Problem 8.1

a)

$$I_{REF} = I_{D1} = I_{D2} \Rightarrow I_{REF} = \frac{k_n}{2}(V_1 - V_2 - V_{TN})^2\left[1+\lambda_n(V_1 - V_2)\right]$$

$$= \frac{k_n}{2}(V_2 - V_{TN})^2\,(1+\lambda_n V_2)$$

$$\Rightarrow V_1 = 2V_2, \quad V_2 = 1.4351V, \quad V_1 = 2.8703V$$

b)

$$I_{D3} = I_{D4} \Rightarrow \frac{k_n}{2}(V_1 - V_3 - V_{TN})^2\,(1+\lambda_n(V_{OUT} - V_3))$$

$$= \frac{k_n}{2}(V_2 - V_{TN})^2\,(1+\lambda_n V_3)$$

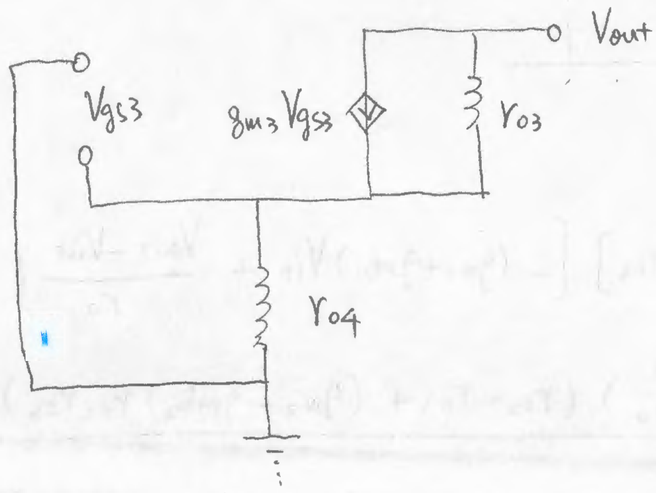
$$\Rightarrow V_{OUT} = \frac{1}{\lambda_n}\left[\frac{(V_2 - V_{TN})^2\,(1+\lambda_n V_3)}{(V_1 - V_3 - V_{TN})^2} - 1\right] + V_3$$

c)

$$M_3: \; V_1 - V_{TN} < V_{OUT} \qquad M_4: \; V_2 - V_{TN} < V_3$$

$$\Rightarrow V_{OUT} > 2.3703V$$

e)

