1. A certain discrete-time LTI system with system mapping S has impulse response h with specification

$$h(n) = \frac{(-5)^{-n}}{7} u(n) \ \text{ for all } \ n \in \mathbb{Z} \ .$$

(a) Suppose that $\overline{x}$ is a bounded signal, and that $S(\overline{x}) = \overline{y}$. Suppose d is another bounded signal that acts as a disturbance at the input. Suppose $\|d\|_\infty = \epsilon$. Find an upper bound on the effect of the disturbance at the output when we use $x = \overline{x} + d$ as input, i.e., find an upper bound on

$$\|S(x) - \overline{y}\|_\infty \ .$$

(b) Can you find a disturbance d with $\|d\|_\infty = \epsilon$ that causes the inequality you found in (a) to hold with equality?

2. In each case, determine whether the convolution of x_1 and x_2 exists. In any instance where it does exist, just say why you know it exists — you don't have to compute it. In any instance where it doesn't exist, please show why by trying to compute it.

(a) x_1 is the signal with specification $x_1(t) = u(t+7)$ for all $t \in \mathbb{R}$ and and $x_2 = u$.
(b) x_1 is the signal with specification $x_1(t) = 7$ for all $t \in \mathbb{R}$ and x_2 is the signal with specification $x_2(t) = e^{3t}u(t)$ for all $t \in \mathbb{R}$.
(c) x_1 is the signal with specification $x_1(t) = \cos t$ for all t and x_2 is the signal with specification $x_2(t) = e^{-3|t|}$ for all $t \in \mathbb{R}$.
(d) $x_1 = u - \text{Shift}_7(u)$ and x_2 is the signal with specification $x_2(t) = e^{3t^2}$ for all $t \in \mathbb{R}$.
(e) x_1 is the signal with specification $x_1(t) = e^{-3t}u(t)$ for all $t \in \mathbb{R}$ and x_2 is the signal with specification $x_2(t) = e^{7t^2}u(-t)$ for all $t \in \mathbb{R}$.

In the remaining six problems, find $x_1 * x_2$. Please try to express your answer to Problems 4, 5, and 7 in the form

$$x_1 * x_2(t) = \begin{cases} f(t) & \text{if } \ t \geq 0 \\ g(t) & \text{if } \ t < 0 \ , \end{cases}$$

which is the same as

$$x_1 * x_2(t) = f(t)u(t) + g(t)u(-t) \ \text{ for all } \ t \in \mathbb{R} \ .$$

3. $x_1 = \text{Shift}_1(u)$ and x_2 is the signal with specification $x_2(t) = e^{-3t}u(t)$, $t \in \mathbb{R}$.

4. x_1 and x_2 are signals with specification $x_1(t) = e^{3t}u(t)$ and $x_2(t) = e^{7t}u(t)$ for all $t \in \mathbb{R}$.

5. x_1 and x_2 are signals with specification $x_1(t) = x_2(t) = e^{-3t}u(t)$ for all $t \in \mathbb{R}$.

6. x_1 and x_2 are signals with specification $x_1(t) = e^{5t}$ and $x_2(t) = u(t)$ for all $t \in \mathbb{R}$.

7. $x_1 = u$ and x_2 is the signal with specification

$$x_2(t) = e^{-3|t|} = \begin{cases} e^{-3t} & \text{if } \ t \geq 0 \\ e^{3t} & \text{if } \ t < 0 \ , \end{cases}$$

8. x_1 and x_2 are signals with specification $x_1(t) = 7 + \cos(3t)$ for every $t \in \mathbb{R}$ and

$$x_2(t) = \begin{cases} \frac{3}{2\pi} & \text{if } \ 0 \le t \le 2\pi/3 \\ 0 & \text{otherwise.} \end{cases}$$