ECE 3250 **PRELIM II** **Fall 2015**

Each part of Problems 1 through 9 is worth 2 points. Problem 10 is worth 18 points. Throughout, $*$ denotes convolution.

1. A certain LTI system has system mapping S satisfying

$$S(x) = \text{Shift}_{-3}(x) + h_0 * x \quad \text{for all} \quad x \in \mathcal{D}_{h_0} \ ,$$

where h_0 is a decent signal and $\mathcal{D}_{h_0}$ is the set of all decent signals x for which $h_0 * x$ exists. You can be certain that
(i) If $h_0(t) = e^{3t}$ for all $t \in \mathbb{R}$ and $x \in \mathcal{D}_{h_0}$, then $\text{Shift}_{13}(x) \in \mathcal{D}_{h_0}$
(ii) If x has finite duration, then $x \in \mathcal{D}_{h_0}$
(iii) If $h_0(t) = e^{3t}u(-t)$ for all $t \in \mathbb{R}$ and x is a decent right-sided signal, then $x \in \mathcal{D}_{h_0}$
(iv) The system is causal when $h_0(t) = 0$ for all $t < 0$

2. S is the system mapping and h is the impulse response of a causal continuous-time LTI system over $\mathbb{F}$. You can be certain that
(i) If $h(t) = 1/t$ for every $t \geq 1$ and $h(t) = 0$ for $t < 1$, then the system is BIBO stable
(ii) If h is a decent signal with finite duration, then the system is BIBO stable
(iii) If h has finite duration and the system is BIBO stable, then $h * x$ has finite duration for every bounded decent signal $x \in \mathbb{F}^{\mathbb{R}}$
(iv) If the system is BIBO stable, then $|S(10^{59}u)(t)| \leq 10^{59}$ for all $t \geq 0$

3. Indicate with "L" the sets of signals closed under finite linear combinations and with "S" the sets of signals closed under time-shifting. (A given set could be neither, either, or both. All signals are complex-valued.)
(i) The set of all decent continuous-time absolutely integrable signals that are zero for $t < 0$
(ii) The set of all decent continuous-time periodic signals that have 19 as a period and average value 13
(iii) The set of all decent finite-duration continuous-time signals
(iv) The set of all decent square-integrable continuous-time signals x that satisfy $\|x\|^2 \leq 10^{59}$

4. Here is the Fourier series for a certain decent continuous-time periodic signal x:

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{1}{k^2 + 7} e^{jk\frac{2\pi}{17}t} \ .$$

Which statements are true of x?
(i) x is bandlimited
(ii) The average value of x is $7/4$
(iii) x has 51 as a period
(iv) x is real-valued

5. Let V be a Hilbert space and $\mathcal{W} = \{w_k : k \in \mathbb{Z}\}$ an orthonormal subset of V. You can be certain that
(i) $\{w_3, w_5, w_7, w_{11}, w_{13}\}$ spans a 5-dimensional subspace of V
(ii) If $v \in V$, then $v = \lim_{N\to\infty} \sum_{k=-N}^{N} \langle v, w_k \rangle w_k$
(iii) If $\langle v, w_k \rangle = 0$ for all $k \in \mathbb{Z}$, then $v = 0$
(iv) Every Cauchy sequence $\{v_k\}$ in V has a limit in V

6. Let V be an inner-product space and let $\{w_1, w_2, w_3, w_4, w_5\}$ be an orthonormal set in V. You can be certain that
(i) If $v \in V$, then $\|v\|^2 \geq \sum_{k=1}^{5} |\langle v, w_k \rangle|^2$ for every $v \in V$
(ii) If $v \in V$, then $v - \sum_{k=1}^{5} \langle v, w_k \rangle \, w_k$ is orthogonal to w_3
(iii) $w_1 + w_2$ is orthogonal to $w_1 - w_2$
(iv) If V has dimension at least 5, there exists some nonzero $v \in V$ such that $\langle v, w_k \rangle = 0$ for $1 \leq k \leq 5$

7. Suppose x is a decent continuous-time signal and $\widehat{X}$ is its Fourier transform. Suppose there exists some $\Omega_m > 0$ such that $\widehat{X}(\Omega) = 0$ when $|\Omega| > \Omega_m$. You can be certain that
(i) x has finite duration
(ii) x is bandlimited
(iii) If $\widehat{X}$ is a decent function of Ω, then then $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \widehat{X}(\Omega) e^{j\Omega t} d\Omega$ for all $t \in \mathbb{R}$
(iv) If y is a decent finite-duration signal with Fourier transform $\widehat{Y}$, then $z = x * y$ exists and its Fourier transform $\widehat{Z}$ satisfies $\widehat{Z}(\Omega) = 0$ when $|\Omega| > 2\Omega_m$

