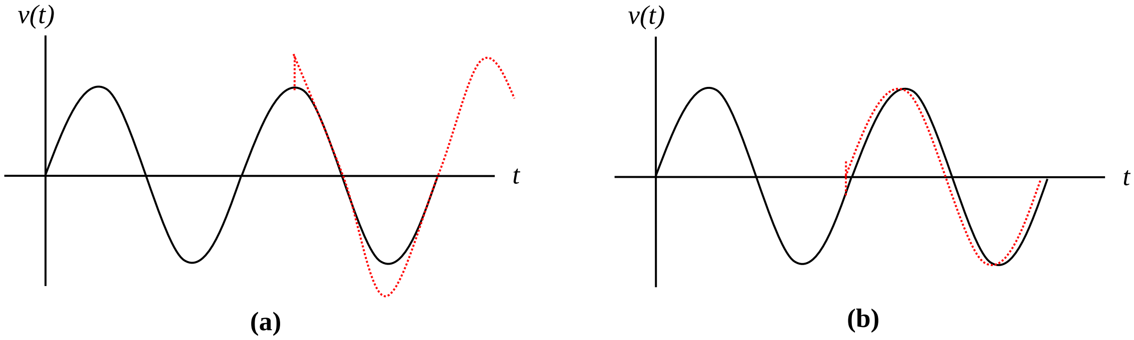


Fig. 9.5. Special cases in perturbation of oscillators: (a) Injection at voltage peak will cause *only* amplitude modulation; (b) Injection at voltage zero-crossing point will cause *only* phase modulation.

9.2.2 Impedance analysis and $1/f^2$ dependence

We will now assume R is the only noise source in this model. This assumption is not too far off, as lossless L and C have to be noiseless. Remember that noises have origins from the 2nd law of thermodynamics where the noise power cannot be retrieved. Any energy conservation system will not have any noise power. We can write down the noise power expression as:

$$\overline{V}_n^2 = 4kTR \cdot BW = 4kTR \cdot \int_0^\infty \left| \frac{Z(f)}{R} \right|^2 df = 4kTR \cdot \frac{1}{4RC} = \frac{kT}{C} \tag{9.1}$$

We can model $Z(f)$ as the typical resonance system with a peak value of R at $\omega_0 = 1/\sqrt{LC}$, where at $Z(f \to 0) = 0$ due to inductance L and $Z(f \to \infty) = 0$ due to capacitance C. The impedance Z and the quality factor Q can be expressed as usual:

$$\frac{1}{Z} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \tag{9.2}$$

$$Q \equiv \frac{P_{stored}}{P_{diss}} = \frac{\omega_0 E_{sig}}{P_{diss}} = \frac{R}{\omega_0 L} = \omega_0 RC \tag{9.3}$$

As the energy stored in L and C alternatively is:

$$E_{sig} = C\overline{V}_{sig}^2 \tag{9.4}$$

The noise-to-signal power for an oscillator can be expressed as: (in dBc, i.e., noise power with respect to the carrier signal)

$$\frac{P_{noise}}{P_{signal}} = \frac{\overline{V}_n^2}{\overline{V}_{sig}^2} = \frac{kT}{E_{sig}} \tag{9.5}$$

By the equi-partition principle, if the noise has two degrees of freedom as in AM and FM, each one will have half of the power in Eq. (9.5) (because the noise has no preference to be in amplitude or phase). Thus the phase noise power in dBc will be:

$$P_{phase\_noise} = \frac{kT}{2E_{sig}} \quad \text{(in dBc)} \tag{9.6}$$

We can recognize from Eq. (9.6) that a significant part of phase noise in an oscillator CAN come from the thermal noise in R, in addition to any other Shot or Flicker noises. Now we have an idea of the total phase noise power in the oscillator, but what will be the spectral distribution? Observing from the integral in Eq. (9.1) with $Z(f)$, we know that the noise will not be white, but focus around ω_0, as shown in Fig. 9.6. What is a good approximation? Unfortunately, no clean analytical form is available. We will use a perturbation analysis on a frequency $\Delta\omega$ higher than ω_0 to build some intuition.

