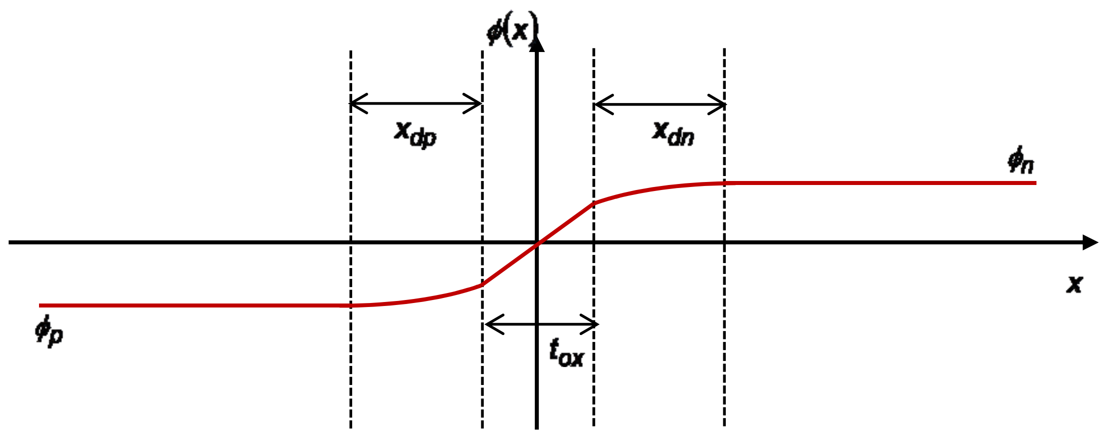
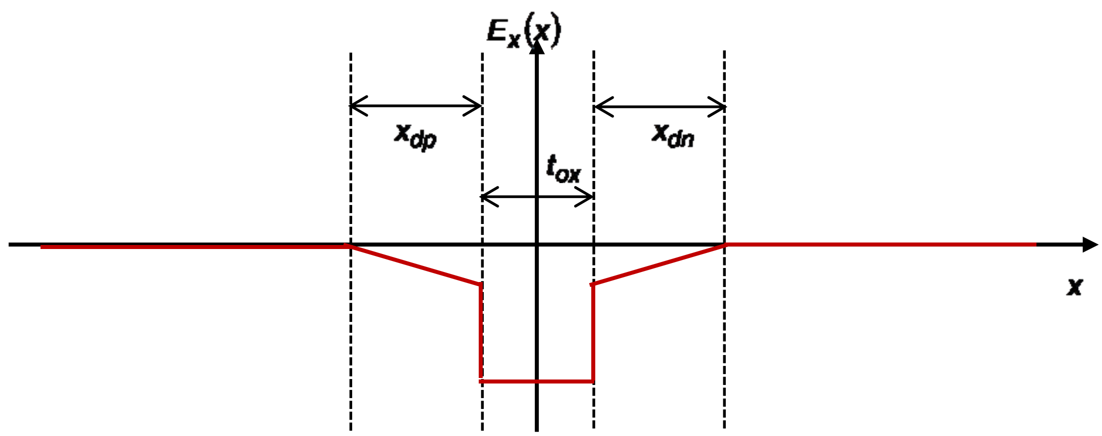
ECE 3150 Homework 5 Solutions

5.2)

a)



b)



c) $$\phi_B = \phi_n - \phi_p = \frac{KT}{q}\log\left(\frac{N_d N_a}{n_i^2}\right) = 0.83 \text{ V}$$

d) The potential drop in all regions must add up to ϕ_B. Therefore:

$$\frac{qN_d x_{dn}^2}{2\varepsilon_s} + \frac{qN_d x_{dn}}{\varepsilon_{ox}} t_{ox} + \frac{qN_a x_{dp}^2}{2\varepsilon_{ox}} = \phi_B$$

