# ECE 4880 RF Systems Fall 2015 

Final Exam Solution

Name:
Net ID: $\qquad$
Load reflection coefficient $\Gamma_{L}$ at the load as: $\Gamma_{L}=\frac{V_{-}}{V_{+}}=\frac{Z_{L} / Z_{o}-1}{Z_{L} / Z_{o}+1}$
$\left[\begin{array}{ll}T_{11} & T_{12} \\ T_{21} & T_{22}\end{array}\right]=\frac{1}{S_{21}}\left[\begin{array}{cc}1 & -S_{22} \\ S_{11} & S_{12} S_{21}-S_{11} S_{22}\end{array}\right] \quad\left[\begin{array}{ll}S_{11} & S_{12} \\ S_{21} & S_{22}\end{array}\right]=\frac{1}{T_{11}}\left[\begin{array}{cc}T_{21} & T_{11} T_{22}-T_{12} T_{21} \\ 1 & -T_{12}\end{array}\right]$
$\left[\begin{array}{ll}Y_{11} & Y_{12} \\ Y_{21} & Y_{22}\end{array}\right]=\frac{Y_{0}}{\left(1+S_{11}\right)\left(1+S_{22}\right)-S_{12} S_{21}}\left[\begin{array}{cc}\left(1-S_{11}\right)\left(1+S_{22}\right)+S_{12} S_{21} & -2 S_{12} \\ -2 S_{21} & \left(1+S_{11}\right)\left(1-S_{22}\right)+S_{12} S_{21}\end{array}\right]$
$\left[\begin{array}{ll}S_{11} & S_{12} \\ S_{21} & S_{22}\end{array}\right]=\frac{1}{A+B Y_{0}+C / Y_{0}+D}\left[\begin{array}{cc}A+B Y_{0}-C / Y_{0}-D & 2(A D-B C) \\ 2 & -A+B Y_{0}-C / Y_{0}+D\end{array}\right]\left[\begin{array}{cc}A & B \\ C & D\end{array}\right]=\frac{1}{Y_{21}}\left[\begin{array}{cc}-Y_{22} & -1 \\ Y_{12} Y_{21}-Y_{11} Y_{22} & -Y_{11}\end{array}\right]$

Thermal noise at room temperature: $-174 \mathrm{dBm}+10 \log _{10}(\mathrm{BW} / 1 \mathrm{~Hz})$



$$
I_{1 d B c o m p}=I I P_{I M 3}-9.64 \mathrm{~dB} \text { when only IM3 is dominant. }
$$

Nonlinear Taylor coefficients: $A_{\text {IIPIM } 2}=\left|\frac{a_{1}}{a_{2}}\right| \quad ; A_{\text {IIPIM } 3}^{2}=\frac{4}{3}\left|\frac{a_{1}}{a_{3}}\right|$

$$
\begin{array}{ll}
\frac{1}{O I P_{I M 3, c a s}}=\frac{1}{g_{2} \cdot O I P_{I M 3,1}}+\frac{1}{O I P_{I M 3,2}} & \text { (IM3 adding coherently) } \\
\frac{1}{O I P_{I M 3, c a s}^{2}}=\frac{1}{\left(g_{2} \cdot O I P_{I M 3,1}\right)^{2}}+\frac{1}{O I P_{I M 3,2}^{2}} \quad \text { (IM3 adding randomly) }
\end{array}
$$

The noise factor of the cascade: $f_{c a s}=f_{1}+\sum_{k=2}^{N} \frac{f_{k}-1}{\prod_{i=1}^{k-1} g_{i}}$
The Leeson empirical approximation for the LO phase noise: $L(\Delta \omega)=\frac{2 F k T}{P_{\text {sig }}} \cdot\left[1+\left(\frac{\omega_{0}}{2 Q \Delta \omega}\right)^{2}\right] \cdot\left(1+\frac{\Delta \omega_{1 / f^{3}}}{|\Delta \omega|}\right)$

Part I. Multiple Choices (MC) and Fill-ins. There can be more than one correct choices in each MC question. Wrong choices can deduct the points, but you will not get a negative mark. Please write your MC answers clearly to the left blank space to minimize grading errors. $\mathbf{4}$ pts each.

1. (Quarter wavelength impedance transform) For a transmission line at quarter wavelength $\lambda / 4$ of a given frequency $\omega_{0}$ with characteristic impedance of $Z_{0}$, the load (complex number) is $Z_{L}$ and the impedance looking into the transmission line can be represented by $Z_{l} . Z_{0}, Z_{L}$ and $Z_{l}$ are all in unit of $\Omega$ ).