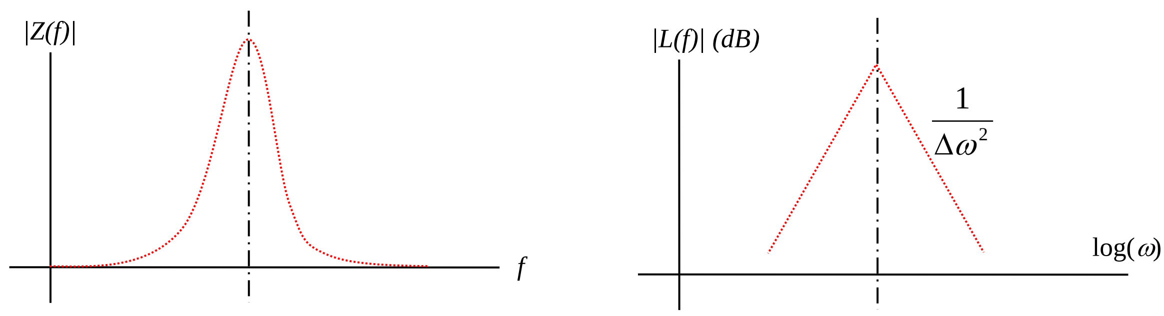


Fig. 9.6. Oscillator modeling by RLC resonators: (a) The impedance behavior; (b) The phase noise power density approximation in a log-log plot.

Close to ω_0 the imaginary part of the impedance in Eq. (9.2) can be approximated by:

$$\text{Im}\left(Z\left(\omega_0 + \Delta\omega\right)\right) \cong -j \cdot \frac{\omega_0 L}{2\left(\frac{\Delta\omega}{\omega_0}\right)} \tag{9.7}$$

With the help in the expression of Q to include R, we can find the magnitude of the impedance:

$$\left|Z\left(\omega_0 + \Delta\omega\right)\right| \cong R \cdot \frac{\omega_0}{2Q\Delta\omega} \tag{9.8}$$

Equation (9.8) reveals two things: (1) $|Z|$ close to ω_0 is inversely proportion to $\Delta\omega$, and Higher Q will have a faster drop in impedance with respect to $\Delta\omega$. As we know the total phase noise power from Eqs. (9.5) and (9.6), we can now estimate the phase noise power density as a function of $\Delta\omega$ close to ω_0 in the following expression by observing Eq. (9.1):

$$\mathcal{L}(\Delta\omega) = \frac{2kT}{P_{sig}} \cdot \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2 \tag{9.9}$$

Notice that $\mathcal{L}$ is typically expressed in unit of dBc/Hz, where "c" reminds us that the phase noise power density is normalized to the oscillating carrier P_{sig}. The $1/\Delta\omega^2$ dependence comes from the thermal noise here, not Flicker. Why is this thermal noise not white? This is due to the thermal noise in R interacting with the resonator, which gives it the specific frequency dependence (a preference to be centered around ω_0).

9.3 Empirical functions for the phase noise in oscillators

However, the Flicker noise will also interact with the oscillator. The Flicker noise from surface conduction interaction is originally very close to zero frequency. If the oscillator is first thought as a delta function in ω_0, then the nearly DC Flicker noise will simply be shifted to around ω_0. However, some further interaction with the thermal noise will happen, similar to how the phase noise spread interacts with any signals, as illustrated in Fig. 9.2. An empirical expression is often invoked (named as Leeson approximation), when the Flicker noise is not too dominant:

$$\mathcal{L}(\Delta\omega) = \frac{2FkT}{P_{sig}} \cdot \left[1 + \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right] \cdot \left(1 + \frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right) \tag{9.9}$$

