

2.1

$$a) \sigma \approx q \mu_p p_0 = 4.8 \text{ S/m or } .048 \text{ S/cm} \quad \left\{ p_0 \approx N_A = 10^{15} / \text{cm}^3 \right.$$

$$\Rightarrow R = \frac{L}{\sigma A} = 2.08 \times 10^5 \Omega$$

$$b) G_L = 10^{23} \frac{1}{\text{cm}^3 \cdot \text{s}} \quad n = n_0 + n' \quad p = p_0 + p'$$

$$n \approx n'$$

$$\frac{dn'}{dt} = -\frac{n'}{\tau_n} + G_L$$

$$\text{In steady state } \frac{dn'}{dt} = 0 \Rightarrow n' = G_L \tau_n = 10^{17} / \text{cm}^3$$

$$\sigma = q [u_p (p_0 + p') + \mu_n (n_0 + n')] \approx q (u_p + u_n) n' \quad \left\{ p \approx n \approx n' \right.$$

$$\approx 2080 \text{ S/m or } 20.8 \text{ S/cm}$$

$$R = \frac{L}{\sigma A} = 480 \Omega$$

$$c) G_L(t) = g_l + \text{Re} \{ g_l e^{j\omega t} \}$$

$$\text{Let } n = n_0 + n'(t) \quad p = p_0 + p'(t)$$

$$\text{and let } u'(t) = n'_{dc} + \text{Re} \{ n'(\omega) e^{j\omega t} \}$$

$$p'(t) = p'_{dc} + \text{Re} \{ p'(\omega) e^{j\omega t} \}$$

DC
part

AC
part

$$\frac{dn'(t)}{dt} = -\frac{n'(t)}{\tau_n} + G_L(t).$$

plug the assumed form of the solution in the equation to get:

$$\operatorname{Re}\{j\omega n'(\omega) e^{j\omega t}\} = - \frac{(n'_{DC} + \operatorname{Re}\{n'(\omega) e^{j\omega t}\})}{\tau_n} + g_e + \operatorname{Re}\{g_e e^{j\omega t}\}$$

Matching time independent terms gives:

$$n'_{DC} = g_e \tau_n$$

Matching time dependant terms give:

$$n'(\omega) = \frac{g_e}{j\omega + \frac{1}{\tau_n}}$$

$$\Rightarrow p'(t) = n'(t) = g_e \tau_n + \operatorname{Re}\left\{ \frac{g_e}{j\omega + \frac{1}{\tau_n}} e^{j\omega t} \right\}$$

$$\begin{aligned} d) \quad I(t) &= \frac{V}{R(t)} = \frac{AV}{L} \sigma(t) = \frac{AV}{L} q \left[\mu_p (p_0 + g_e \tau_n) + \mu_n (n_0 + g_e \tau_n) \right] \\ &= \frac{AV}{L} q (\mu_p + \mu_n) \operatorname{Re}\left\{ \frac{g_e}{j\omega + \frac{1}{\tau_n}} e^{j\omega t} \right\} \end{aligned}$$

$$\Rightarrow I_{DC} = \frac{AV}{L} q \left[\mu_p (p_0 + g_e \tau_n) + \mu_n (n_0 + g_e \tau_n) \right]$$

$$e) \quad i(\omega) = \frac{AV}{L} q (\mu_p + \mu_n) \frac{g_e}{j\omega + \frac{1}{\tau_n}}$$

$$f) \quad |i(\omega)|^2 = \frac{|i(\omega=0)|^2}{1 + \omega^2 \tau_n^2} \Rightarrow |i(\omega)|^2 = \frac{|i(0)|^2}{2} \quad \text{when } \omega = \frac{1}{\tau_n}$$

$$\omega = \frac{1}{\tau_n} = 10^6 \text{ rad/s}$$

Problem 2.2

(a) A is wider.

$$x_{p0} = \sqrt{\frac{2 \epsilon_s \phi_B}{q_0 N_A} \left(\frac{N_d}{N_A + N_d} \right)} \quad x_{n0} = \sqrt{\frac{2 \epsilon_s \phi_B}{q_0 N_d} \left(\frac{N_A}{N_A + N_d} \right)} \quad \epsilon_s = 11.7 \epsilon_0$$

$$\phi_B = \frac{kT}{q} \log \left(\frac{N_A N_d}{N_i^2} \right) \quad N_i = 10^{10} / \text{cm}^3$$

$$\Rightarrow x_{p0} = 0.3 \mu\text{m} \quad x_{n0} = 0.03 \mu\text{m} \quad x_{p0} + x_{n0} = 0.33 \mu\text{m}$$

(b) B's E is the largest.

$$|E_{\text{max}}| = \sqrt{\frac{2 q_0 \phi_B}{\epsilon_s} \left(\frac{N_A N_d}{N_A + N_d} \right)} \quad N_A = 10^{18} / \text{cm}^3 \quad N_d = 10^{17} / \text{cm}^3$$

$$= 1.6 \times 10^5 \text{ V/cm}$$

(c) B.

Sample B has higher doping level and thinner depletion region.

At a certain reverse bias, the electric field inside B is higher.

