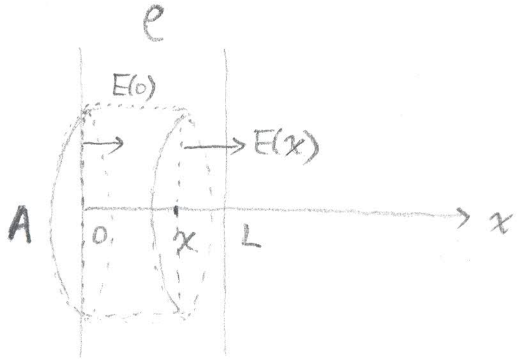
ECE 3150 Solutions HW #1

1. (a)

A nice way to solve this problem is to apply the integral form of Gauss's Law.



We choose a cylinder in the space as indicated on the left. The area of top/bottom is A.

According to Gauss's Law,

$$-E(0)A + E(x)A = \frac{\rho x A}{\varepsilon_0}. \quad \text{for } 0<x<L.$$

$$\Rightarrow \quad E(x) = \frac{\rho x}{\varepsilon_0} + E(0), \quad 0<x<L$$

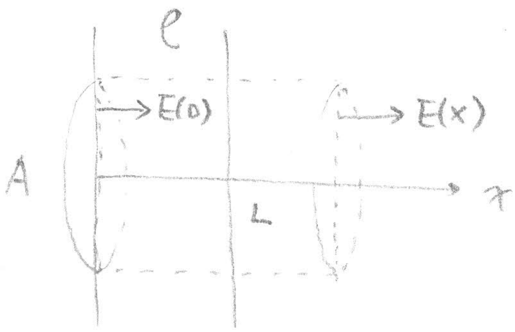
$$-\frac{d\phi}{dx} = E(x) \Rightarrow \phi(x) = -\frac{\rho x^2}{2\varepsilon_0} - E(0)x + \phi(0). \quad 0<x<L$$

$$\phi(L) = -\frac{\rho L^2}{2\varepsilon_0} - E(0)L + \phi(0)$$

(b) Choose a slightly different cylinder for $x>L$

According to Gauss's Law.

$$-E(0)A + E(x)A = \frac{\rho L}{\varepsilon_0}A. \quad \text{for } x>L$$



$$\Rightarrow \quad E(x) = \frac{\rho L}{\varepsilon_0} + E(0), \quad x>L$$

$$-\frac{d\phi}{dx} = E(x) \Rightarrow \int_{\phi(L)}^{\phi(x)} d\phi = -\int_L^x E\,dx$$

$$\Rightarrow \phi(x) = -\frac{\rho L^2}{2\varepsilon_0} - \frac{\rho L}{\varepsilon_0}x + \phi(0). \quad x>L$$

(c) According to the question, we have boundary conditions for the new system that the potential and electric field at $x=0$ are $\phi'(0)$ and $E'(0)$.

Following the same arguments in (a) and (b), we have

$$E(x) = \frac{\rho}{\varepsilon_0} x + E'(0), \quad 0 < x < L$$

$$E(x) = \frac{\rho L + \sigma}{\varepsilon_0} + E'(0), \quad x > L$$

$$\phi(x) = -\frac{\rho x^2}{2\varepsilon_0} - E'(0)x + \phi'(0), \quad 0 < x < L$$

$$\phi(x) = -\frac{\rho L^2}{2\varepsilon_0} - \frac{\rho L + \sigma}{\varepsilon_0}(x-L) - E'(0)x + \phi'(0), \quad x > L$$

2. Sample A: N type. $n_0 = 10^{17}\ cm^{-3}$, $p_0 = 10^{3}\ cm^{-3}$.

$\rho = 0.0391\ \Omega/m$. $\phi_n = 419\, mV$

Sample B: P type. $n_0 = 2\times 10^{4}\ cm^{-3}$. $p_0 = 5\times 10^{15}\ cm^{-3}$

$\rho = 2.0833\ \Omega/m$. $\phi_p = -341\, mV$

Sample C: Intrinsic. $n_0 = 10^{10}\ cm^{-3}$. $p_0 = 10^{10}\ cm^{-3}$

$\rho = 2.84\times 10^{5}\ \Omega/m$. $\phi = 0$

3.

(a) According to the plot

$$\phi_n = -0.03x + 0.42. \quad \text{in Volts} \quad x \text{ is in } \mu m$$

$$n = n_i e^{\frac{q\phi}{KT}} \quad \text{at room temperature}, \ KT \sim 26meV$$

$$n = n_i e^{\frac{\phi}{0.026}}, \quad n_i = 10^{10} \ cm^{-3}$$

$$P = \frac{n_i^2}{n} \quad \text{in } cm^{-3}$$

Matlab plots according to these fomulas are attached in the end.