We can see that Eq. (9.9) will contain a $1/\Delta\omega^3$ term when $\Delta\omega$ is very small. It will then become either $1/\Delta\omega$ or $1/\Delta\omega^2$, depending on the value of $\omega_0/2Q$ and $\Delta\omega_{1/f^3}$, as shown in Fig. 9.7. When $\omega_0/2Q$ is large, it takes a larger $\Delta\omega$ to reduce the middle term to 1, and the dependence by $1/\Delta\omega^2$ will last longer. The parameter F is another parameter to account for the increased phase noise in the $1/\Delta\omega^2$ region due to the interplay between the thermal and Flicker noises.

Notice that when Q is not very large (i.e., the thermal noise from R in the resonator system is significant) and the active energy replenishing unit does not have a lot of Flicker noise (i.e., $\omega_0/2Q >> \Delta\omega_{1/f^3}$), we will have a transition from $1/f^3$ to $1/f^2$ region before the phase noise hits the additive thermal noise floor. On the other hand, when Q is very large (i.e., the thermal noise from R is small or the oscillator itself has very high Q like a crystal oscillator) and the active energy replenishing unit has significant Flicker noise (a surface-channel MOSFET), or equivalently $\omega_0/2Q < \Delta\omega_{1/f^3}$, then we will have a transition from $1/f^3$ to $1/f$ region with a very long tail before the phase noise hits the additive thermal noise floor. This is also shown in Fig. 9.7.

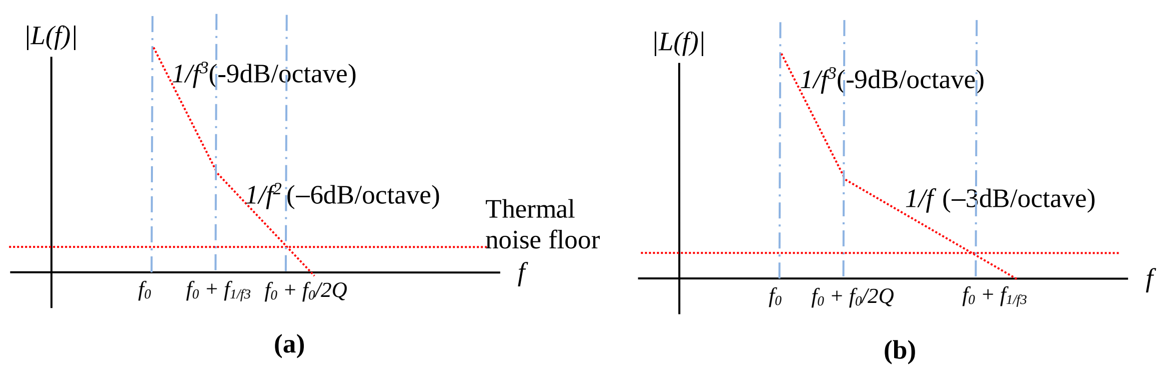


Fig. 9.7. Leeson's empirical model for phase noise: (a) $f_0/2Q > f_{1/f3}$ (higher thermal noise and lower Flicker noise); (b) $f_0/2Q < f_{1/f3}$ (lower thermal noise, i.e., large Q and higher Flicker noise)

In actual measurements when $f_0/2Q$ and $f_{1/f3}$ are not far apart, we often get the phase noise tail to be a function of $1/f^{\alpha}$, where α is between 1 and 2 (a typical value can be 1.8). If one attempts to measure the Flicker noise directly close to DC, we often obtain an α that is between 1 and 2 as well. This is not because Flicker noise is far away from $1/f$, but because the measurement equipment often has a limited Q and some noises as well. It is very difficult for any oscillator to maintain a high Q when the frequency is very low. The thermal noise of the oscillator in the measurement equipment is thus mixed into the given noise response to push α to be significantly larger than 1.