ECE 4880: RF Systems

Fall 2016

Chapter 6: Interplay Between Noise and Nonlinearity

Reading Assignments:

1. W. F. Eagan, *Practical RF System Design*, Wiley, 2003, Chap. 5.

Game Plan for Chap. 6: Interplay between noise and nonlinearity

1. Illustration of the problem caused by noise-nonlinearity interplay
2. Gaussian noise and 2nd-order nonlinearity
3. 3rd-order nonlinearity and its influence on frequency-division modulation
4. Instantaneous spurious-free dynamic range (ISFDR)

6.1 Illustration of noise-nonlinearity interplay

To introduce the interplay between nonlinearity and noise, we will begin with the most serious scenario in a qualitative way, and then develop tangible quantitative treatment from the simplest second harmonic and Gaussian noise to push further. Consider a channel contains data modulation and phase noise from the LO mixing in the transmitter in Fig. 6.1.

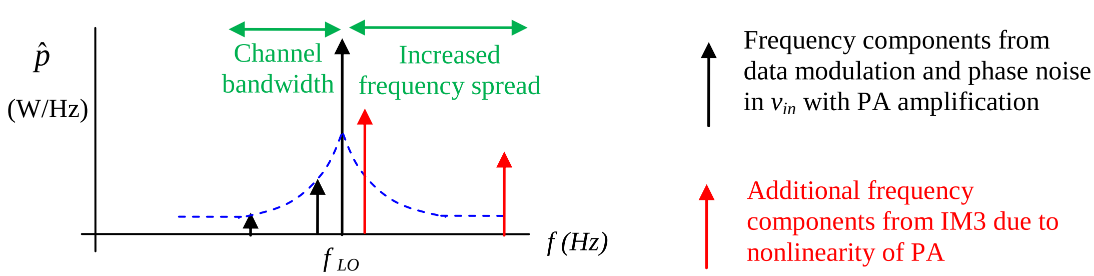

Fig. 6.1. Illustration of interplay between IM3 and bandwidth by data modulation and phase noise. Two frequency components originally in v_{in} are plotted in the black arrow. The IM3 nonlinearity can magnify (depending on a_3 and B/A) and spread them (Δf becomes $2\Delta f$).

The LO carrier frequency is denoted by a large delta-function-like arrow, and the bandwidth has double-side bands from data and phase noise before going through the nonlinearity of the power amplifier. Assume the power amplifier has uniform gain within all frequency in the channel, and the only in-band nonlinearity is IM3 with intermodulation at $(2f_a - f_b)$ and $(2f_b - f_a)$. Let's further assume the LO carrier has much larger magnitude than any other signal in the bandwidth of interest. Now if the carrier is the $B\cos\varphi_b$ term, then for IM3 in Eq. (6.1):

$$v_{out,IM3} = \frac{a_3}{4} 3\left[A^2 B\cos(2\varphi_a - \varphi_b) + AB^2\cos(2\varphi_b - \varphi_a)\right] \qquad (6.1)$$

only the second term of $3a_3/4[AB^2\cos(2\varphi_b - \varphi_a)]$ is important, as $B >> A$. We notice that for the scenario here the LO carrier will have three effects on the frequency components originally in v_{in}:

1. The component at the left side of f_{LO} will now be on the right side, and vice versa (this is true only if the LO signal is much stronger). Please recall that the receiver mixer will see the noise and interference in the image as well (Fig. 4.6).

2. The frequency spread will be two times larger.

3. The IM3 magnitude $3a_3/4\times AB^2$ can be larger or smaller than the original component with A.

Remember that the data that the receiver needs to correctly retrieve is ONLY in the data modulation. The phase noise, together with the IM3 generation, will make the retrieval contain distortion in analog, or bit error in digital.

