ECE 3250 **HOMEWORK ASSIGNMENT XI** **Fall 2015**

1. Find the z-transform (formula for $X(z)$ along with $(\text{ROC})_X$), if the transform exists, for each signal x specified below. If the transform fails to exist, explain why.

(a) $x(n) = 2^{-n}u(n+1)$ for all $n \in \mathbb{Z}$.
(b) $x(n) = n3^{-(n+1)}u(n) - 7^n u(-n-1)$ for all $n \in \mathbb{Z}$.
(c) $x(n) = \delta(n+3) - \delta(n+5)$ for all $n \in \mathbb{Z}$.
(d) $x(n) = z_o^{|n|}$ for all $n \in \mathbb{Z}$, where z_o is a nonzero complex number satisfying $|z_o| < 1$.
(e) $x(n) = 3^n u(n) - 7^{-n}u(-n-1)$ for all $n \in \mathbb{Z}$.

2. Without evaluating the convolution directly, find the z-transform of the signal $x = x_1 * x_2$, where x_1 and x_2 are the signals with specifications

$$x_1(n) = -13^n u(n) \text{ for all } n \in \mathbb{Z}$$

and

$$x_2(n) = n5^{n-1}u(n) + 3(-7)^n u(n) \text{ for all } n \in \mathbb{Z}\,.$$

3. Find in each case the signal x with the given z-transform.

(a) $X(z) = \frac{z^2}{(z+7)(z-5)} \quad 7 < |z| < \infty$.
(b) $X(z) = \frac{z+1}{(z-1)^2(z+3)} \quad 1 < |z| < 3$.
(c) $X(z) = \sum_{k=0}^{5} 2^{-k} z^{-k} \quad 0 < |z| < \infty$.

4. A causal LTI system has impulse response h with specification

$$h(n) = 2^{-n}u(n) - 3^{-n}u(n) \text{ for all } n \in \mathbb{Z}\,.$$

(a) Find the system's transfer function (formula for $H(z)$ along with $(\text{ROC})_H$). Is the system BIBO stable? Explain both in terms of h and in terms of the transfer function.
(b) Find the system's output signal when the input signal is $x = u$.
(c) Find the system's output signal when the input is the constant signal x with specification $x(n) = 1$ for all $n \in \mathbb{Z}$.
(d) Find the system's output signal when the input is the signal x with specification $x(n) = 3^n$ for all $n \in \mathbb{Z}$.

5. A certain causal LTI system has transfer function

$$H(z) = \frac{z^2}{z^2 - .3z + .02} \quad .2 < |z| < \infty\,.$$

(a) Why couldn't $(\text{ROC})_H$ have been $.1 < |z| < .2$?
(b) Is the system BIBO stable? Explain in terms of the poles of the transfer function.
(c) Find the system's output y when the input x has specification $x(n) = 5(7^n)$ for all $n \in \mathbb{Z}$.
(d) Find the system's impulse response h.

6. Find the Direct Form II and Transposed Direct Form II signal flow graphs for the causal LTI system with transfer function

$$H(z) = \frac{z^2 + 2z + 1}{z^2 - \frac{3}{4}z + \frac{1}{8}} \quad \frac{1}{2} < |z| < \infty \, .$$

Also, find a difference equation of the form

$$y(n) + q_1 y(n-1) + q_2 y(n-2) = p_o x(n) + p_1 x(n-1) + p_2 x(n-2)$$

relating the system's input x and output y.

7. Find the Laplace transform (formula for $X(s)$ along with $(\text{ROC})_X$), if the transform exists, for each signal x specified below. If the transform fails to exist, explain why.

(a) $x(t) = e^{3t}u(t-1)$ for all $t \in \mathbb{R}$.
(b) $x(t) = e^{3t}u(t) - e^{-t}u(-t)$ for all $t \in \mathbb{R}$.
(c) $x(t) = e^{-3|t|}$ for all $t \in \mathbb{R}$.

8. A certain LTI system has impulse response h with specification $h(t) = e^{-t}u(t) - e^{3t}u(-t)$.

(a) Find the system's transfer function (formula for $H(s)$ along with $(\text{ROC})_H$).
(b) Find the output y of the system that arises when the input signal x has specification $x(t) = e^t$ for all $t \in \mathbb{R}$.
(c) Suppose we use u as the input to the system. Find the Laplace transform of the output y that arises (formula for $Y(s)$ along with $(\text{ROC})_Y$). You need not find the signal y itself.

9. Let

$$A = \begin{bmatrix} 1 & \sqrt{\frac{2}{3}} \\ 1 & \sqrt{\frac{2}{3}} \\ 1 & \sqrt{\frac{2}{3}} \end{bmatrix} .$$

(a) What is the rank of A?
(b) Find the SVD of A.
(c) Find the Moore-Penrose pseudo-inverse of A.