ECE 4880: RF Systems

Fall 2016

Chapter 3: Nonideal Expressions in Gain and Reflection

Reading Assignments:

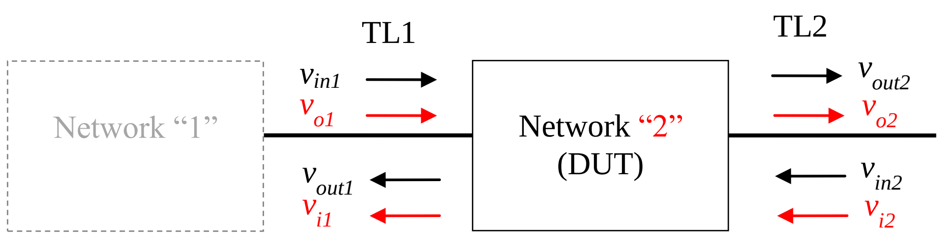
1. W. F. Eagan, *Practical RF System Design*, Wiley, 2003, Chap. 2.
2. S. Amakawa, N. Ishihara and K. Masu, "A Through-only De-embedding Method for On-Wafer Characterization of Multiport Networks", *Intech,* 2010 (pdf file on line), pp. 13 – 17.

Game Plan for Chap. 3: Gain and Reflection

1. Generalized bilateral wave propagation units
2. S parameter matrix and T parameter matrix: Theory and examples
3. Asymptotic form in unilateral wave propagation unit and lumped unit
4. Implication of the use of S and T matrices in signal chain analyses

3.1 Generalized Bilateral Wave Propagation Units

Similar to the transmission line analogy between distributed representation (plane waves as a Maxwell equation solution) and lumped-element representation (I-V solutions of KVL and KCL in the RLC lattice), we can view traveling waves (when the elements and lines are comparable to wavelength) going through processing in a lumped functional block (when the block is thought as a unit much smaller than the wavelength, or only as a topological description). This is the spirit behind the S (scattering) matrix, which derives from free traveling particles going through a scattering (collision). As a 1D traveling waves or plane waves can be superpositioned as V_+ and V_- traveling in opposite directions (notice that this is NOT appropriate in lump-element circuit representation, where the nodal voltage is already the combination of all relevant elements or as a standing wave), we can represent a functional block connecting two traveling waves as:



- Black: Labeled by TL1 and TL2
- Red: Labeled by Network 1 and 2

Fig. 3.1. Definition of traveling-wave voltages connected by a functional block.

Notice that in Fig. 3.1, we just use one line to represent the "signal transmission line". In other drawings, two lines representing Signal (S) and Ground (G) are regarded as equivalent here. Transmission lines can be a coaxial cable, where the center wire is S and the circular shield is G. As integrated circuits or PCB are constructed layer by layer, circular geometry (although an ideal geometry as far as a well-shielded transmission line goes) cannot be made in plane, a co-planar waveguide (CPW) of three metal lines in GSG or a co-planar slotline (CPS) of one metal line S on top of a wider line G beneath is more practical. CPW and CPS are not as well shielded as the coaxial cable, and the line impedance can be affected by a nearby ground plane or even moving human body. Furthermore, CPW and CPS can have small loss to mode conversion (more than the fundamental TEM mode) or radiation (like antenna).

From the module point of view (this is the most straightforward view for the measurement, but do reserve some of your brain capacity for later de-embedding discussions), we can write down the S matrix as:

$$\begin{bmatrix} v_{out1} \\ v_{out2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} v_{in1} \\ v_{in2} \end{bmatrix} \tag{3.1}$$

$$\begin{bmatrix} v_{i1} \\ v_{o2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} v_{o1} \\ v_{i2} \end{bmatrix} \tag{3.2}$$

Each *S* parameter can be a complex number and frequency dependent. From definition, we can have:

$$S_{11} \equiv \left.\frac{v_{out1}}{v_{in1}}\right|_{v_{in2}=0} \equiv \left.\frac{v_{i1}}{v_{o1}}\right|_{v_{i2}=0} \tag{3.3}$$

$$S_{21} \equiv \left.\frac{v_{out2}}{v_{in1}}\right|_{v_{in2}=0} \equiv \left.\frac{v_{o2}}{v_{o1}}\right|_{v_{i2}=0} \tag{3.4}$$

$$S_{12} \equiv \left.\frac{v_{out1}}{v_{in2}}\right|_{v_{in1}=0} \equiv \left.\frac{v_{i1}}{v_{i2}}\right|_{v_{o1}=0} \tag{3.5}$$

$$S_{22} \equiv \left.\frac{v_{out2}}{v_{in2}}\right|_{v_{in1}=0} \equiv \left.\frac{v_{o2}}{v_{i2}}\right|_{v_{o1}=0} \tag{3.6}$$