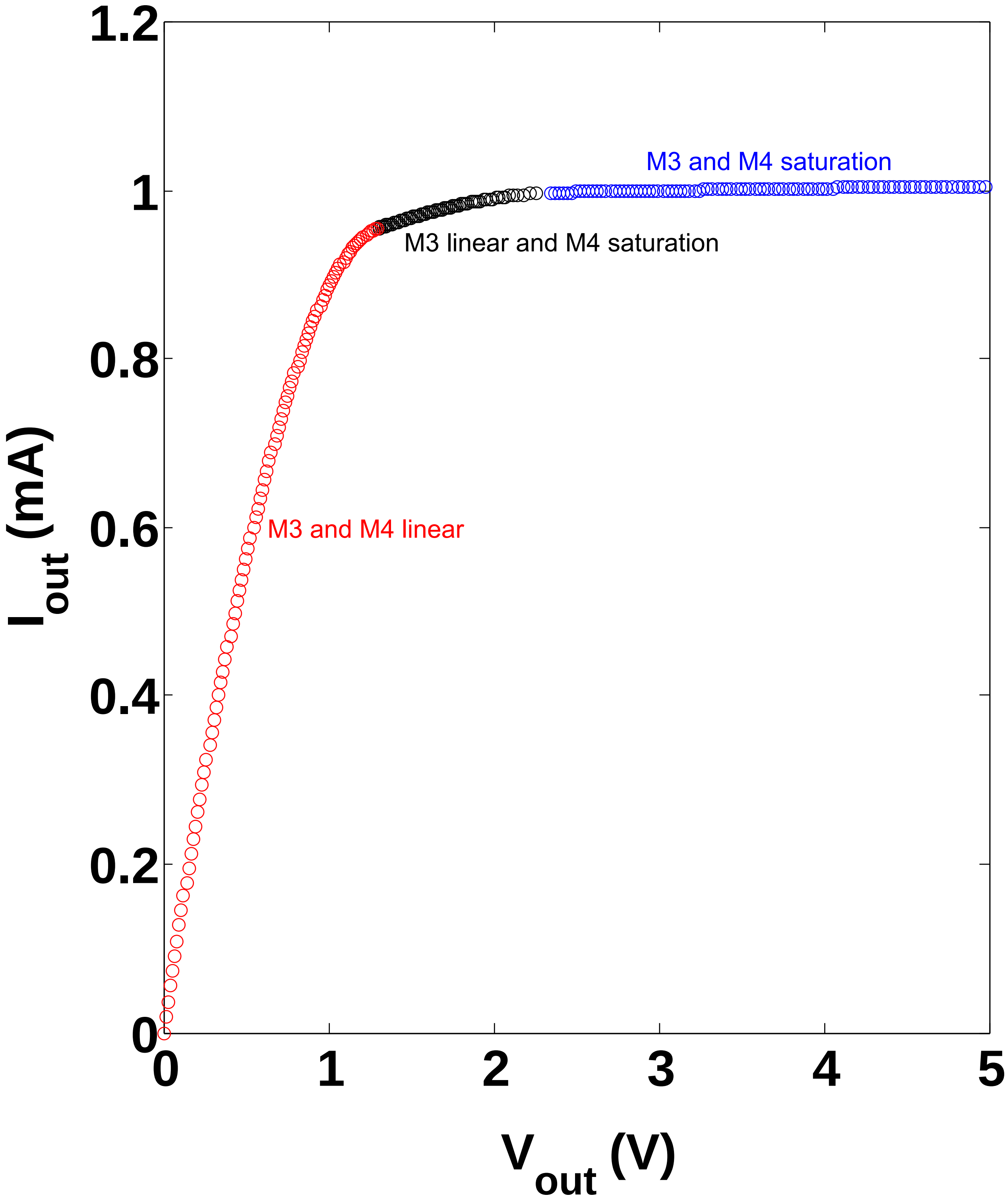


f) $r_{oc} = r_{o3} + r_{o4} + g_{m3} r_{o3} r_{o4}$

g) $g_o = \lambda_n I_D \Rightarrow r_{o3} = r_{o4} = 100000\Omega$.

$$g_{m3} = \sqrt{2k_n I_D(1+\lambda_n V_{DS})} = 0.0021\,S$$

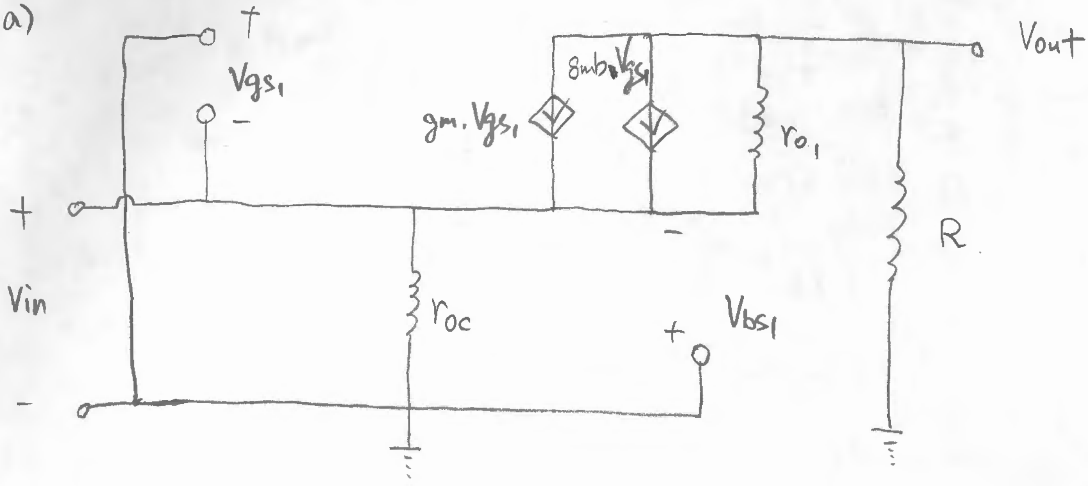
$$r_{oc} = 2.3387 \times 10^5\,\Omega$$



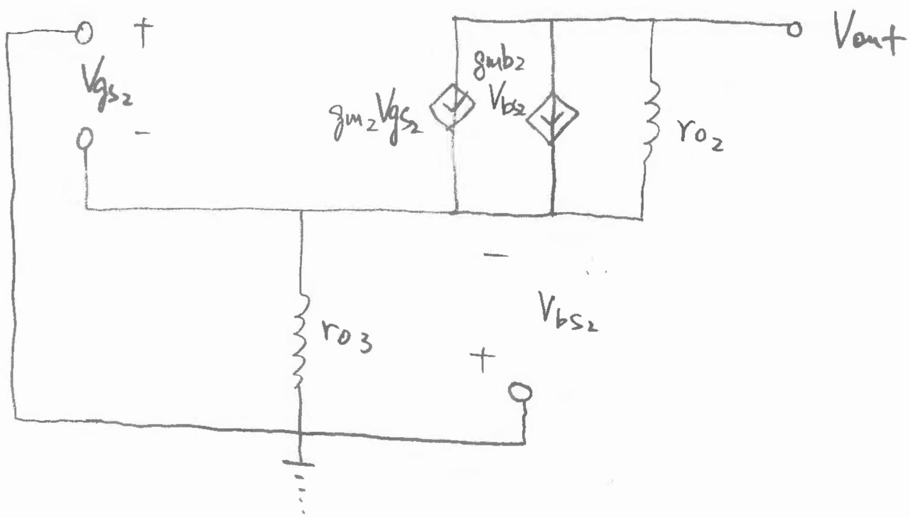
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
I out (mA)
V out (V)
M3 and M4 saturation
M3 linear and M4 saturation
M3 and M4 linear

