ECE 3150 Homework #4 Solutions

4.1

a) charge stored in the N-side $= qA\int_{x_n}^{W_n+x_n} p'(x)\,dx = Q_d A$

$$= qA\frac{n_i^2}{N_d}\frac{W_n}{2}\left(e^{\frac{qV_D}{kT}}-1\right)$$

$$\Rightarrow C_d = \frac{q^2}{kT}A\frac{n_i^2}{N_d}\frac{W_n}{2}e^{\frac{qV_D}{kT}}$$

$$\text{current} = I_D = qA\frac{n_i^2}{N_d}\frac{D_p}{W_n}\left(e^{\frac{qV_D}{kT}}-1\right)$$

$$\Rightarrow r_d^{-1} = \frac{q^2}{kT}A\frac{n_i^2}{N_d}\frac{D_p}{W_n}\left(e^{\frac{qV_D}{kT}}-1\right)$$

b) $$i_d(\omega) = \left(j\omega C_d + \frac{1}{r_d}\right)v_d(\omega) \Rightarrow v_d(\omega) = \frac{r_d\, i_d(\omega)}{1+j\omega C_d r_d}$$

c) $$\omega_{3dB} = \frac{1}{C_d r_d}$$

d) $$\omega_{3dB} = \frac{2D_p}{W_n^2}$$

e) 4 mice

f) $N = RT$

g) $$J_p(x=x_n) = q\frac{n_i^2}{N_d}\frac{D_p}{W_n}\left(e^{\frac{qV_D}{kT}}-1\right)$$

$$Q_d = q\frac{n_i^2}{N_d}\frac{W_n}{2}\left(e^{\frac{qV_D}{kT}}-1\right)$$

$$\tau_{p\text{-diff}} = \frac{Q_d}{J_p(x=x_n)} = \frac{W_n^2}{2D}$$

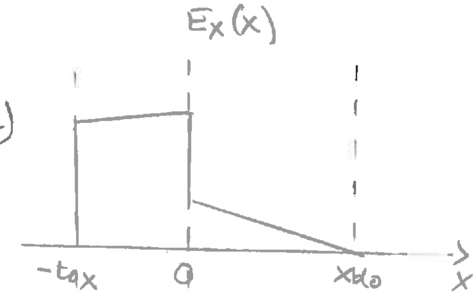
h) $$\omega_{3dB} = \frac{1}{\tau_{p\text{-diff}}}$$

4.3)

a) $\phi_B = \phi_m - \phi_p = \phi_n - \phi_p = KT \log\left(\frac{N_a N_d}{n_i^2}\right) = 0.89\ V$

b) $E_x(x=0^-) = \frac{\varepsilon_s}{\varepsilon_{ox}} E_x(x=0^+) = +\ q\frac{N_a X_{do}}{\varepsilon_{ox}}$

c)



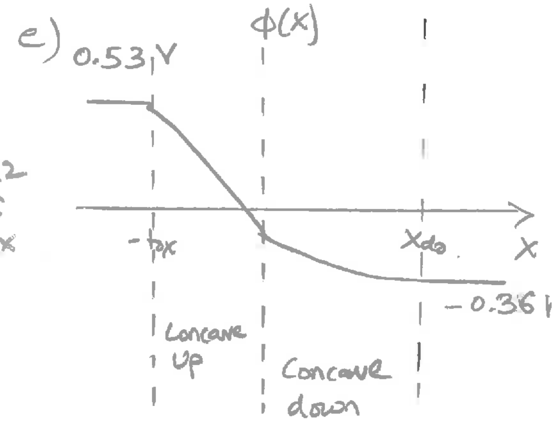
$$\frac{dE_x}{dx} = \frac{\rho_o}{\varepsilon_{ox}} \Rightarrow E_x(x) = q\frac{N_a X_{do}}{\varepsilon_{ox}} + \frac{\rho_o x}{\varepsilon_{ox}}$$

d) $\frac{d\phi}{dx} = -E_x(x) = -q\frac{N_a X_{do}}{\varepsilon_{ox}} - \frac{\rho_o x}{\varepsilon_{ox}}$