Let use an example of a passive bandpass filter to illustrate how *S* parameters make intuitive sense and correspond directly to measurement. Let's assume the filter is bilaterally symmetric (a T or π LC network) with the pass band insertion loss of –0.3dB and stop band rejection of –50dB. We also notice that the small insertion loss in the pass band implies good impedance match, and the large rejection in the stop band implies signal reflection in the stop band with big mismatch in impedance, as the input excitation power has to go somewhere in this passive block. For a symmetric network, by definition, $S_{12} = S_{21}$ and $S_{11} = S_{22}$. From the measurement setup point of view as defined in Eqs. (3.3) – (3.6), S_{21} is excitation at TL1 and measurement at TL2, while S_{11} is excitation and measurement both at TL1. Similarly, S_{12} is excitation at TL2 and measurement at TL1, while S_{22} is excitation and measurement both at TL2. We can see that the module view of v_{in1}, v_{in2}, v_{out1} and v_{out2} is most natural here. Let's also assume the filter is composed of "lumped elements" with no physical size or additional time delay, so we only have to deal with the magnitude of the S parameters.

We can also look at an "imaginary" unilateral case (i.e., signals at TL1 can go through to TL2 according to band pass while signals at TL2 cannot go through to TL1 in any frequency). For S_{12} (excitation from TL2), as there is no signal flowing in TL1, you should measure just thermal noise. For S_{22}, there should be total reflection at TL2 as the block should look like an open circuit for unilateral signal block.

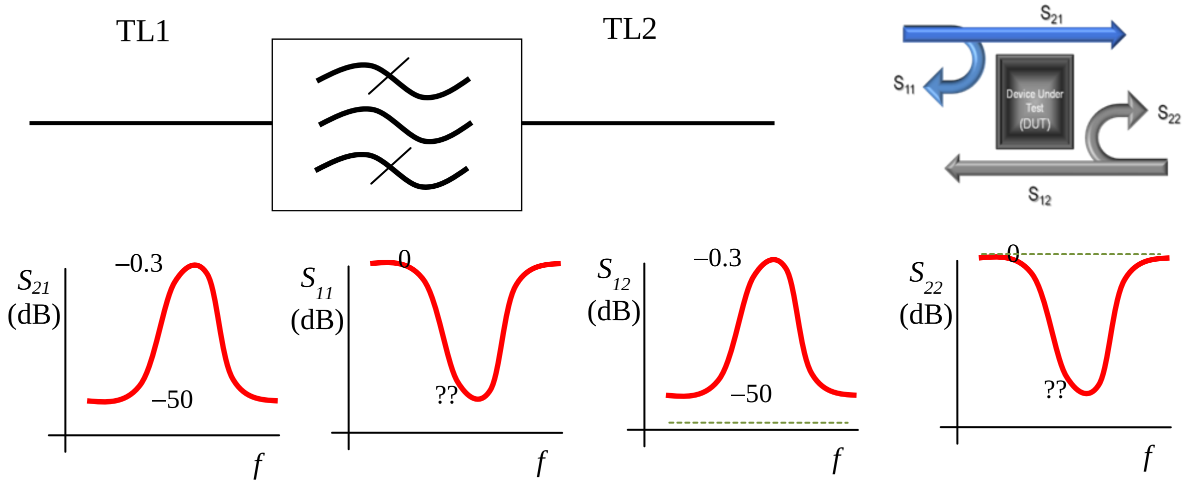


Fig. 3.2. S parameters for a symmetric band pass filter with the pass band insertion loss of −0.3dB and stop band rejection of −50dB. The green dash lines in S_{12} and S_{22} represent ideal unilateral cases where S_{12} will be thermal noise and S_{22} will be total reflection. Notice that we have rough power conversation in this passive block.

Exercise:

Give the S parameter magnitudes of a Class C power amplifier with a gain of 16 dB in the pass band, and a loss of −20dB in the stop band. We assume the gain is unilateral, and the reverse gain is very small and at most −3dB. Notice also that the power does not need to conserve within the signal lines, as there is DC input to the module.

3.2 Normalization of Signals Before More Matrix Representations

Before we go on, we will look at the case of various normalization for convenience in the S parameter measurements. As most device under tests (DUT) have input and output impedance matched at the intended frequency range, we can represent voltage as magnitude, peak-to-peak, RMS, and impedance-normalized RMS. Assume a monotone sinusoidal voltage traveling wave, we can write:

$$v_m \cos(\omega t + \theta) = \mathrm{Re}\left(v_m e^{j(\omega t + \theta)}\right) \qquad (3.7)$$

Or in phasor notation:

$$\hat{v}(t) = \hat{v}_p e^{j\omega t} \quad \text{with} \quad \hat{v}_p = v_m e^{j\theta} \qquad (3.8)$$

Or in RMS notation:

$$\tilde{v}_{RMS} = \frac{v_m}{\sqrt{2}} e^{j\theta} \quad \text{with} \quad v_{RMS} = \frac{\tilde{v}_{RMS}}{\sqrt{Z_0}} \qquad (3.9)$$

All these magnitude v_m, peak-to-peak v_{pp}, phasor $\hat{v}_p$, unnormalized RMS $\tilde{v}_{RMS}$, and normalized RMS v_{RMS} will express the power p in the traveling wave of Eq. (3.7) as:

$$p = \frac{v_m^2}{2Z_0} = \frac{v_{pp}^2}{8Z_0} = \frac{|\hat{v}_p|^2}{2Z_0} = \frac{|\tilde{v}_{RMS}|^2}{Z_0} = |v_{RMS}|^2 \qquad (3.10)$$

Depending on the context, we may use any of these expressions differed by a constant.