Problem 8.2

a)



The circuit for R:

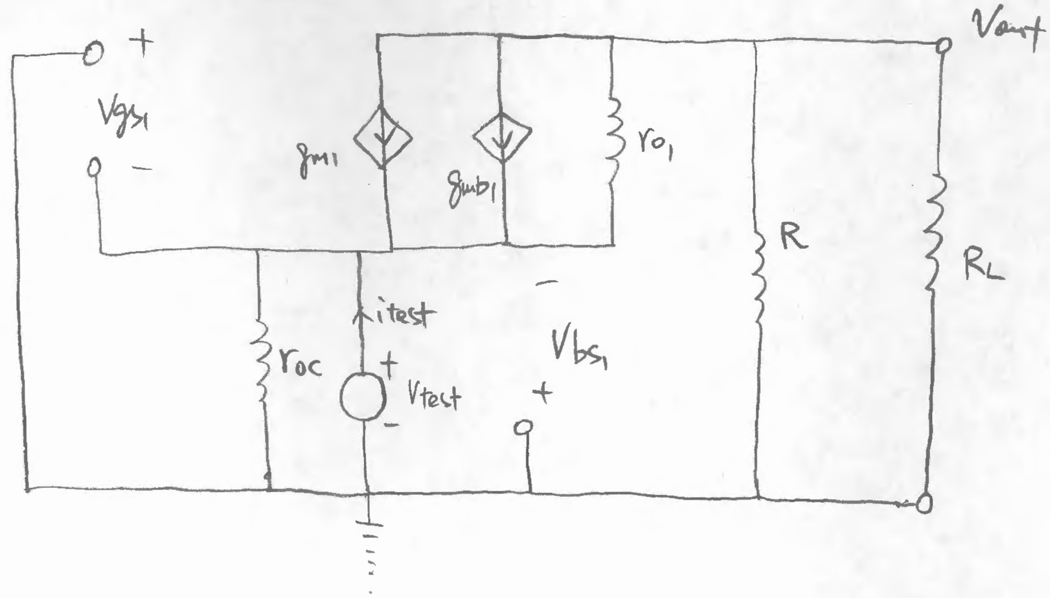


$$R = r_{o2} + r_{o3} + (g_{m2} + g_{mb2}) r_{o2} r_{o3}$$

$$V_{out} = -i_d R = -R \left[-(g_{m1} + g_{mb2}) V_{in} + \frac{V_{out} - V_{in}}{r_o} \right]$$

$$\Rightarrow \quad A_v = \frac{V_{out}}{V_{in}} = \frac{\left(g_{m1} + g_{mb1} + \frac{1}{r_{o1}} \right) R}{1 + \frac{R}{r_o}}$$

b)