(a) $Z_{L}=Z_{l}$.
(b) $Z_{L}=1 / Z_{l}$.
(c) $Z_{L} \cdot Z_{l}=Z_{0}{ }^{2}$.
(d) If $Z_{L}$ is inductive (positive imaginary part), then $Z_{l}$ will be capacitive.
(e) If $Z_{L}$ is open circuited, then $Z_{l}$ will contain a nonzero imaginary part.
(f) If $Z_{L}$ is short circuited, then $Z_{l}=Z_{0}$.

Answer: (c)(d).
Quarter wavelength is half circle in the Smith Chart. Remember that the impedance or admittance is normalized by $Z_{0}$ in the Smith Chart. If $Z_{L}$ is open circuited, then $Z_{l}$ is short circuited.
2. (Discrete transmission lines) For the discrete-element transmission line shown below, if we match the unit elements $L \Delta z$ and $C \Delta z$ to the distributed line characteristic impedance: $Z_{0}=\sqrt{\frac{L}{C}}$ and define $v=\frac{1}{\sqrt{L C}}$. For an AC signal with $f_{0} \ll f_{\text {bragg }}$,


$$
f_{\text {bragg }}=\frac{1}{\pi \sqrt{L \Delta z C \Delta z}}=\frac{1}{\pi \sqrt{L C} \Delta z}=\frac{v}{\pi \Delta z}
$$

(a) The wavelength $\lambda_{0}$ corresponding to $v / f_{0}$ needs to be much larger than $\Delta z$ for voltage waveform propagation.
(b) When we excite a step signal at the left end at $t=0$, this signal can appear at the right end instantaneously as there is no loss by resistance.
(c) We cannot use the Smith Chart to calculate the impedance transformation from load to source as $v$ may not be the speed of light.
(d) For a broadband AC response, the discrete transmission line will behave like a band-pass filter around $f_{\text {bragg }}$.
(e) For a broadband AC response, the discrete transmission line will behave like a low-pass filter with the corner frequency around $f_{\text {bragg }}$.

Answer: (a)(e).

The discrete transmission line will behave exactly like a distributed transmission line (or TEM mode in a waveguide) when $f_{0} \ll f_{\text {bragg }}$. For broadband, it will be like a low-pass filter as higher frequency cannot transmit but only has evanescent mode.
3. (Circulator in a RFID reader) For the RFID transceiver below, the circulator has 1 dB pass loss and -60 dB isolation. The transmission loss from reader to tag at the given distance is -45 dB . The power amplifier (PA) will emit 30 dBm . The low-noise amplifier (LNA) has $I_{1 d B c o m p}$ at $-10 \mathrm{dBm} . f_{L O}=1 \mathrm{GHz}$.


Block diagrams for RFID. Questions 3 8 all used the same parameters and conditions.
(a) The tag will receive about -30 dBm power.
(b) The LNA will receive about -62 dBm power from tags.
(c) The self jamming will be at the level of -30 dBm power.
(d) The LNA will have a signal-to-interference ratio of 15 dB .
(e) The LNA will have a signal-to-interference ratio of -5 dB .
(f) We expect significant signal distortion at the receiver path.

Answer: (b)(c).
Tag received power $=30 \mathrm{dBm}-1 \mathrm{~dB}-45 \mathrm{~dB}=-16 \mathrm{dBm}$ power.
The receiver LNA power from tag $=30 \mathrm{dBm}-1 \mathrm{~dB}-45 \mathrm{~dB}-45 \mathrm{~dB}-1 \mathrm{~dB}=-62 \mathrm{dBm}$.
Self jamming $=30 \mathrm{dBm}-60 \mathrm{~dB}=-30 \mathrm{dBm}$.
SIR $=-62 \mathrm{dBm}-(-30 \mathrm{dBm})=-32 \mathrm{~dB}$.
Even with the self jamming, the signal now is relatively small at -30 dBm away from $I_{1 d B c o m p}=-$ 10 dBm by 20 dB , so the distortion is expected to be small.
4. (PA noise and coherent receiver) The PA above has a gain of 20 dB , noise figure of $8 \mathrm{~dB}, I I P_{I M 2}=$ 25 dBm , and $I I P_{I M 3}=15 \mathrm{dBm}$. The PA output needs an SNR of 20 dB for its commands to be correctly decoded by the tag.
(a) The SNR at the PA input needs to be around 12 dB .
(b) The noise power at the PA input needs to be smaller than -18 dBm .
(c) For thermal noise at $-174 \mathrm{dBm} / \mathrm{Hz}$, the bandwidth needs to be smaller than 10 MHz to satisfy the transmitter SNR requirements.
(d) The noise figure of the downconverter mixer is not critical as long as LNA has reasonable gain.
(e) As LO is shared between the transmitter and the receiver, the phase noise of LO will not distort the baseband information.
(f) As LO is shared between the transmitter and the receiver, the LNA nonlinearity will not cause any DC drift in the direct conversion.