Also note that the equality of the total positive and negative charge in the structure implies that:

$$qN_d x_{dn} = qN_a x_{dp}$$

$$\Rightarrow x_{dn} = x_{dp}$$

The latter follows from $N_d = N_a$. So we get:

$$\frac{q(N_d + N_a)x_{dn}^2}{2\varepsilon_s} + \frac{qN_d x_{dn}}{\varepsilon_{ox}} t_{ox} = \phi_B$$

Solving gives:

$$x_{dn} = 59.8 \text{ nm}$$

e) If a voltage V is applied, then the potential drop in all regions must add up to $\phi_B + V$. Therefore,

$$\frac{q(N_d + N_a)x_{dn}^2}{2\varepsilon_s} + \frac{qN_d x_{dn}}{\varepsilon_{ox}} t_{ox} = \phi_B + V$$

The potential drop in every region will therefore increase or decrease with the applied voltage. If the potential drop on the N-side becomes equal to $2\phi_n$ then the surface of the N-side right next to the oxide will become inverted; i.e. the hole density there will become as large as the bulk electron density, which is approximately N_d. To see this clearly note that on the N-side:

$$p(x_2) = p(x_1)e^{-\frac{q[\phi(x_2) - \phi(x_1)]}{KT}}$$

$$\Rightarrow p(\text{surface}) = p(bulk)\, e^{-\frac{q[\phi(\text{surface}) - \phi(\text{bulk})]}{KT}} = \frac{n_i^2}{N_d} e^{-\frac{q[\phi(\text{surface}) - \phi(\text{bulk})]}{KT}}$$

If $\phi(\text{bulk}) - \phi(\text{surface}) = 2\phi_n$, then:

$$p(\text{surface}) = \frac{n_i^2}{N_d} e^{\frac{q2\phi_n}{KT}} = \frac{\left(n_i e^{\frac{q\phi_n}{KT}}\right)^2}{N_d} = \frac{N_d^2}{N_d} = N_d$$

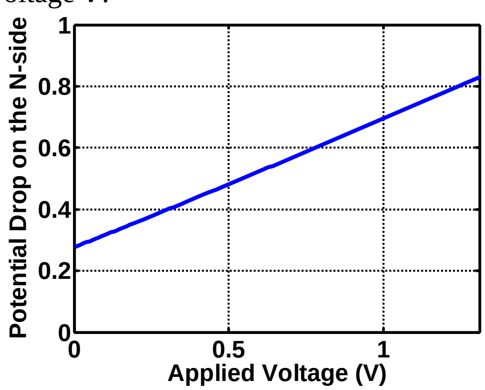
Similarly, if the potential drop on the P-side becomes equal to $-2\phi_p$ then the surface of the P-side right next to the oxide will become inverted; i.e. the electron density there will become as large as the bulk hole density, which is approximately N_a. Since the structure is symmetric, the hole inversion layer on the N-side and the electron inversion layer on the P-side will take place at the same applied voltage. The easiest way to solve this problem is to first find x_{dn} as a function of the applied voltage V using:

$$\frac{q(N_d + N_a)x_{dn}^2}{2\varepsilon_s} + \frac{qN_d x_{dn}}{\varepsilon_{ox}} t_{ox} = \phi_B + V$$

and then figuring out at what voltage value does the potential drop on the N-side, given by:

$$\frac{qN_d x_{dn}^2}{2\varepsilon_s}$$

equals $2\phi_n = 0.83$ V. Below is a matlab plot of the potential drop on the N-side as a function of the applied voltage V.



When V equals 1.31 Volts, the potential drop on the N-side becomes equal to 0.83 Volts.