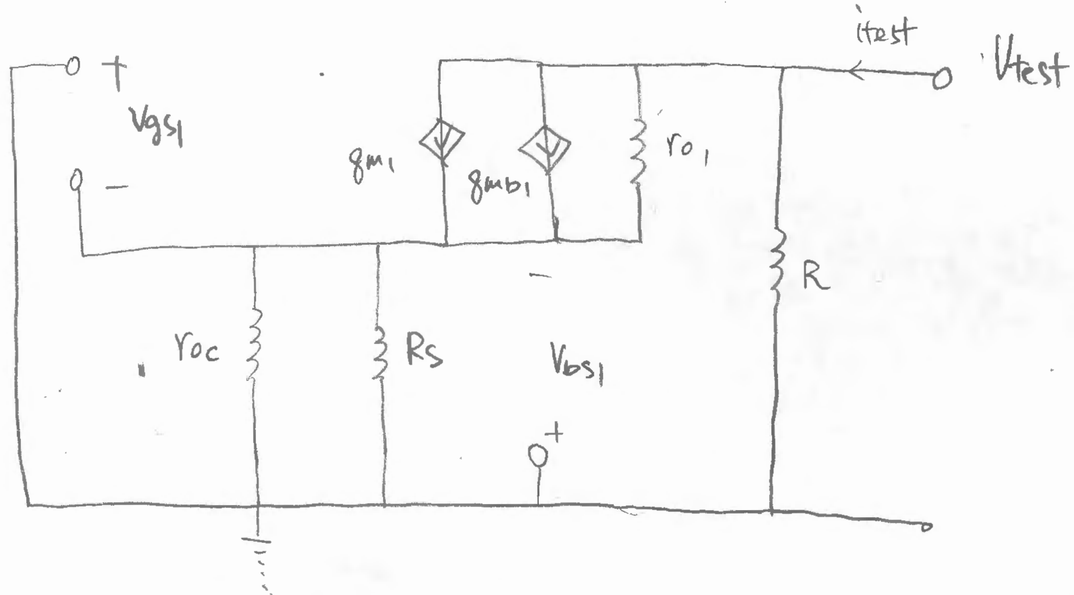


$$i_{test} = (g_{m1} + g_{mb1}) V_{test} - \frac{V_{out} - V_{test}}{r_{o1}} + \frac{V_{test}}{r_{oc}}$$

$$= \left(g_{m1} + g_{mb1} + \frac{1}{r_{oc}}\right) V_{test} - \frac{1}{r_{o1}} \left[\frac{\left(g_{m1} + g_{mb1} + \frac{1}{r_{o1}}\right)(R /\!/ R_L)}{1 + \frac{R /\!/ R_L}{r_{o1}}} - 1 \right] V_{test}$$

$$\Rightarrow R_{in} = \frac{itest}{vtest} = \frac{(g_{m1} + g_{mb1}) r_{o1} + \frac{r_{o1} + R /\!/ R_L}{r_{oc}} + 1}{r_{o1} + R /\!/ R_L}$$

c)



$$i_{test} = i_d + \frac{V_{test}}{R} = \frac{V_{test}}{R} + \frac{V_{test}}{r_{o1}} \Big/ \left[1 + \frac{r_{oc} /\!/ R_S}{r_{o1}} + (g_{m1} + g_{mb1})(r_{oc} /\!/ R_S) \right]$$

$$\Rightarrow R_{out}^{-1} = \frac{itest}{vtest} = \frac{1}{R} + 1 \Big/ \left[r_{o1} + r_{oc} /\!/ R_S + (g_{m1} + g_{mb1}) r_{o1} (r_{oc} /\!/ R_S) \right]$$

8.3)

a) The transconductance of the cascode amplifier is appoximatly the transconductance of M_1. To make

$$g_{m1} = \sqrt{2k_n I_D} \approx 1mS. \Rightarrow W_1 = 200\mu m$$

b)

$M1: R_{out} \approx r_{o1} = (\lambda_n I_D)^{-1} = 200k\Omega$

$M_2: R_{out} \approx \quad r_{o1}$

$M_3: R_{out} \approx \quad r_{o8}$

$M_8: R_{out} \approx r_{o8} = (\lambda_p I_D)^{-1} = 500k\Omega$

c) $R_{out} \approx g_{m2} r_{o1} r_{o2} = 20 M\Omega \Rightarrow g_{m2} = 0.5mS$

d) $g_{m2} = \sqrt{2k_n I_D} = 0.5mS \Rightarrow W_2 = 50\mu m$

e) $R_{out} = g_{m3} r_{o3} r_{o8} = 100M\Omega \Rightarrow g_{m3} = 0.4mS$

f) $g_{m3} = \sqrt{2k_p I_D} = 0.4mS \Rightarrow W_3 = 64\mu m$

g) $W_4 = W_3 = 64\mu m$, $W_5 = W_2 = 50\mu m$, $W_9 = W_8 = W_3 = 64\mu m$

i) $V_1: \; -I_{D1} = -\frac{W_9}{2L}\mu_p C_{ox}(V_1 - V_{DD} - V_{TP})^2 \Rightarrow V_1 = 3.5V$

$V_2: \; -I_{D1} = -\frac{W_4}{2L}\mu_p C_{ox}(V_2 - V_1 - V_{TP})^2 \Rightarrow V_2 = 2V$

$V_3 = V_2 = 2V$

j) $V_4 = V_1 = 3.5V$.

For V_6:

$$I_{D1} = \frac{W_5}{2L}\mu_n C_{ox}(V_3 - V_6 - V_{TN})^2 \Rightarrow V_6 = 0.6V$$

For V_7:

$$I_{D1} = \frac{W_6}{2L}\mu_n C_{ox}(V_7 - V_{TN})^2 \Rightarrow V_7 = 1.2V$$

h)

$$I_{D1} = \frac{W_7}{2L}\mu_n C_{ox}(V_7 - V_{TN})^2 \Rightarrow W_7 = W_6 = W_1 = 200\mu m$$

k) $V_{BIAS} = V_7 = 1.2V$

l)

$$R = \frac{V_{DD} - V_7}{I_{REF}} = 38000\Omega$$