BC: $\phi(x=0) = \phi_p + q\frac{N_a}{2\varepsilon_s} X_{do}^2$

$$\Rightarrow \phi(x) = \phi_p + q\frac{N_a}{2\varepsilon_s} X_{do}^2 - q\frac{N_a X_{do}}{\varepsilon_{ox}} x - \frac{\rho_o x^2}{2\varepsilon_{ox}}$$

e)



f) $\phi(x=-t_{ox}) = \phi_M$

$$\Rightarrow \phi_p + q\frac{N_a}{2\varepsilon_s} X_{do}^2 + q\frac{N_a}{\varepsilon_{ox}} X_{do} t_{ox} - \frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}} = \phi_M$$

$$\Rightarrow q\frac{N_a}{2\varepsilon_s} X_{do}^2 + q\frac{N_a}{\varepsilon_{ox}} X_{do} t_{ox} = \phi_B + \frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}}$$

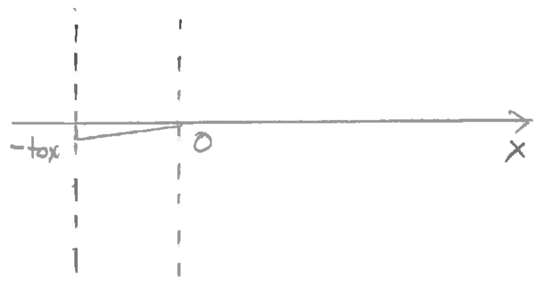
$$\Rightarrow X_{do} = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_a}\right)\left(\phi_B + \frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}}\right)}$$

g) when $V_{GB} \neq 0$ $\quad X_{do} = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_a}\right)\left(\phi_B + V_{GB} + \frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}}\right)}$

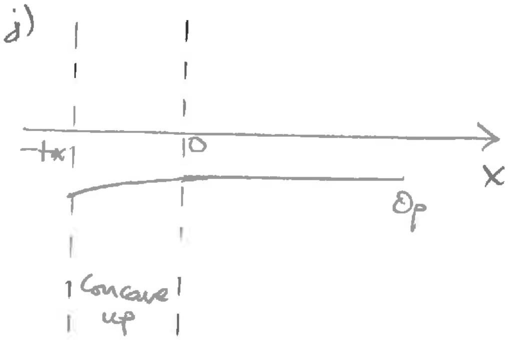
$X_{do} = 0$ when $V_{GB} = -\phi_B - \frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}} = V_{FB}$

h) In flatband, since there is no charge in the semiconductor, the gate charge must be equal and opposite to the oxide charge $\Rightarrow Q_G = -\rho_o t_{ox}$

i)



j)



k) $Q_G = -\rho_o t_{ox} + C_{ox}(V_{GB} - V_{FB})$

l) $X_{do} = -\dfrac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\dfrac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\dfrac{2\varepsilon_s}{qN_a}\right)(V_{GB} - V_{FB})}$

m) $Q_G = -\rho_o t_{ox} + qN_a X_{do}$

n) From part (f) $V_{GB} - V_{FB} = \dfrac{qN_a}{2\varepsilon_s} X_{do}^2 + \dfrac{qN_a X_{do} t_{ox}}{\varepsilon_{ox}}$

$$= (\phi_s - \phi_p) + \frac{\sqrt{2q\varepsilon_s N_a(\phi_s - \phi_p)}}{C_{ox}}$$

Same as when $\rho_o = 0$

$$\Rightarrow \quad V_{TN} = \underbrace{V_{FB}}_{\text{depends on } \rho_o} - 2\phi_p + \frac{\sqrt{2q\varepsilon_s N_a(-2\phi_p)}}{C_{ox}}$$

o) $V_{TN}(\rho_o) - V_{TN}(\rho_o = 0) = -\dfrac{\rho_o t_{ox}^2}{2\varepsilon_{ox}}$

p) $Q_G = \sqrt{2q\varepsilon_s N_a(-2\phi_p)} - \rho_o t_{ox} + C_{ox}(V_{GB} - V_{TN})$