(b) $$E = -\frac{d\phi}{dx} = 300 \ V/cm$$

(c) $$J_{drif} = n q_e \mu E \quad \text{where} \quad n = n_i e^{\frac{\phi}{0.026}} \quad q_e = 1.6 \times 10^{-19} C$$

$$\mu = 1000 \ cm^2/Vs \quad E = 300 \ V/cm$$

(d) $$J_{diff} = q_e D_e \frac{dn}{dx} \quad D_e = \frac{\mu KT}{q_e} \quad \text{where}$$

$$\mu = 1000 \ cm^2/Vs \quad KT = 0.026eV, \quad q_e = 1.6 \times 10^{-19} C$$

Matlab plots are attached in the end.

4

$$\frac{kT}{q} = \frac{D}{\mu} \Rightarrow \mu = \frac{qD}{kT}$$

$$R = \rho\frac{l}{A} \Rightarrow \rho = \frac{RA}{l}. \qquad \sigma = \frac{1}{\rho} = \frac{l}{RA}$$

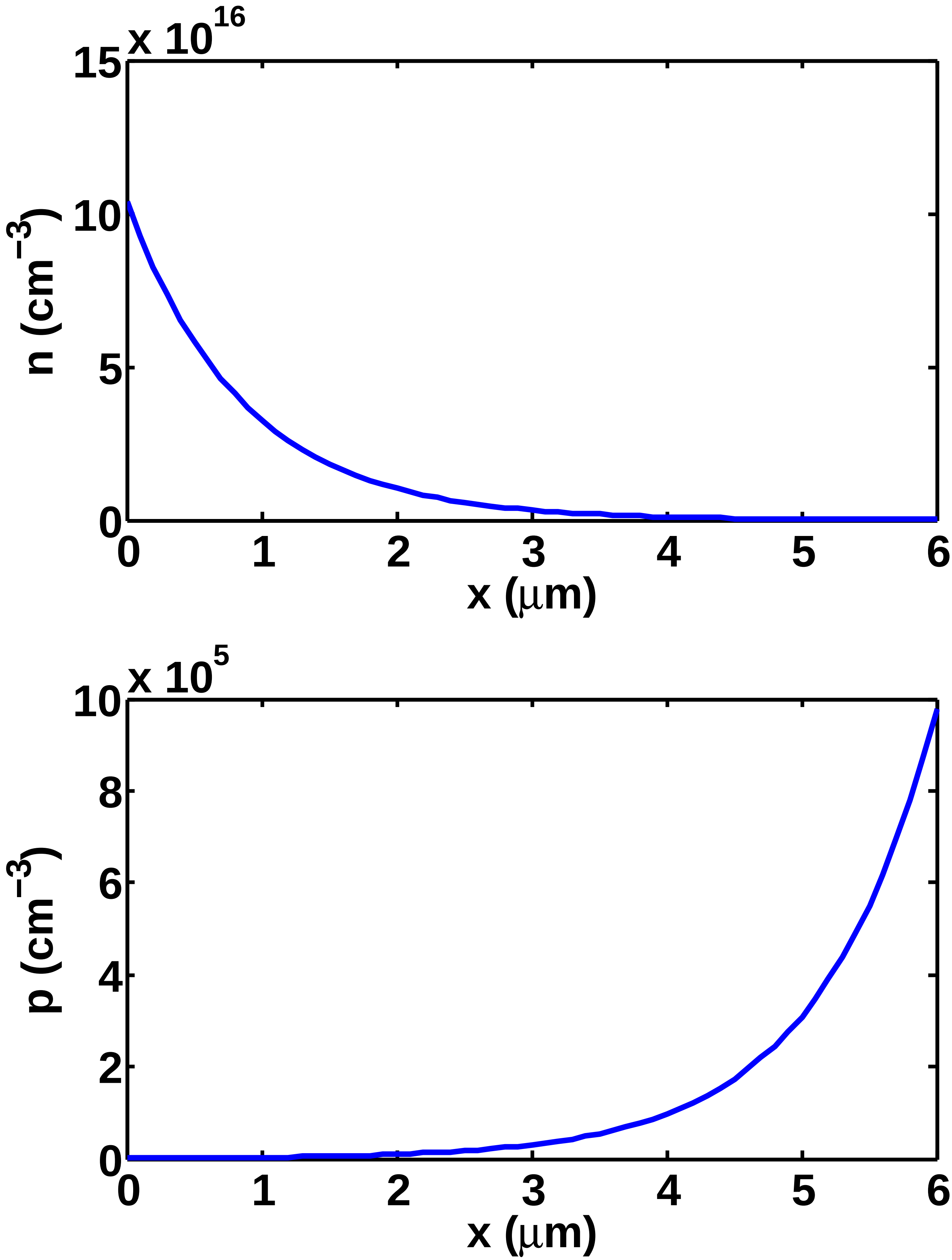
$$\sigma = p\mu q \Rightarrow p = \frac{\sigma}{\mu q} = \frac{\frac{l}{RA}}{\frac{qD}{kT}\cdot q}$$

