ECE 3250 **HOMEWORK ASSIGNMENT III** **Fall 2015**

For many of the problems, you might find the following geometric-series identities useful. In the equations, γ is a real or complex number.

$$\sum_{m=0}^{M} \gamma^m = \begin{cases} M+1 & \text{if } \gamma = 1 \\ \frac{1-\gamma^{M+1}}{1-\gamma} & \text{if } \gamma \neq 1\ . \end{cases}$$

Furthermore,

$$\sum_{m=0}^{\infty} \gamma^m = \frac{1}{1-\gamma}$$

when $|\gamma| < 1$.

In first six problems, find $x_1 * x_2$. Please try to express your answers to all but Problems 4 and 6 in the form

$$x_1 * x_2(n) = \begin{cases} f(n) & \text{if } n \geq 0 \\ g(n) & \text{if } n < 0\ , \end{cases}$$

which is the same as

$$x_1 * x_2(n) = f(n)u(n) + g(n)u(-n-1) \ \text{ for all } \ n \in \mathbb{Z}\ .$$

1. $x_1 = u$ and x_2 is the signal with specification $x_2(n) = 7^n u(n)$, $n \in \mathbb{Z}$.

2. x_1 and x_2 are signals with specification $x_1(n) = 7^{-n}u(n)$ and $x_2(n) = 3^n u(n)$ for all $n \in \mathbb{Z}$.

3. x_1 and x_2 are signals with specification $x_1(n) = x_2(n) = 3^n u(n)$ for all $n \in \mathbb{Z}$.

4. x_1 and x_2 are signals with specification $x_1(n) = u(-n-1)$ and $x_2(n) = 3^{-n}$ for all $n \in \mathbb{Z}$.

5. $x_1 = u$ and x_2 is the signal with specification

$$x_2(n) = 3^{-|n|} = \begin{cases} 3^{-n} & \text{if } n \geq 0 \\ 3^n & \text{if } n < 0\ , \end{cases}$$

6. x_1 and x_2 are signals with specification

$$x_1(n) = (-1)^n = \begin{cases} 1 & \text{if } n \text{ is even} \\ -1 & \text{if } n \text{ is odd}\ . \end{cases}$$

and

$$x_2(n) = \begin{cases} \frac{1}{6} & \text{if } 0 \leq n \leq 5 \\ 0 & \text{otherwise.} \end{cases}$$

7. In each case, verify that the convolution of x_1 and x_2 fails to exist.

(a) $x_1 = u$ and x_2 is the signal with specification $x_2(n) = 3^{-n}$ for every $n \in \mathbb{Z}$.

(b) $x_1 = u$ and x_2 is the signal with specification

$$x_2(n) = 5^{-n}u(-n-1) = \begin{cases} 5^{-n} & \text{when } n < 0 \\ 0 & \text{when } n \geq 0\ . \end{cases}$$

8. In each case, without trying to calculate $x_1 * x_2$, explain why you know $x_1 * x_2$ exists.

(a) x_1 is the signal with specification $x_1(n) = e^{n^2}$ for every $n \in \mathbb{Z}$ and $x_2 = u - \text{Shift}_3(u)$.

(b) $x_1 = u$ and x_2 is the signal with specification

$$x_2(n) = \begin{cases} \cos^2(n) & \text{when } n \geq -17 \\ 0 & \text{when } n < -17 \,. \end{cases}$$

(c) $x_1 = 3u$ and x_2 is the signal with specification $x_2(n) = 1/(|n|!)$ for every $n \in \mathbb{Z}$.

9. Let $h \in \mathbb{R}^{\mathbb{Z}}$ be the signal with specification:

$$h(n) = \begin{cases} (1+r)^n & \text{if } \ n \geq 0 \\ 0 & \text{if } \ n < 0 \,, \end{cases}$$

where $r \in (0,1)$. Let $x \in \mathbb{R}^{\mathbb{Z}}$ be the signal with specification

$$x(n) = \begin{cases} P & \text{if } \ N_o \leq n \leq N_o + 359 \\ 0 & \text{otherwise,} \end{cases}$$

where $P > 0$ and N_o is a nonnegative integer.

(a) Find $\text{Conv}_h(x)$, i.e. find an explicit formula for $\text{Conv}_h(x)(n)$ for every $n \in \mathbb{Z}$.

(b) Suppose you work at a mundane job for thirty years starting in month N_o of your life. You get paid 273 dollars at the beginning of each month and never get a raise or salary reduction. You never spend any of the money you make because you're independently wealthy, but you put all your earnings in a special bank account that earns interest compounded monthly at a monthly rate of .07/12. The interest you earn in month n goes into your account at the beginning of month $n+1$. Find the balance in your bank account on the second day of month n as a function of n, where n ranges over all of $\mathbb{Z}$.