Answer: (b)(d).
$N F=S N R_{\text {in }} / S N R_{\text {out }}$, and therefore $\mathrm{SNR}_{\text {in }}$ needs to be at least 28 dB . Through any amplifier, SNR will degrade, not improve. As the input signal and noise will both be amplified, and there are additional noises in the amplifier circuits.

The input signal power to PA needs to be 10 dBm . Therefore the noise power needs to be $10 \mathrm{dBm}-$ $28 \mathrm{~dB}=-18 \mathrm{dBm}$.

Thermal noise total power at 10 MHz bandwidth will be: $-174 \mathrm{dBm}+70 \mathrm{~dB}=-104 \mathrm{dBm}$, which is far away from -18 dBm . Therefore, no bandwidth limitation due to noise here.

Coherent receiver (shared LO for transmitter and receiver; hearing echoes) will not need synchronization between the transmitter and the receiver, but if there is phase noise in LO, it will pollute the signal twice, instead of being cancelled. LNA nonlinearity will cause DC drift, regardless of LO synchronization.
5. (PA nonlinear parameters) Use the same PA in question 4 for questions 5 and 6. Fill in the following PA nonlinear parameters, which are independent of the input level. Give both numerical values AND units. Assume nonlinearity is caused by gain saturation and $Z_{0}=50 \Omega$.

| $O I P_{\mathrm{H} 2}$ | OIP $_{\mathrm{H} 3}$ | $a_{2}$ | $a_{3}$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |

gain $=20 \mathrm{~dB}, I I P_{I M 2}=25 \mathrm{dBm}$ and $I I P_{I M 3}=15 \mathrm{dBm}$.
$a_{1}=10^{20 / 20}=10 ; A_{\text {IIPIM } 2}=\left|\frac{a_{1}}{a_{2}}\right| ; A_{\text {IIPIM } 2}=5.6 \mathrm{~V} .\left|a_{2}\right|=1.78 \mathrm{~V}^{-1}$.
$A_{\text {IIPIM } 3}^{2}=\frac{4}{3}\left|\frac{a_{1}}{a_{3}}\right| ; A_{\text {IIPIM3 }}=1.8 \mathrm{~V} .\left|a_{3}\right|=4.1 \mathrm{~V}^{-2}$.

| OIP $_{\mathrm{H} 2}$ | OIP $_{\mathrm{H} 3}$ | $a_{2}$ | $a_{3}$ |
| :---: | :---: | :---: | :---: |
| 51 dBm | 39.77 dBm | $-1.78 \mathrm{~V}^{-1}$ | $-4.1 \mathrm{~V}^{-2}$ |

6. (PA nonlinear responses) What would be the correct nonlinear output responses in dBm when the PA output is at 30 dBm ?

| $p_{\text {IM2 }}$ | $p_{\text {H2 }}$ | $p_{\text {IM3 }}$ | $p_{\text {H3 }}$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |

$$
\begin{aligned}
& p_{\text {in }}=10 \mathrm{dBm} ; p_{\text {out }}=30 \mathrm{dBm} . \\
& p_{\text {IIM } 2}=30 \mathrm{dBm}-(25 \mathrm{dBm}-10 \mathrm{dBm})=15 \mathrm{dBm} . p_{\text {H2 }}=p_{\text {IM } 2}-6 \mathrm{~dB}=9 \mathrm{dBm} . \\
& p_{\text {IM } 3}=30 \mathrm{dBm}-(15 \mathrm{dBm}-10 \mathrm{dBm}) \times 2=20 \mathrm{dBm} . p_{\text {H3 }}=p_{\text {IM } 3}-9.54 \mathrm{~dB}=10.46 \mathrm{dBm} .
\end{aligned}
$$

| $p_{I M 2}$ | $p_{H 2}$ | $p_{I M 3}$ | $p_{H 3}$ |
| :---: | :---: | :---: | :---: |
| 15 dBm | 9 dBm | 20 dBm | 10.46 dBm |