3.3 Other Network Module Parameters and Representations

3.3.1 S parameters and T parameters

In RF systems, there is often more than one module component in the carrier frequency: for transmitters, mixer, (possible filter), power amplifier and antenna; for receivers, antenna, LNA, (possible filter), and mixers. One of the most inconvenient problems for S parameters is that they cannot be cascaded directly as matrix multiplication, as shown in Fig. 3.3. By observing Eqs. (3.1) and (3.2), we can understand that in order to cascade functional block with module matrix multiplication, we need to reorder the variables so that variables of TL1 and TL2 are their own vectors.

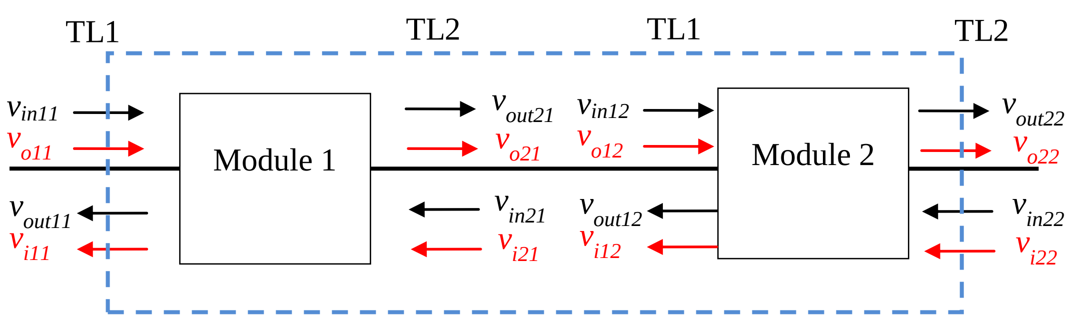


Fig. 3.3. Cascading functional blocks. We have added one more index to denote the traveling waves related to the Module number. We need matrix representations that can deal with module cascade intuitively.

This is where the T matrix comes in. We will define the T matrix for Module 1 and Module 2 in Fig. 3.3. Note that the last subscript in the v signals denotes the module number and is omitted below, so that the notation is more comparable to Eq. (3.2). Also, we will use the variable names from the signal flow in view of the modules, so that T matrix can be cascaded.

$$\begin{bmatrix} v_{o1} \\ v_{i1} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} v_{o2} \\ v_{i2} \end{bmatrix} \qquad (3.11)$$

By definition and as an example,

$$T_{22} \equiv \left. \frac{v_{i1}}{v_{i2}} \right|_{v_{o2}=0} \qquad (3.12)$$

Now we will derive the relation between the T and S parameters. From Eq. (3.2), in order to evaluate v_{i1} against v_{i2} at $v_{o2} = 0$, we have:

$$\begin{aligned} &S_{21}v_{o1} + S_{22}v_{i2} = 0 \\ &v_{i1} = S_{11}v_{o1} + S_{12}v_{i2} = -\frac{S_{11}S_{22}}{S_{21}}v_{i2} + S_{12}v_{i2} \end{aligned} \qquad (3.13)$$

Rearranging, we can write:

$$T_{22} = S_{12} - \frac{S_{11}S_{22}}{S_{21}} \qquad (3.14)$$

Through very similar procedures, we can find the relations for all T parameters as functions of S parameters as:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{S_{21}} \begin{bmatrix} 1 & -S_{22} \\ S_{11} & S_{12}S_{21} - S_{11}S_{22} \end{bmatrix} \qquad (3.15)$$

Or vice versa:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{T_{11}} \begin{bmatrix} T_{21} & T_{11}T_{22} - T_{12}T_{21} \\ 1 & -T_{12} \end{bmatrix} \qquad (3.17)$$

From the definition of Eq. (3.11), we can see the T matrix can be multiplied together for two modules in Fig. 3.3. With the last subscript denotes the module number, we can write the T parameters for the cascaded Module 1 and Module 2 as:

$$\begin{bmatrix} v_{o11} \\ v_{i11} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} v_{o22} \\ v_{i22} \end{bmatrix} = \begin{bmatrix} T_{111} & T_{121} \\ T_{211} & T_{221} \end{bmatrix} \underbrace{\begin{bmatrix} T_{112} & T_{122} \\ T_{212} & T_{222} \end{bmatrix} \begin{bmatrix} v_{o22} \\ v_{i22} \end{bmatrix}}_{\begin{bmatrix} v_{o12} \\ v_{i12} \end{bmatrix} = \begin{bmatrix} v_{o21} \\ v_{i21} \end{bmatrix}} \qquad (3.16)$$

Exercise:

For the symmetrically bilateral passive bandpass filter, draw the T parameters as a function of frequency. Observe if the T matrix is still symmetrical? You should now appreciate S parameters for individual components and T parameters for cascading!!!