where $l = 10\times10^{-6}\,m$. $R = 100\,\Omega$. $A = 10^{-12}\,m^2$. $q = 1.6\times10^{-19}\,C$

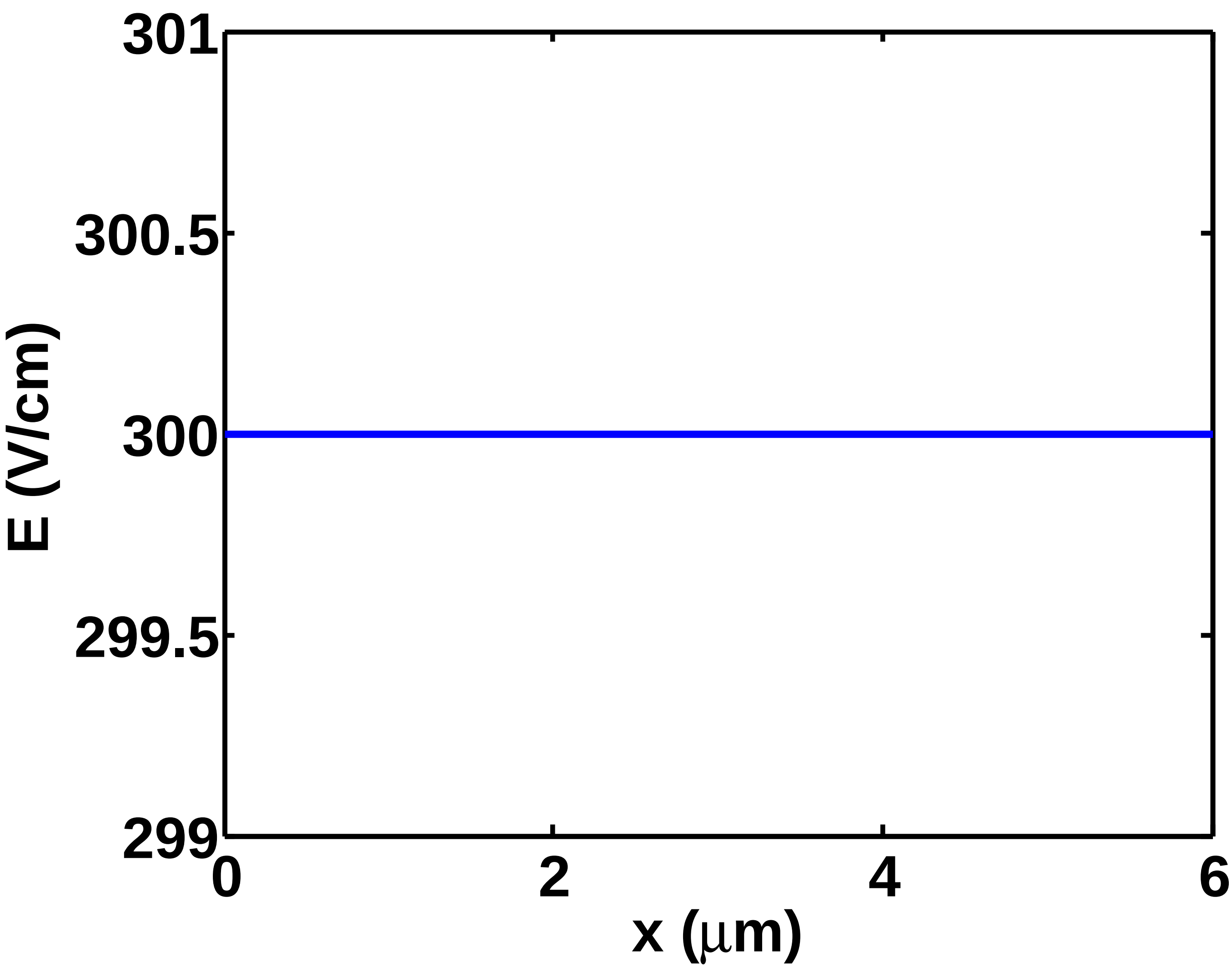
$D = 27\times10^{-4}\,m^2/s$. $\frac{kT}{q} = 0.026\,V$