5.1)

a) $V_{FB} = +0.5$ V

b) $V_{TN} = -0.6$ V

c) $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 8\times10^{-15}\ F/\mu m^2 = 8\times10^{-3}\ F/m^2$

$\Rightarrow t_{ox} = 4.3\ nm = 43\ \text{Å}$

d) N-type (PMOS capacitor)

e) Just before threshold; $C \approx 2\ fF/\mu m^2$

$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_b} \Rightarrow C_b = \frac{8}{3}\ fF/\mu m^2 \qquad C_b = \frac{\varepsilon_s}{X_{dmax}}$

$X_{dmax} = \sqrt{\frac{2\varepsilon_s(+2\phi_n)}{qN_d}} \Rightarrow \sqrt{\frac{qN_d\varepsilon_s}{2(2\phi_n)}} = C_b = \frac{8}{3}\ fF/\mu m^2$

$\Rightarrow N_d \approx 8\times10^{17}\ 1/cm^3$

f) Analysis similar to that in problem 4.3 gives the result: $V_{TP}(\rho_0) - V_{TP}(\rho_0=0) = -\frac{\rho_0 t_{ox}^2}{2\varepsilon_{ox}} = -0.25$ V

$\Rightarrow \rho_0 = +9.33\times10^{-1}\ C/cm^3$

5.3)

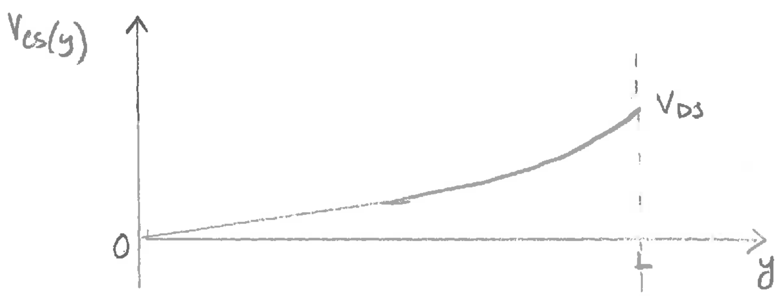
a) $$I_D = W\mu_n C_{ox}\left(V_{GN} - V_{TN} - V_{CS}(y)\right)\frac{dV_{CS}(y)}{dy}$$

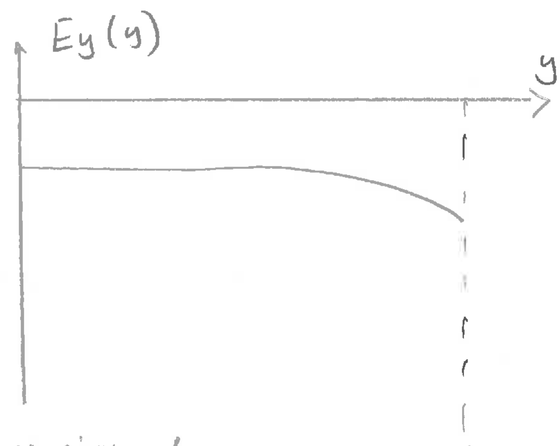
$$\Rightarrow \int_0^y I_D\,dy = \int_0^{V_{CS}} W\mu_n C_{ox}\left(V_{GN} - V_{TN} - V_{CS}\right)dV_{CS}$$

$$I_D y = W\mu_n C_{ox}\left[V_{GN} - V_{TN} - \frac{V_{CS}(y)}{2}\right]V_{CS}(y)$$

$$\Rightarrow V_{CS}(y) = (V_{GS} - V_{TN}) - \sqrt{(V_{GS} - V_{TN})^2 - \frac{2y I_D}{W\mu_n C_{ox}}}$$

b) $$E_y = -\frac{dV_{CS}}{dy} = \frac{-\frac{I_D}{W\mu_n C_{ox}}}{\sqrt{(V_{GS} - V_{TN})^2 - \frac{2y I_D}{W\mu_n C_{ox}}}}$$





The field is not uniform because the inversion layer charge density is not uniform along the channel. As the inversion layer charge density decreases going towards the drain end, the field increases, but the product gives the constant current (i.e. independent of position).

c) As $V_{DS} \to V_{GS} - V_{TN}$ and $I_D \to \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_{TN})^2$ the field at $y = L$, $E_y(y=L)$, approaches infinity — because the inversion layer charge density at $y = L$ is approaching zero.