3.3.2 S parameters and Y parameters: Concepts and relations

Remember that S and T parameters relate the traveling waves on TL1 and TL2. The T matrix is similar to the ABCD matrix in the EM solution, but we will leave that for your own review. If we treat the traveling wave as a standing wave or in the lump-element circuit, we can transform the traveling waves to *I* and *V* on TL1 and TL2 by the equivalent lump-element circuits with Y parameters in Eq. (3.17) and Fig. 3.4. Notice that the voltage is the two traveling waves adding up (similar to the standing wave, or SWR) and the current is the two traveling waves subtracting (opposite directions of travel). Notice that we have assumed that the input and output ports have Y_0 as the impedance.

$$
\begin{aligned}
I_1 &= Y_0\left(v_{o1} - v_{i1}\right)\\
V_1 &= v_{o1} + v_{i1}\\
I_2 &= Y_0\left(v_{i2} - v_{o2}\right)\\
V_2 &= v_{o2} + v_{i2}
\end{aligned}
\qquad
\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{3.18}
$$

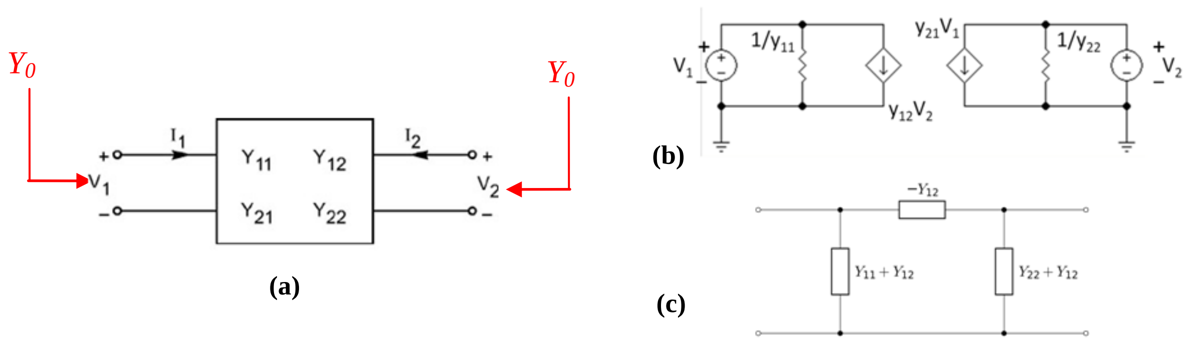


Fig. 3.4. Y parameter network. Remember now the ports are in view of lump-element circuits, instead of traveling wave. (a) The current and voltage relate to the traveling waves by Eq. (3.17). (b) The general linearized Y network. (c) The network when $Y_{12} = Y_{21}$.

With some algebraic manipulation, we can derive the following relationship between the Y and S parameters as:

$$
\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{Y_0}{(1+S_{11})(1+S_{22}) - S_{12}S_{21}} \begin{bmatrix} (1-S_{11})(1+S_{22}) + S_{12}S_{21} & -2S_{12} \\ -2S_{21} & (1+S_{11})(1-S_{22}) + S_{12}S_{21} \end{bmatrix}
$$

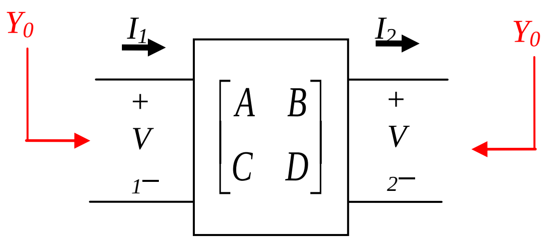
$$
\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{(1+Z_0Y_{11})(1+Z_0Y_{22}) - Z_0^2Y_{12}Y_{21}} \begin{bmatrix} (1-Z_0Y_{11})(1+Z_0Y_{22}) + Z_0^2Y_{12}Y_{21} & -2Z_0Y_{12} \\ -2Z_0Y_{21} & (1+Z_0Y_{11})(1-Z_0Y_{22}) + Z_0^2Y_{12}Y_{21} \end{bmatrix} \tag{3.19}
$$

For completeness, you can write down the Z parameters under very similar assumptions and derivation. We have (and intuitively so):

$$
[Y] = [Z]^{-1}; \quad \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{Z_{11}Z_{22} - Z_{12}Z_{21}} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix} \tag{3.20}
$$

Although you can treat S, T, Y and Z parameters to be algebraic manipulations, they offer views of the functional module in fundamentally different ways. S and T parameters are formulated with traveling waves on the transmission lines, with S parameters giving you intuitive insights in DUT measurements and with T parameters giving you the convenience of cascading. Y and Z parameters are formulated with voltages and currents on circuit nodes in lumped-element circuits. Transmission lines only approaches lumped-elements in frequencies significantly lower than the Bragg frequency, or when the lattice elements are much shorter than the wavelength. Therefore, Y and Z parameters have additional frequency assumptions in mind!