Problem 2.3

(a)

$$\frac{A^2}{C_j^2} = \frac{2}{q_e N_a \epsilon_s} (\phi_B - V_D) \quad \text{since } N \text{ side is heavily doped}$$

So we only consider the depletion region on P side.

Reading from the plot.

$$\begin{cases} 7 \times 10^{26} = \frac{2}{q_e N_a \epsilon_s A^2} (\phi_B + 5) \\ 1.06 \times 10^{26} = \frac{2}{q_e N_a \epsilon_s A^2} (\phi_B - 0) \end{cases}$$

$$\Rightarrow \phi_B = 0.89 \text{ V.}$$

$$(b) \quad q_e = 1.6 \times 10^{-19} \text{ C} \quad \epsilon_s = 11.7 \epsilon_0 \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$A = 100 \mu\text{m}^2.$$

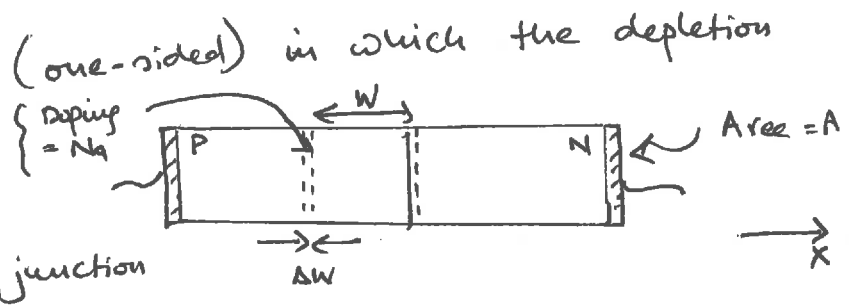
$$N_a = 10^{17} / \text{cm}^3.$$

$$\phi_B = \frac{kT}{q_e} \log\left(\frac{N_a N_d}{N_i^2}\right) \quad N_i = 10^{10} / \text{cm}^3, \quad kT = 25.8 \text{ meV}$$

$$\Rightarrow N_d = 9.4 \times 10^{17} / \text{cm}^3$$

Another look at part (c)

c) Consider a PN junction (one-sided) in which the depletion region width is W .



If the voltage across the junction is changed by ΔV_D , the depletion region width would change by ΔW , then the charge per unit area would change by $-q N_A \Delta W = \Delta Q$. The junction Electric field would ~~change~~ change

by $\Delta E_x = \frac{\Delta Q}{\epsilon_s} = -\frac{q N_A \Delta W}{\epsilon_s}$. But $\Delta V_D = \Delta E_x W$. Therefore,

$$W \frac{\partial W}{\partial V_D} = W \frac{\Delta W}{\Delta V_D} = W \frac{\Delta W}{\Delta E_x W} = \frac{\Delta W}{\Delta E_x} = \frac{\Delta W}{-q N_A \Delta W} = -\frac{\epsilon_s}{q N_A}$$

Note that the doping N_A above is the doping right at the edge of the depletion region on the P-side. Therefore,

$$\frac{\partial (1/C_j^2)}{\partial V_D} = \frac{2}{\epsilon_s^2 A^2} W \frac{\partial W}{\partial V_D} = -\frac{2}{\epsilon_s^2 A^2} \frac{\epsilon_s}{q N_A} = -\frac{2}{q \epsilon_s A^2 N_A}$$

The slope of the curve $1/C_j^2$ vs V_D is inversely related to the doping at the edge of the depletion region on the P-side (the lightly doped side).

Therefore, when V_D is decreased below $-5V$ the depletion region edge on the P-side moves into a region in which the doping is less than $10^{17} \frac{1}{cm^3}$ by a factor of 2 - so it is $5 \times 10^{16} \frac{1}{cm^3}$.

Problem 2.4

(a) At zero reverse bias, we have

$$\frac{A^2}{C^2} = \frac{2}{q_s \epsilon_s N_A} \phi_B \quad \text{since } N_A \ll N_D.$$

$$\text{and } \phi_B = \frac{kT}{q_s} \log \frac{N_A N_D}{N_i^2}$$

In order to know A from the above expression, we need to know C as well.

For the RLC circuit at resonance frequency 1 GHz, the capacitance needs to satisfy

$$f = \frac{1}{2\pi\sqrt{LC}} = 1 \text{ GHz} \Rightarrow C = 5 \times 10^{-12} \text{ F}$$

Then we get

$$A = 1.3 \times 10^5 \mu\text{m}^2$$

(b) The breakdown voltage of silicon $E = 3 \times 10^5 \text{ V/cm}$. According to the expression of electric field inside depletion region,

$$|E_{\text{max}}| = \sqrt{\frac{2q_s (\phi_B - V_D)}{\epsilon_s} \left(\frac{N_A N_D}{N_A + N_D} \right)},$$

The maximum reverse bias is $V_D = -2.94 \times 10^3 \text{ V}$.

$$\frac{A^2}{C'^2} = \frac{2}{q_s \epsilon_s N_A} (\phi_B - V_D)$$

$$\Rightarrow C' = 7.1 \times 10^{-14} \text{ F}, \quad f' = \frac{1}{2\pi\sqrt{LC'}} = 8.4 \text{ GHz}$$