8. Let S the system mapping of a certain causal continuous-time system with decent impulse response h. As usual, $\mathcal{D}_h$ is the set of all decent signals x for which $h * x$ exists. You can be certain that
(i) If h has finite duration, then the system has a frequency response
(ii) If the system has frequency response $\widehat{H}$ and $x(t) = 1 + \cos(13t)$ for all $t \in \mathbb{R}$, then

$$S(x)(t) = 1 + \frac{1}{2}\left(\widehat{H}(13)e^{j13t} - \widehat{H}(-13)e^{-j13t}\right) \quad \text{for all} \quad t \in \mathbb{R}$$

(iii) If h is absolutely integrable, then the system has a frequency response
(iv) If the system has a frequency response, $x \in \mathcal{D}_h$, and $S(x)$ is bandlimited, then x is also bandlimited

9. Let x_c be a decent continuous-time signal with Fourier transform $\widehat{X}_c$ that's a decent function of Ω. Throughout this problem, assume $T > 0$ is such that the discrete-time signal x with specification

$$x(n) = x_c(nT) \quad \text{for all} \quad n \in \mathbb{Z}$$

has a DTFT $\widehat{X}$. Referring to the equations on the board, you can be certain that
(i) Equation (D) holds regardless of the value of T but only when x_c is bandlimited
(ii) $\widehat{X}(\omega) = \widehat{X}(\omega + 2\pi)$ for all $\omega \in \mathbb{R}$
(iii) Equation (D) holds when $2\pi/T > 2\Omega_m$ and x_c is bandlimited to within Ω_m (i.e. $\widehat{X}(\Omega) = 0$ when $|\Omega| > \Omega_m$)
(iv) x_R is always bandlimited to within π/T regardless of the value of T
(v) x_R is bandlimited to within $\pi/2T$ regardless of the value of T provided x_c is bandlimited to within $\pi/3T$
(vi) x_R is bandlimited to within $\Omega_m/7$ if $2\pi/T > 14\Omega_m$ and x_c is bandlimited to within Ω_m
(vii) $x_R(t) = x_c(t)$ for all $t \in \mathbb{R}$ regardless of the value of T
(viii) If $x_R(t) = e^{j\Omega_1 t}$ for all $t \in \mathbb{R}$ for some $\Omega_1 \geq 0$, then $x_c(t) = e^{j\Omega_o t}$ for all $t \in \mathbb{R}$ for some $\Omega_o \geq 0$
(ix) If $x_c(t) = e^{j\Omega_o t}$ for all $t \in \mathbb{R}$ for some $\Omega_o \geq 0$, then $x_R(t) = e^{j\Omega_1 t}$ for all $t \in \mathbb{R}$ for some $\Omega_1 \geq 0$

10. The system mappings S_1 and S_2 for two continuous-time LTI systems with respective input spaces X_1 and X_2 are

$$S_1(x)(t) = \frac{1}{14}\int_{t-14}^{t} x(\tau)d\tau \quad \text{for all } \; x \in X_1 \; \text{ and } \; t \in \mathbb{R} \; ,$$

where X_1 is the set of all decent signals, and

$$S_2(x) = h_2 * x \quad \text{for all } \; x \in X_2 \; ,$$

where $h_2(t) = e^{-3t}u(t)$ for every $t \in \mathbb{R}$ and $X_2 = \mathcal{D}_{h_2}$ is the set of all decent signals x for which $h_2 * x$ exists.

(a) Find h_1, the impulse response of the first system.
(b) Find $S_1(x)(t)$ for all $t \in \mathbb{R}$ when $x(t) = \sin(2\pi t/7) - 13 + \cos(2\pi t/14)$

(c) Find $S_2(S_1(x))(t)$ for all $t \in \mathbb{R}$ when $x(t) = 13$ for all $t \in \mathbb{R}$. (Suggestion: find $S_1(x)$ first.)
(d) Find $S_2(x)(t)$ for all $t \in \mathbb{R}$ when $x(t) = 5e^{-t}u(t)$ for all $t \in \mathbb{R}$.