Although Y and Z matrices give a good link to the circuit view, they cannot be multiplied when modules are concatenated, just as in the case of S parameters. We can use a similar strategy in the I-V variables, as shown in Fig. 3.5
:



$$I_1 = Y_0\left(v_{o1} - v_{i1}\right)$$
$$V_1 = v_{o1} + v_{i1}$$
$$I_2 = Y_0\left(v_{o2} - v_{i2}\right)$$
$$V_2 = v_{o2} + v_{i2}$$

Fig. 3.6. The ABCD matrix for cascading the impedance network.

We will still assume that the ports 1 and 2 have impedance of Y_0, and we define the ABCD matrix as:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (3.21)$$

Please be careful that I_2 is defined in the **opposite direction** with the Y and Z matrix to enable cascading.

We can convert the ABCD matrix to S, T, Y and Z matrices for convenient views. For example, the conversion between Y and ABCD matrices are shown below. More conversion formula can be found in the Appendix.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{-1}{Y_{21}} \begin{bmatrix} Y_{22} & 1 \\ Y_{11}Y_{22} - Y_{12}Y_{21} & Y_{11} \end{bmatrix} \qquad (3.22)$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{B} \begin{bmatrix} D & BC - AD \\ -1 & A \end{bmatrix} \qquad (3.23)$$

3.3.3 De-embedding procedures of RF component measurements

Until less than 15 years ago, most RFIC and MMIC researchers look at the measurements of device-under test (DUT) with additional “error” network from the measurement setup, as shown in Fig. 3.5. You will notice that there are a lot of parasitic elements and connections to and from the GSG (ground-signal-ground) connection of the DUT on chip, which can significantly distort the S parameters seen at the network analyzer. If DUT is on a PCB (such as from Mini Circuits), the parasitic elements are simpler (due to better impedance match along the signal path), but some distortion is still unavoidable, especially for conditions where impedance match is not possible (e.x., the stop band in a passive filter).

To isolate the contribution of the parasitic network, the conventional way (valid for more than 60 years) is to use up to four or five dummy structures called SOL-T. S (short), O (open) and L (load) are performed on each single port, and T (through) is performed between the two ports in consideration. Between the signal and ground terminals of a single port, S (short) is a shorting metal line shoring, O (open) is a perceived open circuit (to avoid any capacitance or magnetic coupling), and L (Load) is a standard 50Ω load. Between the two ports, T (Through) is a direct cascade of two lines from the two ports as though

DUT is infinitesimally small. Two parasitic networks (with many RLC components in the lump-element network) from Port 1 to the left of DUT and from Port 2 to the right of DUT can be assumed, and the network analyzer measured the S parameters of all networks (DUT, SOL for each of Port 1 and Port 2, and T between Port 1 and Port 2). An optimization scheme is run to separate the response from DUT and the left and right parasitic networks. For single-port SOL, we obtain S11 (notice Port 2 is not connected) as a complex number, and for two-port T, we obtain the full four S parameters as complex numbers. We therefore obtain 10 = 3 + 3 + 4 complex numbers for each frequency. The unknown is the S or T parameters for the left and right parasitic networks as shown in Fig. 3.5(e), which has 8 complex numbers in total. Therefore, SOL-T is an over-specified problem, and will need some optimization procedure. We can use five lumped elements in each of the left and right parasitic network to avoid the over-specification, but then further assumption is needed for the connectivity topology assumption. Very often when the assumed parasitic network topology is not correct or sufficient, the parasitic component values will be frequency dependent.