6.2 Gaussian noise and 2nd-order nonlinearity

Now let's use the simplest case of 2nd-order nonlinearity with constant noise in the bandwidth of interest. We will use both the positive and negative frequencies (positive and negative traveling waves) here for the general case, as the cosine function in the two-tone analysis in Chap. 5 contains both positive and negative frequencies:

$$\cos(\omega_a t + \theta_a) = \frac{e^{j(\omega_a t+\theta_a)} + je^{-j(\omega_a t+\theta_a)}}{2} \qquad (6.2)$$

In Fig. 6.2, a Gaussian noise in the bandwidth *BW* (to distinguish from the magnitude of the two-tone signal) will be considered first (Gaussian noise is the white noise sampled in a given band). Remember that the 2nd-order nonlinear term gives:

$$v_{out,2nd} = a_2 v_{in}^2 = a_2\left\{\underbrace{\frac{A^2+B^2}{2}}_{DC} + \underbrace{\frac{A^2}{2}\cos 2\varphi_a + \frac{B^2}{2}\cos 2\varphi_b}_{H2} + \underbrace{AB[\cos(\varphi_a - \varphi_b) + \cos(\varphi_a + \varphi_b)]}_{IM2}\right\} \qquad (6.3)$$

This contains a DC term, 2nd harmonic and intermodulation 2 (IM2), as plotted together with the fundamental $a_1 v_{in}$ in Fig. 6.2. Notice that IM2 will cause a triangular shape in spectrum. This is due to multiplication in the real-time space resulting in the convolution in the spectral space, or we can view it from the density of components. The only component to give $2f_c + BW$ is the pair at $f_c + BW/2$ and $-f_c - BW/2$ while at $2f_c$, any two components such as $f_c + \Delta f$ and $-f_c + \Delta f$ with difference at $2f_c$ will contribute.

The IM2 component of the two-tone signals of $v_A(\omega) = A\cos\varphi_a = A\cos(\omega_a t+\theta_a)$ and $v_B(\omega) = B\cos\varphi_b = B\cos(\omega_b t+\theta_b)$ at any given frequency f can thus be written as the convolution:

$$v_{IM2}(f) = a_2 \cdot v_A(f) \otimes v_B(f) = a_2 \cdot \int_{-\infty}^{\infty} v_A(x) v_B(f-x) dx \qquad (6.4)$$

In this general definition, we can see the triangular spectrum for IM2 in Fig. 6.2 can be generated with a sliding v_B offset by f upon v_A, where both v_A and v_B are the input spectrum. The peak around $f = 0$ will be two times of the peak around $f = 2f_c$. Notice that in this practice, we have calculated the convolution ignoring the possible phase difference, i.e., we have considered all IM2 terms in a coherent state. Therefore, this establishes an upper bound for IM2, as any incoherence will decrease the resulting magnitude. If all phases of the IM2 terms are random, then we will approximately have half of the power of the coherent case.

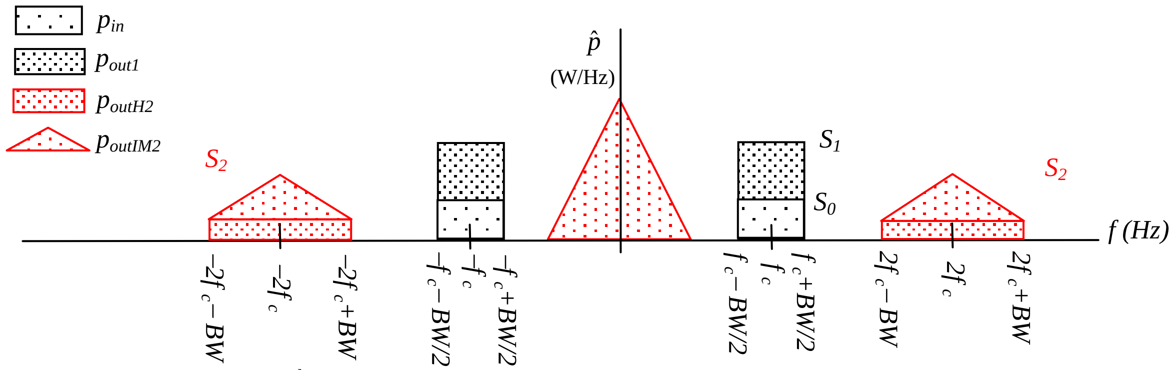

Fig. 6.2. Interplay of 2nd-order nonlinearity and Gaussian noise. We will model the nonlinear voltage transfer function here as $v_{out} = a_1 \cdot v_{in} + a_2 \cdot v_{in}^2$. The input v_{in} is a flat spectrum centered around f_c with S_0 magnitude BW bandwidth.

We can make one more complication to approach a more realistic signal. Assume that we have a large monotone component at f_c and $-f_c$ at the input, as shown in Fig. 6.3. We can use the superposition principle to add in the convolution for this delta function. The direct 2nd-harmonic delta-like and two rectangular boxes will add up with the triangular form to give the final shape of noise-IM2 spectrum around $2f_c$.

