

---

ECE 3150: Microelectronics

Spring 2016

---

Homework 2

Due on Feb. 11, 2016 at 7:00 PM

---

**Suggested Readings:**

a) Lecture notes

**Important Note:**

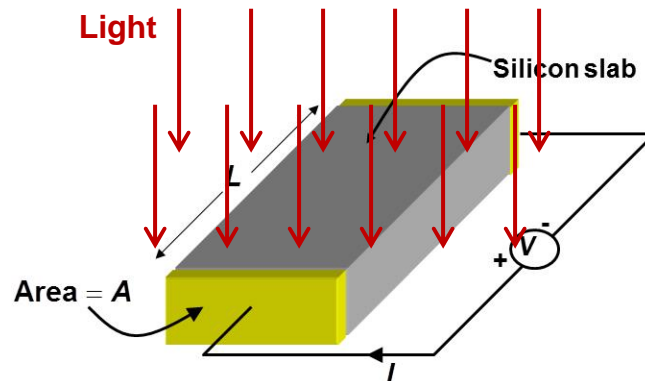
**1) MAKE SURE THAT YOU INDICATE THE UNITS ASSOCIATED WITH YOUR NUMERICAL ANSWERS. OTHERWISE NO POINTS WILL BE AWARDED.**

**2) Lab 1 is scheduled for the week of Feb. 15**

**3) Unless noted otherwise, always assume room temperature.**

**Problem 2.1: (Photoconductors)**

A photoconductor is a device that changes its resistance when exposed to light.



Consider a device consisting of a piece of P-doped silicon ( $N_a = 10^{15} \text{ cm}^{-3}$ ) with an area  $A$  equal to  $1.0 \text{ sq-}\mu\text{m}$  and length  $L$  equal to  $1 \mu\text{m}$ . The electron and the hole mobilities are  $1000$  and  $300 \text{ cm}^2/\text{V-s}$ , respectively. The minority carrier lifetime  $\tau_n$  is  $1 \mu\text{s}$ .

a) What is the resistance of the device under no light illumination?

b) Suppose the device is exposed to light which illuminates the device uniformly. Light illumination results in electron-hole generation rate  $G_L$  of  $10^{23} \text{ 1}/(\text{cm}^3\text{-s})$ . Find the resistance of the device under this light illumination? Assume that the light was turned on in the remote past.

In many schemes used for optical communications, light intensity is modulated in time. We will assume in the parts that follow that the light induced generation rate  $G_L(t)$  is time-dependent and the time dependence is sinusoidal at a frequency  $\omega$  and can be expressed in terms of a phasor:

$$G_L(t) = g_\ell + \text{Re}\{g_\ell e^{j\omega t}\}$$

c) As a result of the time-dependent illumination, the excess electron and hole concentrations are time-dependent and can be written as,

$$n'(t) = n'_{DC} + \text{Re}\{n'(\omega) e^{j\omega t}\}$$

$$p'(t) = p'_{DC} + \text{Re}\{p'(\omega) e^{j\omega t}\}$$

Find expressions that relate the phasors  $n'(\omega)$  and  $p'(\omega)$  to the phasor  $g_\ell$  in steady state.

The current in the circuit, under the time-dependent illumination, is also time-dependent and given by,

$$I(t) = I_{DC} + \text{Re}\{i(\omega) e^{j\omega t}\}$$

d) Find an expression for  $I_{DC}$ ?

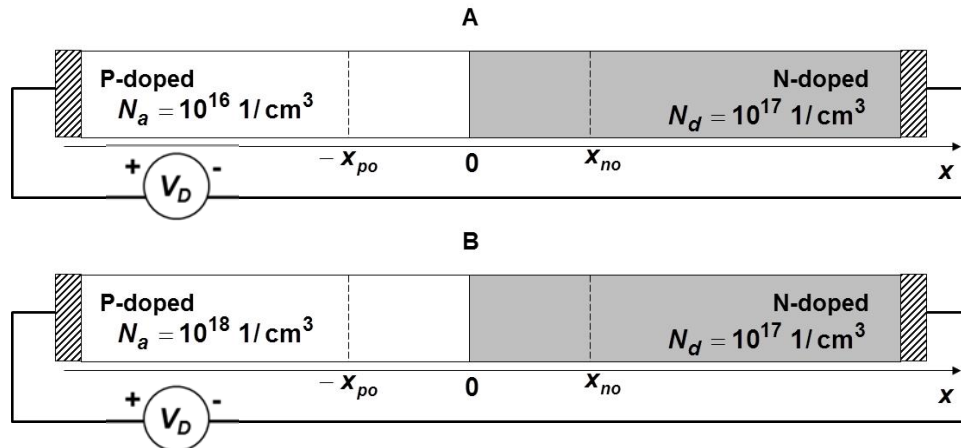
e) Find an expression that relates the current phasor  $i(\omega)$  to the phasor  $g_\ell$ .

e) You would have noticed in part (e) that the magnitude  $|i(\omega)|$  of the current phasor decreases with the frequency  $\omega$ . The magnitude  $|i(\omega)|$  is the largest when  $\omega \approx 0$ . At what frequency  $\omega$  (in rad/s) would the squared-magnitude  $|i(\omega)|^2$  of the current phasor decrease to half its value at  $\omega \approx 0$ ? Hint: try to relate this frequency to the minority carrier lifetime.

