ECE 3250 **HOMEWORK ASSIGNMENT VIII** **Fall 2015**

1.

(a) Find the frequency response $\widehat{H}$ of the LTI system that has output

$$t \mapsto \left(e^{-t} - 2e^{-3t} + e^{-5t}\right) u(t)$$

when its input is $t \mapsto 8e^{-5t}u(t)$.

(b) Find the impulse response h of the ideal low-pass filter with cutoff frequency Ω_2.

(c) Find the impulse response h of the ideal bandpass filter with lower cutoff frequency Ω_1 and upper cutoff frequency Ω_2.

2. Let $\widehat{H}_1$ and $\widehat{H}_2$ be the frequency responses of ideal low-pass and band-pass filters. Specifically, let

$$\widehat{H}_1(\Omega) = \begin{cases} 1 & \text{if } |\Omega| \leq 3700\pi \\ 0 & \text{otherwise} \end{cases}$$

and

$$\widehat{H}_2(\Omega) = \begin{cases} 1 & \text{if } 1737\pi \leq |\Omega| \leq 7140\pi \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that the LTI system you obtain by cascading the two filters (i.e., input x enters the low-pass filter whose output goes through the band-pass filter, leading to system output y) has a frequency response $\widehat{H}$, and then find $\widehat{H}$.

(b) Write out as a linear combination of cosines and constants the responses of the low-pass filter and the band-pass filter to the A-440 with Fourier series

$$\sum_{k=-\infty}^{\infty} \frac{1}{k^2+1} e^{jk880\pi t} \quad \text{for all } \ t \in \mathbb{R}\ .$$

Also find the fundamental period of each output signal.

(c) Write out as a linear combination of cosines the response of the system in (a) to the signal in (b). Also find its fundamental period.

(d) Find as a time function the response of the low-pass filter to the signal x whose Fourier transform is

$$\widehat{X}(\Omega) = \begin{cases} 3 & \text{if } |\Omega| \leq 4100\pi \\ 0 & \text{otherwise.} \end{cases}$$

(e) Find the Fourier transform $\widehat{Y}$ of the response y of the low-pass filter to the signal x whose Fourier transform is

$$\widehat{X}(\Omega) = \frac{7}{3+j\Omega} \quad \text{for all } \ \Omega \in \mathbb{R}\ .$$

You don't have to find the signal y.

3. Someone tells you that the input signal x and output signal y of a certain continuous-time LTI system are related by the differential equation

$$D^2y(t) + \alpha Dy(t) + 2y(t) = x(t) \quad \text{for all } \ t \in \mathbb{R}\ .$$

(a) Find the system's frequency response $\widehat{H}$ as a function of Ω.

(b) Sketch the magnitude $|\widehat{H}(\Omega)|$ and phase $\phi(\Omega)$ as a functions of Ω for $\Omega \geq 0$ when $\alpha = 3$ and when $\alpha = 1$. Recall that

$$\phi(\Omega) = \tan^{-1}\left(\frac{\mathrm{Im}\{\widehat{H}(\Omega)\}}{\mathrm{Re}\{\widehat{H}(\Omega)\}}\right) \quad \text{for all } \ \Omega \in \mathbb{R}\ .$$

4. Find the frequency response of the continuous-time LTI system whose impulse response h has specification

$$h(t) = \left(e^{-7t} - e^{-11t}\right) u(t) \ \text{ for all } \ t \in \mathbb{R}\ .$$

Also find the system's output signal $S(x)$ when the input x has specification

$$x(t) = 4e^{j13t} \ \text{ for all } \ t \in \mathbb{R}$$

without doing any convolution.

5. Given a causal BIBO stable system with system mapping S, impulse response h, and frequency response $\widehat{H}$, consider driving the system with the x that has specification

$$x(t) = e^{j\Omega_o t}u(t) \ \text{ for all } \ t \in \mathbb{R}$$

where Ω_o is some given frequency. Let y be the output signal that arises.

(a) You can express y as the sum of two terms as follows:

$$y(t) = y_{\mathrm{tr}}(t) + \widehat{H}(\Omega_o)e^{j\Omega_o t} \ \text{ for all } \ t \in \mathbb{R}\ .$$

Find a formula involving h for the signal y_{tr}.

(b) Show that $y_{\mathrm{tr}}(t) \to 0$ as $t \to \infty$. Explain why people call y_{tr} the *transient response* to x and the other term in y the *steady-state* response to x.

6. Let z be a real-valued continuous-time signal whose spectrum is narrowband around frequency Ω_o in the sense that $\widehat{Z}(\Omega) = 0$ unless Ω is within Ω_m of $\pm\Omega_o$, where Ω_o is large and Ω_m is relatively small. The goal of this problem is to show that you can write

$$z(t) = x(t)\cos(\Omega_o t) + y(t)\sin(\Omega_o t) \ \text{ for all } \ t \in \mathbb{R}\ ,$$

where x and y are two real-valued signals bandlimited to within Ω_m.

(a) Let q be the (possibly complex-valued) signal whose Fourier transform has specification

$$\widehat{Q}(\Omega) = \begin{cases} 2\widehat{Z}(\Omega - \Omega_o) & \text{when } |\Omega| \leq \Omega_m \\ 0 & \text{otherwise.} \end{cases}$$

Show that

$$z(t) = \mathrm{Re}\{q(t)e^{j\Omega_o t}\} \ \text{ for all } \ t \in \mathbb{R}\ .$$

(Suggestion: because z is real-valued, $\widehat{Z}(-\Omega) = \overline{\widehat{Z}(\Omega)}$ for all $\Omega \in \mathbb{R}$ by 10.12 in the monograph. Next show that $\overline{q}$ has Fourier transform with specification $\overline{\widehat{Q}(-\Omega)}$ for all $\Omega \in \mathbb{R}$. Now write the real part above as half of something plus its complex conjugate and apply the Frequency-shift rule 10.6 in the monograph.)

(b) Use the result of (a) to write z in two ways: $z(t) = r(t)\cos(\Omega_o t + \phi(t))$ for all $t \in \mathbb{R}$ and $z(t) = x(t)\cos(\Omega_o t) + y(t)\sin(\Omega_o t)$ for all $t \in \mathbb{R}$, where r is nonnegative and x and y are bandlimited to within Ω_m. By this I mean express r, x, y, and $\phi(t)$ in terms of q. Make sure to explain why x and y, known as the *quadrature components* of z, are bandlimited to within Ω_m.

7. Find the frequency response of the discrete-time LTI system whose impulse response h has specification

$$h(n) = \left(7^{-n} - 13^{-n}\right) u(n) \ \text{ for all } \ n \in \mathbb{Z} \ .$$

Please simplify any infinite sums you encounter by using geometric series. Also find the system's output signal $S(x)$ when the input x has specification

$$x(n) = 11 e^{j3n} \ \text{ for all } \ n \in \mathbb{Z}$$

without doing any convolution.