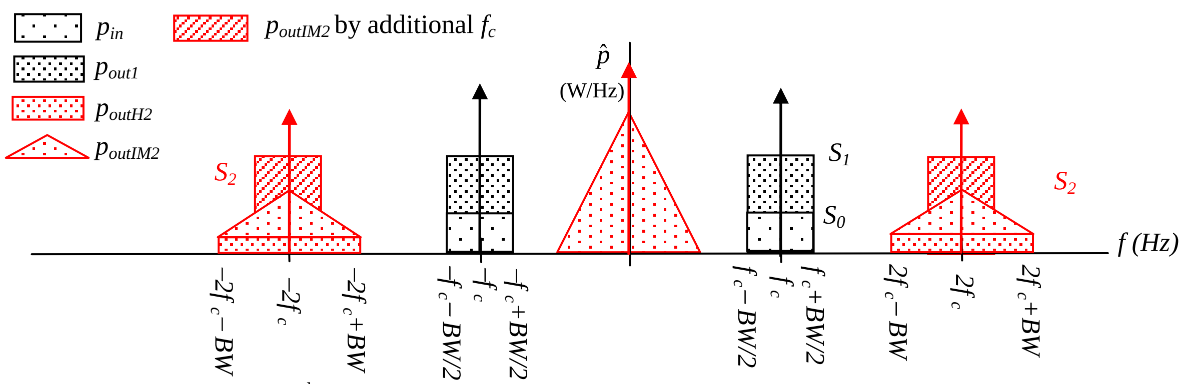

Fig. 6.3. Interplay of 2nd-order nonlinearity and Gaussian noise and a delta-like function at f_c. Notice the additional components in comparison with Fig. 6.2

Equation (6.4) describes the multiplication of two signals in the time domain and convolution in the frequency domain. We should see it soon again when we treat the phase noise in LO.

6.3 Gaussian noise and 3rd-order nonlinearity

Now we can take a "simplified" look at the 3rd-order nonlinearity. You can probably imagine this will be rather complicated not only because more terms in Eq. (5.10), but also because the two-time convolution to get any analytical expressions. As the phase coherence can cause even more complications, we will only have qualitative treatment here. For IM3 in Fig. 6.4, using the slide-window point of view, we can see that the output spectrum of the input in Fig. 6.2 will have a parabolic shape with 3×*BW* spread.

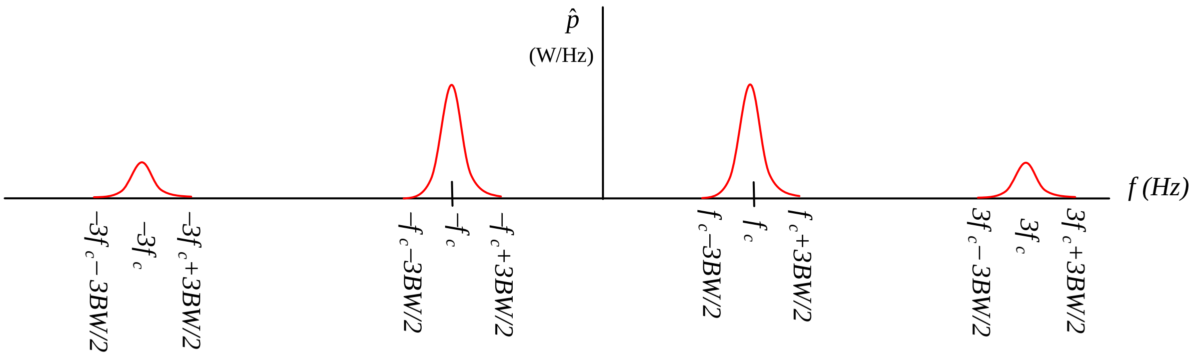

Fig. 6.4. Interplay between IM3 and noise. The peak around f_c is about 3 times higher than the peak around $3f_c$. Three main consequences: image frequency generation, spread-over spectrum and waveform dependence on signal strength.