**Lesson:** The minority carrier lifetime in this photoconductor device sets the upper limit for the frequencies at which this device can be used for optical communications.

## Problem 2.2: (PN Junctions)

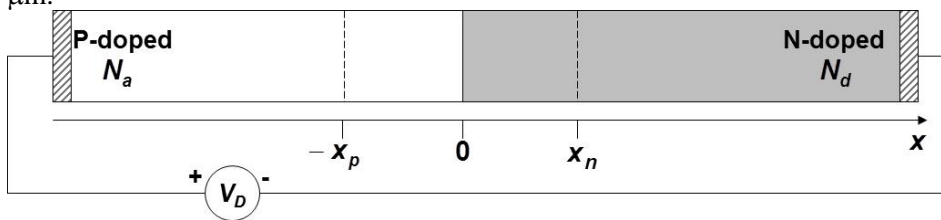
Consider two PN diodes, A and B, as depicted below. Suppose the junction area for each diode is 1.0 square- $\mu\text{m}$ .



- a) At  $V_D = 0$  Volts, which diode has the wider total depletion region width ( $x_{no} + x_{po}$ ) and what is its value (in microns)?
- b) At  $V_D = 0$  Volts, in which diode the magnitude of the maximum electric field in the depletion region is the largest and what is its value (in V/cm)?
- c) Under a reverse bias  $V_D < 0$ , which diode will breakdown first (i.e. at a smaller magnitude of the negative bias).

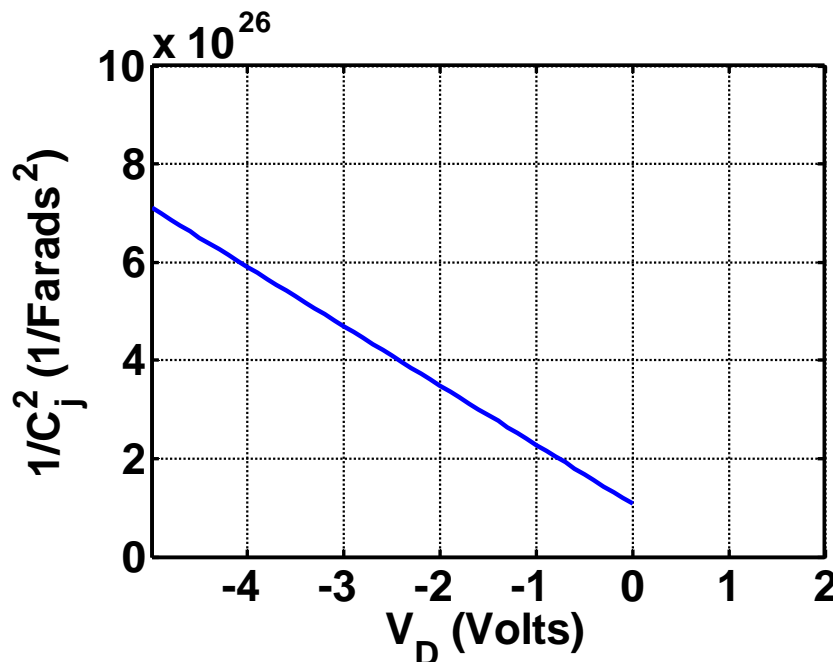
### Problem 2.3: (PN Junction Capacitances)

Consider a PN junction diode, as depicted below. Suppose the junction area of the diode is  $100.0 \text{ square-}\mu\text{m}$ .



Unfortunately, the N- and P- dopings of the PN-diode are unknown. However, it is known that N-side is much more heavily doped than the P-side.