7. (Phase noise in LO) You make noise power density measurements for the above LO around 1 GHz and obtain the following table.

| $\Delta f$ | 10 Hz | 100 Hz | 1 kHz | 10 kHz | 100 kHz | 1 MHz |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Noise power <br> density $(\mathrm{dBm} / \mathrm{Hz})$ | -73 | -104 | -124 | -145 | -165 | -164 |

(a) Only thermal noise is dominant over the entire range of measured $\Delta f$.
(b) The corner $1 / \beta^{3}$ frequency is significantly lower than the LO half bandwidth $f_{0} / 2 Q$.
(c) The surface-oriented Flicker noise dominates over thermal noise at $\Delta f=10 \mathrm{kHz}$.
(d) We can extrapolate the noise power density at 1 Hz to be around $-63 \mathrm{dBm} / \mathrm{Hz}$.
(e) We need to consider the DC drift issue for the RFID transceiver.

Answer: (b)(e).
There is clearly $1 / f^{2}$ to $1 / \boldsymbol{f}^{3}$ transition between $10-100 \mathrm{~Hz}$, so the Flicker corner frequency should be in that range. The extrapolated noise power density at 1 Hz would be around $-43 \mathrm{dBm} / \mathrm{Hz}$. As the noise density at low $\Delta f$ is significant, there would be DC drift issues in the RFID transceiver.
8. (Nonlinearity and noise interplay) The received signal after the downconverter mixer contains many spurious frequency elements (spurs) that make the A/D has high bit error rate. To reduce the number and magnitude of the spurious frequency components,
(a) Decrease $\mathrm{IIP}_{\text {IM3 }}$ of the PA.
(b) Use a larger LO magnitude to boost the signal level to PA.
(c) Reduce the LO phase noise by adding a good-quality bandpass filter.
(d) Increase the noise figure of the receiver mixer.
(e) Increase the $\mathrm{IIP}_{\mathrm{H} 2}$ of the LNA.
(f) Add a bandpass filter between the tag antenna and the tag.

Answer: (c)(e).
Spurs are caused by the interplay of the noise and nonlinearity. We will need to decrease either.
Increase IIP will decrease the nonlinearity. Larger LO can cause the mixer nonlinearity to kick in. Add a good-quality bandpass filter before LO injection will help, but this is often hard as LO is often used to select the frequency. Filters with large tunable range is much harder than tunable LO.

Adding a bandpass filter to the tag antenna will cause minimal effect in reducing spurs, as the signal is already weak at the tag.
9. (Use of RF parameter matrices) For the S, T, Y, Z and ABCD matrices in the RF module representation, which choice(s) below are correct? Unilateral means output will not affect input.
(a) The matrix representation is for linear, small-signal calculation as a function of frequency.
(b) All S and T parameters are dimensionless.
(c) ABCD parameters are dimensionless.
(d) For unilateral amplifiers, $S_{12}$ and $Y_{12}$ are 0 .
(e) For unilateral amplifiers, the S and T matrices are not defined in the reverse signal direction.
(f) For unilateral amplifiers, the determinant of the ABCD matrix is 0 .
(g) For unilateral amplifiers, the determinant of the $S$ matrix is always 0 .

Answer: (a)(b)(d)(f).
10. (Cascade of RF modules) Two RF amplifiers are cascaded to obtain a larger gain. Amp1 has $g_{1}=$ $10 \mathrm{~dB} ; N F_{1}=2 \mathrm{~dB} ; O I P_{I M 3}=25 \mathrm{dBm}$. Amp2 has $g_{2}=20 \mathrm{~dB} ; N F_{2}=10 \mathrm{~dB} ; O I P_{I M 3}=15 \mathrm{dBm}$. Assume the worst case of IM3 combination in the cascade.

| Cascade | $g_{\text {cas }}(\mathrm{dB})$ | $N F_{\text {cas }}(\mathrm{dB})$ | $O I P_{I M 3, c a s}(\mathrm{dBm})$ |
| :---: | :--- | :--- | :--- |
| Amp1 $\rightarrow$ Amp2 |  |  |  |
| Amp2 $\rightarrow$ Amp1 |  |  |  |

The worst case in IM3 combination is adding coherently.