6.4 Spur-free dynamic range

As a figure of merits for the RF transceiver hardware, we often would like to compare them by the instantaneous spur-free dynamic range (ISFDR), which is used for technology choices. "Instantaneous" means no further adaptive algorithmic techniques like gain control, noise cancellation or orthogonality have been applied yet. "Spur-free" describes the upper signal limit where nonlinearity-caused "spurs" (often IM3 and desensitization limited) are still insignificant. ISFDR is then set by the difference of upper and lower input signal limits, where the lower signal limit is set by "thermal noise" so that the transceiver output can still recognize an intended signal, and the upper limit by the transceiver nonlinearity where distortion is still no larger than the tolerable thermal noise. ISFDR has NOT considered the ambient interference, self jamming or other non-white noises either, but ISFDR shows a balance presented by noise and nonlinearity for the module cascade at hand.

If we assume the upper limit of spurs is mainly by IM3, we can see from Fig. 6.5 the "spur-free" limit can be guaranteed when the input p_{inmax} does not generate more IM3 spurs than the output noise p_{outmin} by p_{inmin} (noise limited). By the triangular relation in Fig. 6.5 of the dB – dB plot, we can write (all in dB, so p is capitalized as a reminder):

$$ISFDR = P_{in\max} - P_{in\min} = \frac{2}{3}(IIP3 - P_{in\min}) \qquad (6.5)$$

where *IIP3* is the input power at the intercept point of fundamental and IM3 signals.

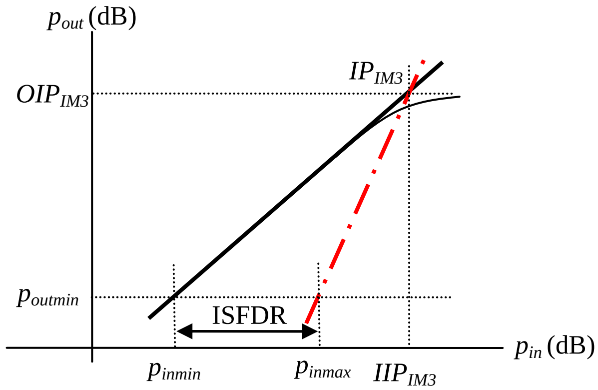

Fig. 6.5. Illustration of instantaneous spur-free dynamic range (ISFDR) for a given RF module. If the minimum recognizable input power p_{inmin} is limited by the noise at its output p_{outmin}, the maximum p_{inmax} without significant IM3 spurs is when $p_{outIM3} = p_{outmin}$. We then define ISFDR = $p_{inmax} - p_{inmin}$.

Example 1: For a Wi-Fi receiver (IEEE 802.11n) with 40MHz bandwidth, cascade noise figure at 8dB and IIP3 = –3dBm (rather low, set usually by the LNA), we can calculate the p_{inmin} and ISFDR as:

$$P_{in\min} = -174dBm + 76dB + 8dB = -90dBm$$

- –174dBm: Thermal noise power per Hz at room temperature.
- 76dB: $10\log_{10}(40\times10^6)$
- 8dB: Additional noise by the cascade noise figure. Notice that the noise factor is defined as the ratio of SNR_{in}/SNR_{out}, and hence we can obtain the noise-limited p_{inmin} by adding the noise figure to the thermal noise floor. 6 – 8dB is a typical noise figure for RF receivers, as LNA often has noise figure around 3 – 6 dB, and a gain of 15 – 20 dB to mitigate the larger noise figure in the following modules.

$$ISFDR = \frac{2}{3}(IIP3 - P_{in\min}) = 58dB\,.$$

This is a typical dynamic range for Wi-Fi receivers.

Example 2: For a GPS receiver with 4kHz bandwidth, cascade noise figure at 8dB and IIP3 = –3dBm (notice that most RF components are similar, but the filter bandwidth is much smaller), we can calculate the p_{inmin} and ISFDR as:

$$P_{in\min} = -174dBm + 36dB + 8dB = -130dBm$$

This is a typical GPS receiver sensitivity. When your RF transceiver is designed properly, you will find your system is limited by the fundamental thermal noise. We then have,

$$ISFDR = \frac{2}{3}(IIP3 - P_{in\min}) = 85dB\,.$$

This is a typical dynamic range for GPS. Notice that the bandwidth is a fundamental choice among data rate, multiple-access, noise and nonlinearity-based spurs. We can make a receiver to be very sensitive by lowering the bandwidth (76dB in Example 1) or decreasing the noise temperature (–174dBm in Example 1), but the dynamic range will not increase as large. As suggested by Eq. (6.5), for every dB increase in receiver sensitivity, the dynamic range will increase only 2/3 dB with a constant IIP3. The other 1/3 dB is given to the noise-nonlinearity interplay.