By putting in all these numbers, we get

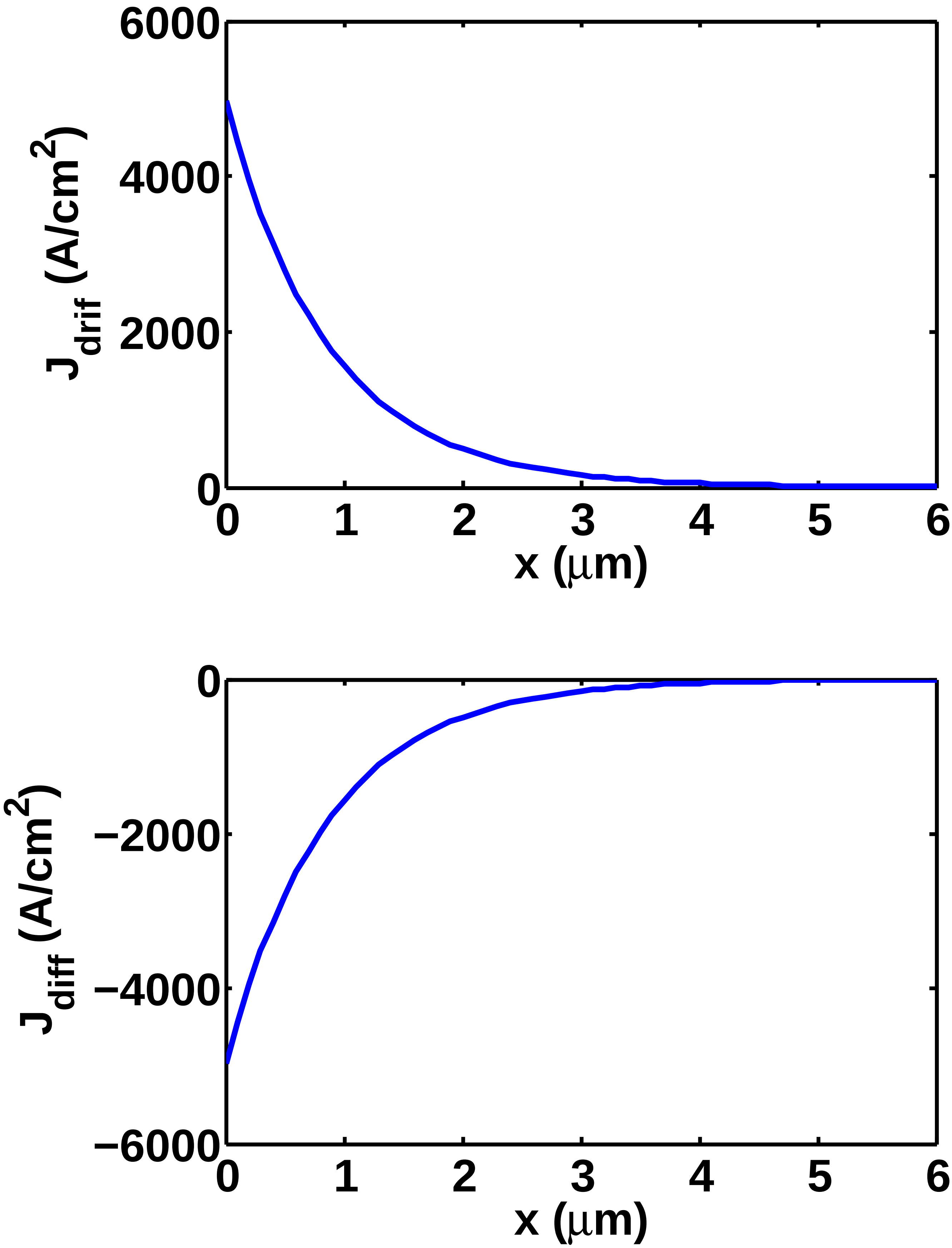
$$p = 6\times10^{24}\,m^{-3} = 6\times10^{18}\,cm^{-3}$$



x 10^16
15
10
5
0
n (cm^−3)
0 1 2 3 4 5 6
x (μm)
x 10^5
10
8
6
4
2
0
p (cm^−3)
0 1 2 3 4 5 6
x (μm)



301
300.5
300
299.5
299
E (V/cm)
0
2
4
6
x (μm)


6000
4000
2000
0
J_{drif} (A/cm^2)
0
1
2
3
4
5
6
x (μm)
0
−2000
−4000
−6000
J_{diff} (A/cm^2)
0
1
2
3
4
5
6
x (μm)