| Cascade | $g_{\text {cas }}(\mathrm{dB})$ | $N F_{\text {cas }}(\mathrm{dB})$ | OIP $_{\text {IM3,cas }}(\mathrm{dBm})$ |
| :---: | :---: | :---: | :---: |
| Amp1 $\rightarrow$ Amp2 | 30 | 3.95 | 15 |
| Amp2 $\rightarrow$ Amp1 | 30 | 10 | 22 |

Part II. Analysis questions. Please box your final answers clearly. For brief explanation, use 1 - 2 sentences only.
11. (Quadrature hybrid amplifier) For the quadrature hybrid amplifier below with $Z_{0}=50 \Omega$, the individual amplifiers can be described by:

$$
\left[\begin{array}{ll}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{array}\right]_{\text {amp1 }}=\left[\begin{array}{cc}
0.1 & 0 \\
10 & 0.1
\end{array}\right]
$$



RF quadrature hybrid

a) What are the input and output impedance of Amp1 in $\Omega$ ? (2 pts)
$S_{11}=\Gamma_{\text {Lin }}=\frac{V_{-}}{V_{+}}=\frac{Z_{\text {in }} / Z_{o}-1}{Z_{\text {in }} / Z_{o}+1}=0.1$. We can solve to get $Z_{\text {in }}=61 \Omega . Z_{\text {out }}$ is equal to $Z_{\text {in }}$ as $S_{11}=S_{22}$.
b) First assume that Amp1 and Amp2 are identical in every aspect. What are the S parameters for the block in the dash-line box? Remember that S parameters are complex numbers. ( $\mathbf{4} \mathbf{~ p t s}$ )

As all reflections will be cancelled out at $p_{\text {in }}$ and $p_{\text {out }}$ due to the $-180^{\circ}$ phase shift of the two paths. The voltage gain magnitude will remain the same as Amp1 and Amp2 have -3 dB input, and then are combined. However, there is a $-90^{\circ}$ phase shift.

$$
\left[\begin{array}{ll}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{array}\right]_{\text {hybrid }}=\left[\begin{array}{cc}
0 & 0 \\
-10 j & 0
\end{array}\right]
$$

c) The cross capacitance in Amp1 and Amp2 has caused small leakage to have nonzero $S_{12}=-40 \mathrm{~dB}$. $p_{\text {out }}$ is connected to an antenna and leaks in a signal of -20 dBm . Will this signal pollute $p_{\text {in }}$ ? If so, what is the leakage signal power in dBm at $p_{\text {in }}$ ? ( $\mathbf{4} \mathbf{~ p t s}$ )

The signal from the reverse two paths will add up coherently, both with $-90^{\circ}$ phase shift, so it will pollute $p_{\text {in }}$.
$p_{\text {out }}=-20 \mathrm{dBm} ; v_{\text {out }}=0.032 \mathrm{~V}$.
$v_{\text {pin }}=\underbrace{\frac{0.032}{\sqrt{2}} \cdot 0.01 \cdot \frac{-j}{\sqrt{2}}}_{\text {path1 }}+\underbrace{\frac{-0.032 j}{\sqrt{2}} \cdot 0.01 \cdot \frac{1}{\sqrt{2}}}_{\text {path2 }}=-3.2 \times 10^{-4} j$
Leakage power $=\frac{\left|v_{\text {pin }}\right|^{2}}{2 Z_{0}}=1.02 \times 10^{-9} \mathrm{~W}=-60 \mathrm{dBm}$.
d) Now ignore the small effect of $S_{12}$. If you measure the $S$ parameters for Amp2 and find that they are slightly different as below. Estimate the S parameters for the block in the dash-line box now? Assume the quadrature hybrid is still ideal. ( $\mathbf{4} \mathbf{p t s}$ ) For $p_{i n}=0 \mathrm{dBm}$, estimate the power dissipated in Iso $_{\text {in }}$ and Iso out. ( $\mathbf{4} \mathbf{~ p t s ) ~}$
$\left[\begin{array}{ll}S_{11} & S_{12} \\ S_{21} & S_{22}\end{array}\right]_{\text {amp } 2}=\left[\begin{array}{cc}0.1 & 0 \\ 8 & 0.15\end{array}\right]$
For $S_{11}$, it is still matched, so we will have $S_{11}=0$.
We will need to trace the two different paths for $S_{21}$ and $S_{22}$.
$S_{21}=\underbrace{\frac{-j}{\sqrt{2}} \cdot 10 \cdot \frac{1}{\sqrt{2}}}_{\text {path1 }}+\underbrace{\frac{1}{\sqrt{2}} \cdot 8 \cdot \frac{-j}{\sqrt{2}}}_{\text {path2 }}=-9 j$
$S_{22}=\underbrace{\frac{1}{\sqrt{2}} \cdot 0.1 \cdot \frac{1}{\sqrt{2}}}_{\text {path }}+\underbrace{\frac{-j}{\sqrt{2}} \cdot 0.15 \cdot \frac{-j}{\sqrt{2}}}_{\text {path2 }}=-0.025$
$\left[\begin{array}{ll}S_{11} & S_{12} \\ S_{21} & S_{22}\end{array}\right]_{\text {hybrid }}=\left[\begin{array}{cc}0 & 0 \\ -9 j & -0.025\end{array}\right]$
$V_{+}$at pin will be 0.32 V for $0 \mathrm{dBm} p_{i n}$.
At Iso $o_{i n}$, we have the voltage as: $V_{- \text {isoin }}=\underbrace{0.32 \frac{-j}{\sqrt{2}} \cdot 0.1 \cdot \frac{1}{\sqrt{2}}}_{\text {path1 }}+\underbrace{0.32 \frac{1}{\sqrt{2}} \cdot 0.1 \cdot \frac{-j}{\sqrt{2}}}_{\text {path } 2}=-0.032 j$