4.2)

The electrical power dissipated in the external resistor is also the electrical power output of the solar cell:

$$P_{out} = I^2 R = -IV_D = -\left(I_o\left(e^{qV_D/KT} - 1\right) - |I_L|\right)V_D$$

Recall that:

$$I = \left(I_o\left(e^{qV_D/KT} - 1\right) - |I_L|\right)$$

$$I = -\frac{V_D}{R}$$

And since the external quantum efficiency is 100%, we must have:

$$I_L = -q\frac{P_{inc}}{\hbar\omega}$$

The above can be used to calculate I_L given P_{inc}.

We need to maximize the power output. The power output depends on the voltage V_D which in turn depends on the external resistor R. There is an optimal value of the resistor that gives the maximum output power. So we set:

$$dP_{out}/dR = 0$$

This gives,

$$\frac{dP_{out}}{dR} = -\frac{dI}{dV_D}\frac{dV_D}{dR}V_D - I\frac{dV_D}{dR} = -\frac{dI}{dV_D}\frac{dV_D}{dR}V_D + \frac{V_D}{R}\frac{dV_D}{dR} = 0$$

$$\Rightarrow \frac{qI_o}{KT}e^{qV_D/KT} = \frac{1}{R}$$

The optimal value of the resistor is related to the voltage via the above relation. In general the voltage V_D is not related to the resistor R by the above relation – only the optimal value of the resistor is related to the corresponding voltage V_D by the above relation.

a) We use the relation:

$$P_{out} = I^2 R = -IV_D = -\left(I_o\left(e^{qV_D/KT} - 1\right) - |I_L|\right)V_D$$

and plot it as a function of R. Note the above only explicitly depends on the voltage V_D but not R. But we also know that the optimal value of R must be related to the corresponding voltage by the relation:

$$\frac{qI_o}{KT}e^{qV_D/KT} = \frac{1}{R}$$

So for every R we can calculate the voltage using the above relation and then calculate P_{out}, and then plot P_{out} as a function of R and find the value of R that maximizes P_{out}. For P_{inc} equal to 0.1 mW, I get maximum P_{out} for R equal to ~5.8 kΩ and a maximum conversion efficiency P_{out}/P_{inc} of 8%.

b) For P_{inc} equal to 1 mW, I get maximum P_{out} for R equal to ~710 Ω and a maximum conversion efficiency P_{out}/P_{inc} of 10.3%.

c) For P_{inc} equal to 10 mW, I get maximum P_{out} for R equal to ~84.5 Ω and a maximum conversion efficiency P_{out}/P_{inc} of 12.6%.

d) The optimal power conversion efficiency is better for larger P_{inc} and, therefore, larger I_L. The optimal power conversion efficiency stems from a competition between the power dissipation in the external resistor and the forward biasing of the diode from the potential drop across this resistor. The power dissipation I^2R varies quadratically with the current whereas the potential drop $|IR|$ across the resistor varies linearly with the current. This suggests that increasing the current $|I|$, and therefore $|I_L|$, ought to be a better strategy for increasing the power output. Of course, the problem is that the forward bias current goes exponentially with the potential drop so one does not win as much. In actual solar cell systems, solar concentrators are often used to increase P_{inc} per cell.

e) The reasoning in part (d) suggests that minimizing the forward bias current is a good strategy for improving the power conversion efficiency. If the temperature is lowered, everything else being equal, the forward bias current will increase. Therefore, lowering the temperature is not a good idea for improving the power conversion efficiency in our simple model.

f) The simplest thing to do would be to reduce the I_o of the diode by design (while maintaining 100% external quantum efficiency). Reducing I_o would reduce the unwanted forward bias current for the cell.