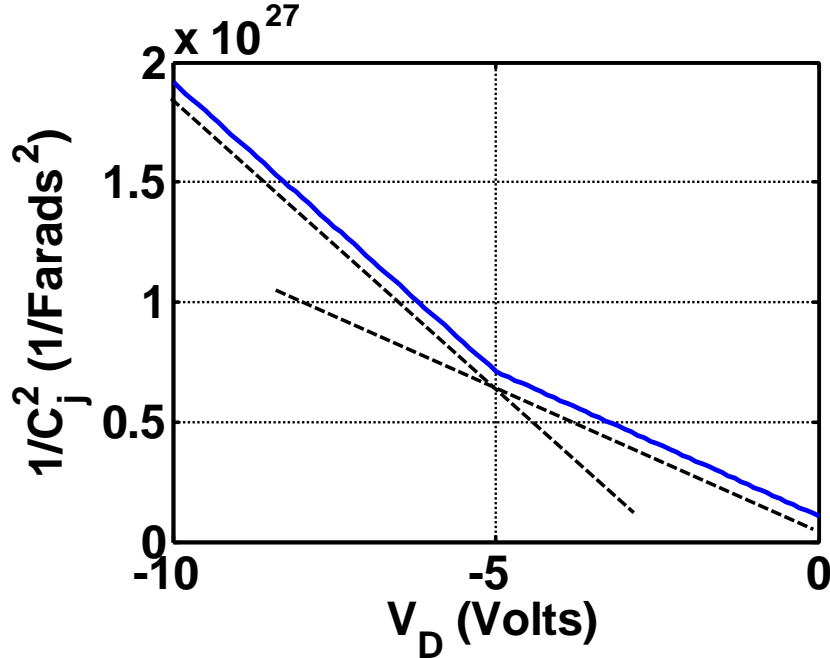
An undergraduate student comes up with a technique to figure out the dopings. He measures the depletion region capacitance of the PN diode in reverse bias, at different values of the reverse bias. He then plots the inverse square of the capacitance (i.e.  $1/C_j^2$ ) as a function of the reverse bias voltage and obtains the following curve:



- a) From the data shown above, figure out the built in voltage  $\phi_B$ .

b) From the information given, the data shown above, and your answer in part (a) above, figure out the dopings,  $N_a$  and  $N_d$ , on both sides of the junction.

Enamored by his experimental success, the undergraduate student decides to extend the negative voltage range of his data and obtains the following plot.



You can see that for values of  $V_D$  less than -5 Volts, the data changes slope (the magnitude of the slope increases by a factor of two).

c) Can you explain why the slope changes in the data shown above? What information can you obtain about the doping from the change in the slope in the data shown above? Explain your answers.

**Hint:** This problem will test your skills with electrostatics. The quantity of interest here is:

$$\frac{\partial(1/C_j^2)}{\partial V_D}$$

Imagine what will happen if the doping were to change slightly as a function of position (i.e. if  $N_d$  or  $N_a$  or both were a function of position:  $N_d(x)$  or  $N_a(x)$ ). The capacitance  $C_j$  can always be written

as  $\epsilon_s A/W$ , where  $W$  is the width of the depletion region. So in order to find  $\frac{\partial(1/C_j^2)}{\partial V_D}$ , you need to find

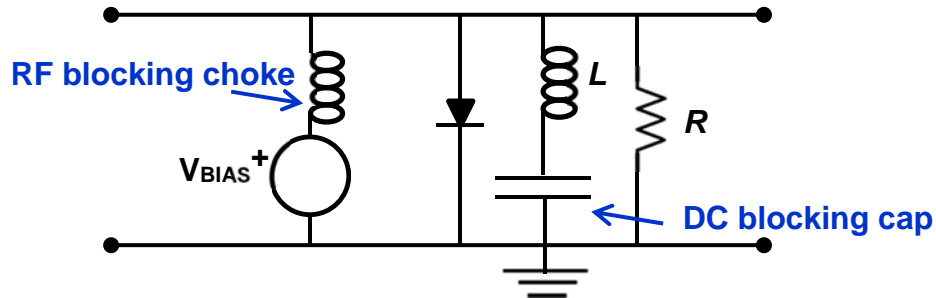
the value of  $W(\partial W/\partial V_D)$  and relate it to the doping right at the “edges” of the depletion region.

**Lesson:** Capacitance measurement (or capacitance spectroscopy) is a very useful tool to characterize the doping in semiconductor devices.

## Problem 2.4: (PN Diode Varactors)

Varactors are tunable capacitors. Tunable capacitors are important for tunable electrical filters and oscillators. Reverse biased PN diodes are commonly used as varactors. The junction depletion capacitance can be tuned by adjusting the reverse bias (and, therefore, the depletion region width).

The circuit for a LC oscillator with a silicon PN diode varactor is shown below. At the frequencies of interest, the blocking choke is an open circuit and the blocking capacitor is a short circuit, so both these elements can be ignored when the resonance frequency of the circuit is being calculated. The reverse bias on the diode is adjusted by the DC biasing voltage source.



The inductance of the inductor is  $L = 5 \text{ nH}$ . The dopings in the diode are:  $N_a = 10^{14} \text{ 1/cm}^3$  and  $N_d = 10^{16} \text{ 1/cm}^3$ .

- What ought to be the junction area (in  $\text{sq-}\mu\text{m}$ ) of the PN diode varactor so that the resonance frequency of the circuit is around 1 GHz under a reverse bias of zero voltage on the diode?
- If the resonant frequency of the circuit is tuned by adjusting the reverse bias, what is the maximum resonance frequency that can be attained before the junction breakdown occurs? Assume the value of the junction area you found in part (a).