The power dissipation at the resistor is then: $p_{\text {isoin }}=\frac{\left(V_{- \text {isoin }}\right)^{2}}{2 Z_{0}}=10.0 \mu \mathrm{~W}=-20 \mathrm{dBm}$.
At Iso $o_{\text {out, }}$, we have the voltage as: $V_{\text {isoout }}=\underbrace{0.32 \frac{-j}{\sqrt{2}} \cdot 10 \cdot \frac{-j}{\sqrt{2}}}_{\text {path1 }}+\underbrace{0.32 \frac{1}{\sqrt{2}} \cdot 8 \cdot \frac{1}{\sqrt{2}}}_{\text {path } 2}=-0.32$
The power dissipation at the resistor is then: $p_{\text {isoout }}=\frac{\left(V_{\text {isoout }}\right)^{2}}{2 Z_{0}}=1.024 \mathrm{~mW}=0.10 \mathrm{dBm}$.
e) If both Amp1 and Amp2 have $I I P_{\mathrm{H} 2}=31 \mathrm{dBm}$ (but their S parameters are slightly different like in part (d)), for $p_{\text {in }}=0 \mathrm{dBm}$, estimate the $2^{\text {nd }}$ harmonic H 2 power at $p_{\text {out }}$. Assume that H 2 will make the total output voltage at Amp1 and Amp2 smaller (i.e., $a_{2}$ is negative). ( $\mathbf{4} \mathbf{~ p t s ) ~ C o m p a r e ~ y o u r ~}$ answer of the H 2 power if $p_{\text {in }}=0 \mathrm{dBm}$ is just fed into Amp1. ( $\mathbf{4} \mathbf{~ p t s ) ~}$

The S parameters are defined with small signals in the linear system, but we can still calculate the $2^{\text {nd }}$ harmonic voltage in the path separately.

Given $I I P_{H 2}=31 \mathrm{dBm}$ and $I I P_{I M 2}=25 \mathrm{dBm}$, we know $A_{\text {IIPH2 }}=5.6 \mathrm{~V}$ and can calculate $a_{2}$
$\left|a_{1}\right|=\left|a_{2}\right| A_{\text {IIPIM } 2}$.
For Amp1, $a_{2}=-1.78\left(\mathrm{~V}^{-1}\right)$
For Amp2, $8 \cdot 11.2=\left|a_{2}\right|(11.2)^{2} ; a_{2}=-1.43\left(\mathrm{~V}^{-1}\right)$
$v_{\text {in }}=0.32 \mathrm{~V}$ at $p_{\text {in }}=0 \mathrm{dBm}$.
$V_{H 2}=\underbrace{-1.78 \cdot\left(0.32 \cdot \frac{-j}{\sqrt{2}}\right)^{2} \cdot \frac{1}{\sqrt{2}}}_{\text {path1 }}+\underbrace{(-1.43)\left(0.32 \frac{1}{\sqrt{2}}\right)^{2} \cdot \frac{-j}{\sqrt{2}}}_{\text {path2 }}=0.060+0.051 j$
$p_{\text {outH } 2}=\frac{|0.060-0.051 j|^{2}}{100}=60 \mu W=-12 \mathrm{dBm}$.
If only Amp1 is used, we know that the linear gain is $20 \log _{10}(10)=20 \mathrm{~dB}$, and thus,
$v_{H 2}=-1.78 \times 0.32^{2}=-0.18 \mathrm{~V} . p_{\text {out } \mathrm{H} 2}=324 \mu \mathrm{~W}=-4.9 \mathrm{dBm}$.
We can see that $p_{\text {outH2 }}$ in the quad hybrid is about 6 dB lower indeed. The difference comes from the slight mismatch in Amp1 and Amp2. This is mainly from the 3dB lower input power to Amp1 and Amp2 when we use the quadrature hybrid.