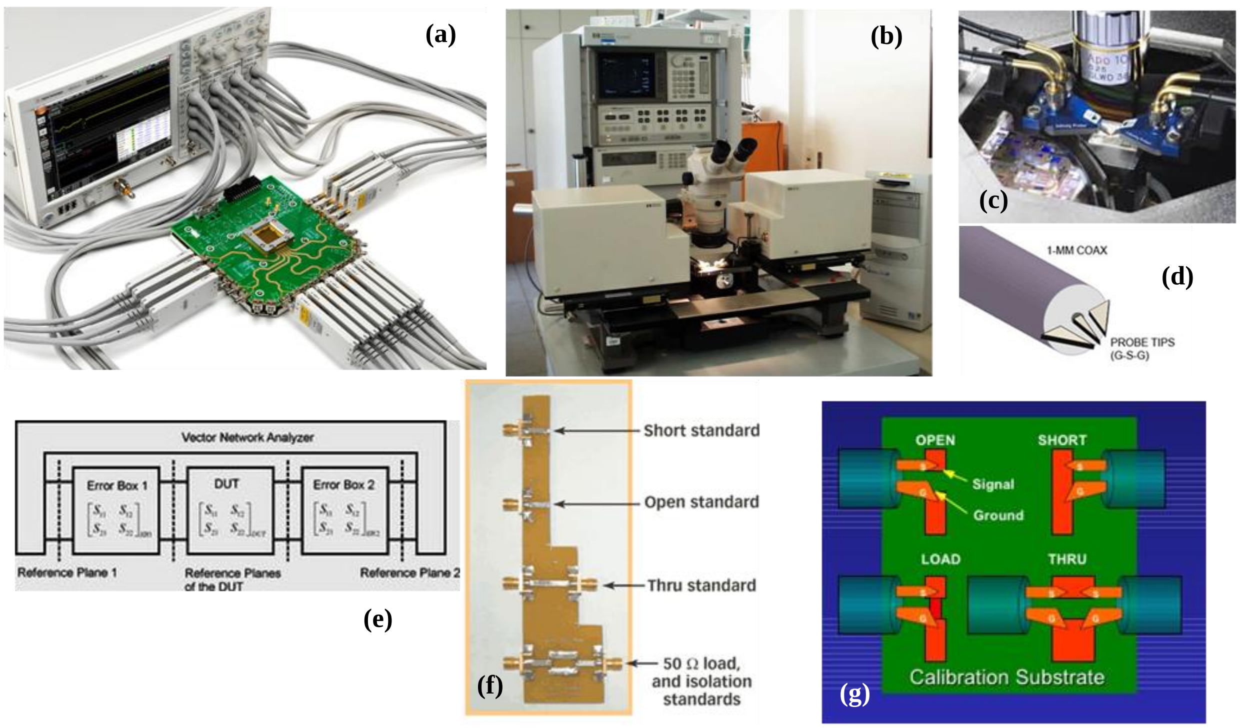


Fig. 3.5. Conventional network analyzer measurements for functional blocks: (a) DUT on PCB; (b) DUT on chip (and the large microwave probe station); (c) the GSG microwave probes for connection to DUT on die; (d) Schematics for connection between coaxial cables and GSG pads; (e) The "error" network sandwiching the DUT; (f) PCB SOLT tests; (g) On-chip SOLT tests.

However, if you observe the setup from a distance, you will soon recognize that it is not appropriate to force a lump-circuit network to the connection. As for the frequency of interest, the network has much larger physical dimension than the wavelength! The manipulation of the lumped elements in the standing-wave point of view (as well as the Y and Z parameters) is very confusing.

Most network analyzer companies such as Agilent (now spin off the microwave part to Keysight) used to provide SOLT characterization kits, but had issues with the "calibration substrate", as the parasitic network needs to go all the way up to DUT connections (and different DUTs on PCB or chip have different connections). From about 15 years ago, the "traveling wave" distributive network has emerged to be the right view to model the “error” network, instead of an assumed lumped-element network. It actually simplifies the representation AND measurements, and has become the more popular microwave “de-embedding” method. “De-embedding” here means the elimination of the influence of the left and right “error network” in Fig. 3.5(e) and Fig. 3.6, which is embedded in the overall network analyzer measurement procedure.

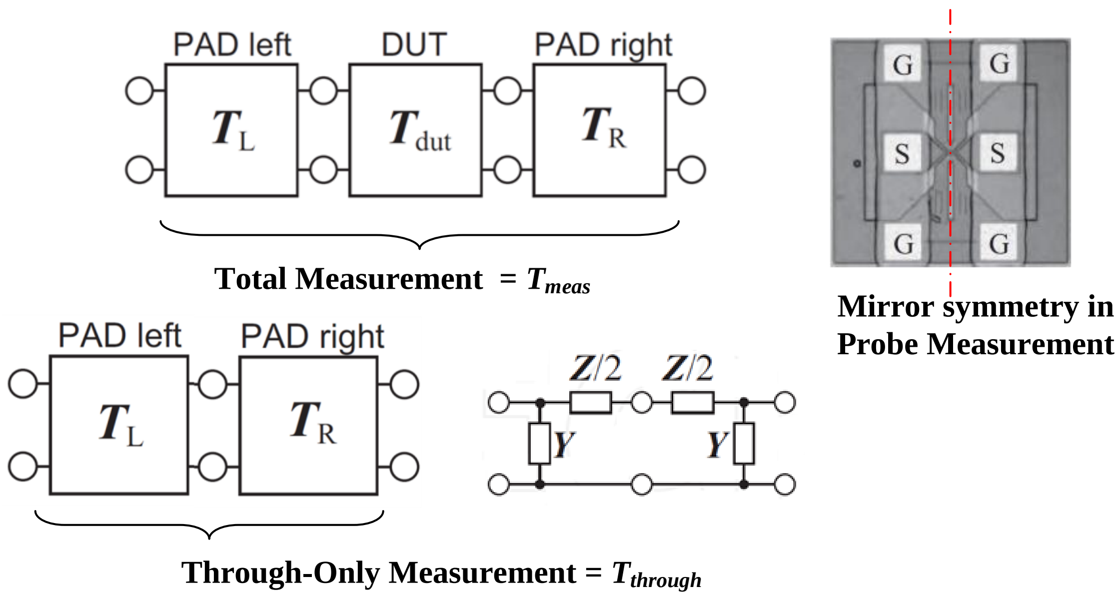


Fig. 3.6. Distributive view of “Through-Only” de-embedding. The TL and TR can be decomposed from the through-only measurements by considering symmetry in the physical connection.