For careful students, you may ask why $p_{\text {outH2 } 2}$ is off from the other possible calculation of $2 \times p_{\text {out }}-$ $I I P_{H 2}=2 \times 10 \mathrm{dBm}-31 \mathrm{dBm}=-11 \mathrm{dBm}$ ? (Notice that dBm is in power, so the previous equation should be read as power $2 /$ power $=$ power for the unit consistency). This is exactly 6 dBm lower! 6 dBm implies that the voltage has been off by 2 times somewhere. The discrepency originates from the definition how H2/IM2 can be related to $a_{2}$ ! When we derive the relation of
$\left|a_{1}\right|=\left|a_{2}\right| A_{\text {IIPIM } 2}$, two signals of $A$ and $B$ have been used, which indeed means that the available input power is doubled. For a single signal, actually we should have used $\left|a_{1}\right|=\left|a_{2}\right| A_{\text {IIPH2 }}$, which makes the two estimates exactly the same.

If you hope to dig further, a complete resolution of the "terminology" is listed in Appendix H of Egan's book.
12. (Direct-conversion Q-ary transceivers) A direct-conversion Wi-Fi transceiver with 4-bit per symbol Q-ary modulation is shown below.

a) The receiver signal power $P_{R}$ can be modeled by $P_{R}=P_{T} K\left(\frac{d_{0}}{d}\right)^{\gamma}$, where $P_{T}$ is the transmitted power, $K$ and $d_{0}$ are empirical constants, and the parameter $\gamma$ represents the multi-path effect. $\gamma=$ 2 for line-of-sight channels, and the typical three-hop indoor path has $\gamma=6$. By using $P_{T}=$ 30 dBm for both your router and mobile unit, you measure $P_{R}$ for the line-of-sight and typical indoor three-hop situations. Fill in the table below. (4 pts)

| Distance between unit to <br> router | 0.3 m | 1 m | 3 m | 10 m | 30 m |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Receiver signal power at <br> line of sight (dBm) |  | -20 dBm |  |  |  |
| Receiver signal power for <br> three-hop indoor (dBm) |  | -30 dBm |  |  |  |

As $10 \times \log _{10} 3 \cong 5$, we can finish the table quickly as:

| Distance between unit to <br> router | 0.3 m | 1 m | 3 m | 10 m | 30 m |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Receiver signal power at <br> line of sight (dBm) | -10 dBm | -20 dBm | -30 dBm | -40 dBm | -50 dBm |
| Receiver signal power for <br> three-hop indoor $(\mathrm{dBm})$ | 0 dBm | -30 dBm | -60 dBm | -90 dBm | -120 dBm |

b) For the baseband channel bandwidth of 20 MHz and $36 \mathrm{Mb} / \mathrm{s}$ bit rate, how would you set the lowpass filter corner frequency interfacing with the data converter? What should be the A/D data converter sampling frequency? Briefly explain. (4 pts)

I would set the low-pass filter corner frequency to 40 MHz (tolerance). The sampling frequency needs to be at least 20 MHz . As there are 4 bits per symbol, this is more than sufficient for $36 \mathrm{Mb} / \mathrm{s}$ (error bits, synchronization, pilot overhead, etc.)
c) As the data converter has relatively high frequency, we choose 10 bits for A/D. With no additional variable-gain amplifier, we can only use the LO signal strength to tune the gain in the receiver path. Assume the mixer gain is proportional to the LO voltage magnitude. To accommodate the dynamic range in $P_{R}$ for the indoor three-hop situation, what is the LO voltage tuning ratio? ( $\mathbf{4} \mathbf{~ p t s ) ~}$