From Fig. 3.6, we can derive the relationship of the DUT and de-embedding through structures as:

$$\begin{aligned} T_{meas} &= T_L T_{DUT} T_R \\ T_{through} &= T_L T_R \end{aligned} \qquad (3.24)$$

By taking the mirror geometry in the physical structure (we will need to use the mirror impedance method), we can determine the unique T_L and T_R from $T_{through}$. From measuring $S_{through}$ (which will be converted to $T_{through}$), we get 4 complex numbers. In general, T_L and T_R will need 8 complex numbers for full description. However, due to the mirror symmetry, there are actually only 4 independent complex numbers to be determined with an assumed network for T_L and T_R. Therefore, the problem is not over-specified. We can then finally have T_{DUT}, clean and simple as:

$$T_{DUT} = T_L^{-1} T_{meas} T_R^{-1} \qquad (3.25)$$

As the T matrix preserves the traveling wave view in its left and right TL1 and TL2 port, further cable consideration can be taken into account in a straightforward manner! We have made network analyzer for a long time, and finally converge to the “Through-Only” (TL) de-embedding without a black-box optimization procedure. My guess is the confusion originated from the lumped-element and distributive views! We have tried to preserve the “assumed parasitic circuit network” in the lump-element view for way too long!

That said, for the DUT itself in the representation of the S or T parameters, it is an "instantaneous" functions joining TL1 and TL2. When the cables and pads are "de-embedded" out, we can still assume some circuit network (an instantaneous transfer and reflection function for the traveling wave) whose parameters will be calibrated from the de-embedded S or T parameters. S-O-L-T structures inside the two GSG pads or PCB connections will be helpful when multiple circuit elements need to be calibrated. Notice that in least-square-fitting of any function, the number of equation is always equal to the number of variables. For a given function f_i that depends on random variables x_j and parameters p_k, we can define least-square-deviation (LSD) as:

$$LSD = \sum_i \left(f_i\left(x_j; p_k\right) - \hat{f}_i\left(\hat{x}_j\right)\right)^2 \qquad (3.26)$$

where $\hat{f}_i\left(\hat{x}_j\right)$ is the i-th measured function (say S parameters here) for given values of random variables x_j (say the nodal voltage or current or frequency). The k-th parameter p_k will be determined by:

$$\forall k, \qquad \frac{\partial LSF}{\partial p_k} = 0; \qquad \frac{\partial^2 LSF}{\partial p_k^2} > 0 \qquad (3.27)$$

3.4 Implication of the use of S and T matrices in signal chain analyses

3.4.1 Unilateral signal chain and feedthrough capacitor

Although the T matrix is convenient for cascading in a signal chain of traveling waves joined by the function modules, it cannot represent unilateral modules in the reverse signal direction for obvious reasons. For S parameters, this is simply either $S_{21} = 0$ or $S_{12} = 0$, depending on the signal direction allowed. For numerical simulation of a signal chain, such singularity in the T parameters is mitigated by adding a small cross capacitor, similar to adding a large resistance for DC stability to floating node in SPICE simulation. The network parameters of adding such feedthrough capacitance can be derived according to Fig. 3.7.

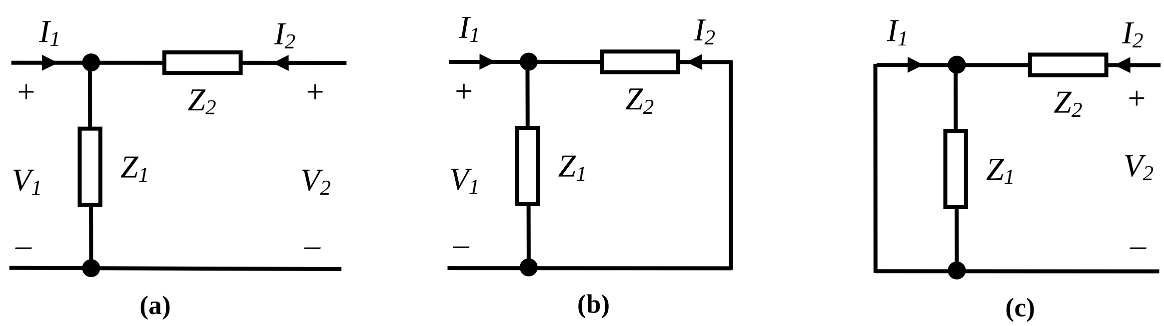


Fig. 3.7. Adding a feedthrough element to the reverse signal transfer in unilateral modules. (a) A generalized L network; (b) Finding $I_1(V_1)$ and $I_2(V_1)$; (c) Finding $I_1(V_2)$ and $I_2(V_2)$.

By using the definition of the Y parameters, we can write down: $Y_{11} = (I_1/V_1)_{V2=0}$; $Y_{21} = (I_2/V_1)_{V2=0}$; $Y_{12} = (I_1/V_2)_{V1=0}$; $Y_{22} = (I_2/V_1)_{V1=0}$;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_1} + \frac{1}{Z_2} & -\frac{1}{Z_2} \\ -\frac{1}{Z_2} & \frac{1}{Z_2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (3.28)$$

If we have $Z_1 = \infty$ in the ideal reverse of the unilateral unit and $Z_2 = 1/j\omega C$ for the cross capacitor, we then have:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = j\omega C \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad (3.29)$$

Looking back, we can confirm ourselves that the Y parameters come from linearization of small monotone AC excitation, which directly corresponds to the inverted-π circuits in Fig. 3.8:

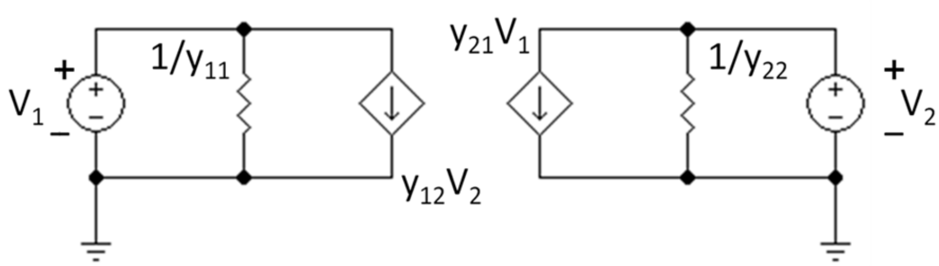


Fig. 3.8. Equivalent inverted-π circuits for the Y parameters by definition.

3.4.2 Insertion gain/loss and the S and T parameters

Finally, we can look back at the simple gain definition when we treat the ideal signal chain. Assuming TL1 and TL2 have matched impedance to the module under consideration in Fig. 3.1. The transfer gain is:

$$g = \frac{\left(\frac{v_{o2}^2}{Z_0}\right)}{\left(\frac{v_{o1}^2}{Z_0}\right)} = |S_{21}|^2 = \frac{1}{|T_{11}|^2} \qquad (3.30)$$

The "insertion gain" or "insertion loss" has slightly different definition, especially under the impedance mismatch condition.

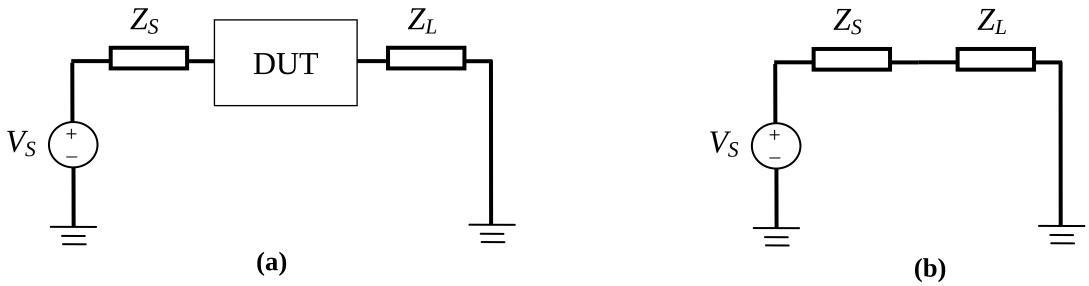


Fig. 3.9. The insertion loss is the ratio of the power delivered to Z_L with (a) or without (b) DUT inserted.

For example, a passive bandpass filters has a passband insertion loss of −0.3dB and a stopband insertion loss of −60dB as shown in Fig. 3.2. In the stop band, the impedance of DUT is NOT matched, and the use of insertion loss is clearer than the transfer gain.

Appendix: Conversion Tables for Matrix Representations

(From Pozar’s book)

	S	Z	Y	$ABCD$
S_{11}	S_{11}	$\frac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\frac{2Z_{12}Z_0}{\Delta Z}$	$\frac{-2Y_{12}Y_0}{\Delta Y}$	$\frac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\frac{2Z_{21}Z_0}{\Delta Z}$	$\frac{-2Y_{21}Y_0}{\Delta Y}$	$\frac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\frac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\frac{Y_{22}}{\lvert Y\rvert}$	$\frac{A}{C}$
Z_{12}	$Z_0\frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\frac{-Y_{12}}{\lvert Y\rvert}$	$\frac{AD-BC}{C}$
Z_{21}	$Z_0\frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\frac{-Y_{21}}{\lvert Y\rvert}$	$\frac{1}{C}$
Z_{22}	$Z_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\frac{Y_{11}}{\lvert Y\rvert}$	$\frac{D}{C}$
Y_{11}	$Y_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{22}}{\lvert Z\rvert}$	Y_{11}	$\frac{D}{B}$
Y_{12}	$Y_0\frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{12}}{\lvert Z\rvert}$	Y_{12}	$\frac{BC-AD}{B}$
Y_{21}	$Y_0\frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{21}}{\lvert Z\rvert}$	Y_{21}	$\frac{-1}{B}$
Y_{22}	$Y_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{11}}{\lvert Z\rvert}$	Y_{22}	$\frac{A}{B}$
A	$\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{\lvert Z\rvert}{Z_{21}}$	$\frac{-1}{Y_{21}}$	B
C	$\frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{1}{Z_{21}}$	$\frac{-\lvert Y\rvert}{Y_{21}}$	C
D	$\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	D

$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$; $|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$; $\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21}$; $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$; $Y_0 = 1/Z_0$.