We need to accommodate 120 dB difference in receiving power, and the range for $\mathrm{A} / \mathrm{D}$ is at $20 \log _{10} 2^{10}$ $\cong 60 \mathrm{~dB}$. That is, LO needs to have a voltage ratio of 60 dB , or 1,000 . This is rather impractical, and often an explicit VGS will be needed.
d) During the full duplex mode in frequency division, the transmitter and the receiver on the mobile unit are on different channels, which are set up during the initial polling stage. A nearby transmitter of another mobile unit, which can have up to 0dBm leaking to LNA, can however cause received signal desensitization through the LNA nonlinearity. The gain of the LNA is 15 dB . Assume the main nonlinearity of LNA is $3^{\text {rd }}$ order. At $P_{T}=30 \mathrm{dBm}$, estimate the required IIP $_{\text {Im }}$ of the LNA. ( $\mathbf{6 ~ p t s}$ ) The desensitization amplitude can be estimated by:
Amplitude $\left(f_{a}\right)=a_{1} A+\frac{a_{3}}{4}\left(3 A^{3}+6 A B^{2}\right) \cong A\left(a_{1}+\frac{3 a_{3}}{2} B^{2}\right)$. Use $Z_{0}=50 \Omega$. The signal $A$ represents the intended channel which is desensitized by $B$.

To stay away from leakage desensitization, $a_{1}>\left|\frac{3 a_{3}}{2}\right| B^{2}$. The power of the leakage signal is at 0 dBm , and the magnitude is $B=0.32 \mathrm{~V}$. This will give $\left|a_{3}\right|<\frac{2 a_{1}}{3 B^{2}}=105\left(\mathrm{~V}^{-2}\right)$.
$A_{\text {IIPIM } 3}^{2}=\frac{4}{3}\left|\frac{a_{1}}{a_{3}}\right|>0.40 ; A_{\text {IIPIM } 3}>0.63(\mathrm{~V}) . I I P_{\text {IM3 }}>4.0 \mathrm{~mW}$ or 6 dBm.
e) For the DC drift concerns, which of the following choice(s) will increase the DC drift at the receiver A/D converter? ( $\mathbf{4} \mathbf{~ p t s ) ~}$
(a) Increasing the $\mathrm{LO}_{\mathrm{R}}$ leakage to the antenna.
(b) Increasing the $\mathrm{LO}_{\mathrm{T}}$ leakage to antenna.
(c) Increasing the noise figure of the $\mathrm{LO}_{\mathrm{T}}$ mixer.
(d) Decreasing $\mathrm{IIP}_{\mathrm{IM} 3}$ of the PA.
(e) Decreasing $\mathrm{IIP}_{\mathrm{IM} 2}$ of the LNA.
(f) $\mathrm{LO}_{\mathrm{R}, \mathrm{I}}$ and $\mathrm{LO}_{\mathrm{R}, \mathrm{Q}}$ are not exactly differed by $90^{\circ}$.

Answer: (a)(e)(f)

The transmitter is at a different frequency, so $\mathrm{LO}_{\mathrm{T}}$ and PA will not affect the DC drift. They can surely affect the bit error rate by noise and nonlinearity interplay along the entire signal chain.
13. (Dual radio receiver) Assume that your future boss should ask you to add a satellite S-band radio receiver (similar to XM) on top of the existing FM receiver ( $88-108 \mathrm{MHz}$ ) with the minimal cost. The satellite radio uses bandwidth $2.320 \mathrm{GHz}-2.332 \mathrm{GHz}$. The satellite transmitter has agreed to conform to the FM baseband and intermediate frequency at $B W_{b b}=200 \mathrm{kHz}$ and $f_{I F}=10.7 \mathrm{MHz}$, respectively.
a) How many stations in total can be accommodated in the dual radio? (2 pts)
$B W_{S}=12 \mathrm{MHz} ; B W_{F M}=20 \mathrm{MHz}$. There can be in total 160 stations.
b) Do we need image rejection filters between the LO and IF mixers for the radio receiver? Briefly explain. ( $\mathbf{4} \mathbf{~ p t s ) ~}$

As $B W_{S}, B W_{F M}<2 f_{I F}$ for both bands, no in-band signal will be the image for the desired signal.
c) Draw the dual-radio receiver block diagram with an A/D converter of sampling rate at 400 kHz . Use an RF switch for toggling between the FM and S bands. Please be as realistic as possible about the bandwidth of modules while minimizing the hardware cost. ( $\mathbf{6} \mathbf{~ p t s )}$

The RF part (antenna, band filter, mixer and LO) will need to be separated for S and FM bands. Likely we will need two LO's to avoid the tuning range to be too large.


