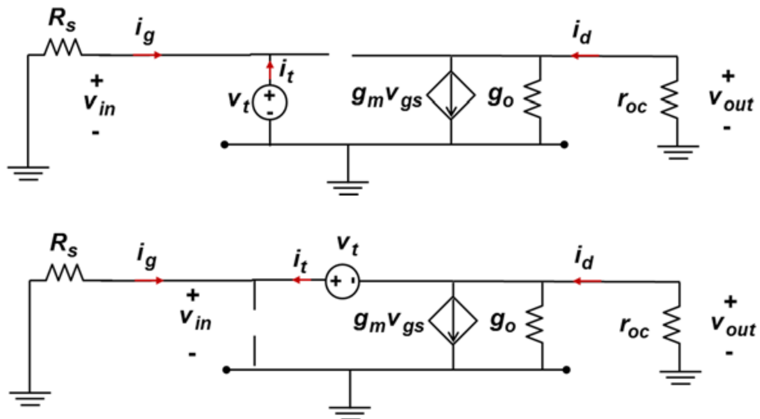
ECE Homework 11 Solutions

Problem 11.1:

a)



For C_{gs} use the top circuit above:

$$\frac{v_t}{i_t} = R_s$$

$$\Rightarrow \tau_a = R_s C_{gs}$$

For C_{gd} use the bottom circuit above:

$$\frac{v_{out} + v_t}{R_s} + g_m(v_{out} + v_t) + \frac{v_{out}}{(r_o \,||\, r_{oc})} = 0$$

$$\Rightarrow v_{out} = -v_t \frac{(r_o \,||\, r_{oc}) + g_m(r_o \,||\, r_{oc})R_s}{R_s + (r_o \,||\, r_{oc}) + g_m(r_o \,||\, r_{oc})R_s}$$

$$\Rightarrow \frac{v_t}{i_t} = R_s + (r_o \,||\, r_{oc}) + g_m(r_o \,||\, r_{oc})R_s$$

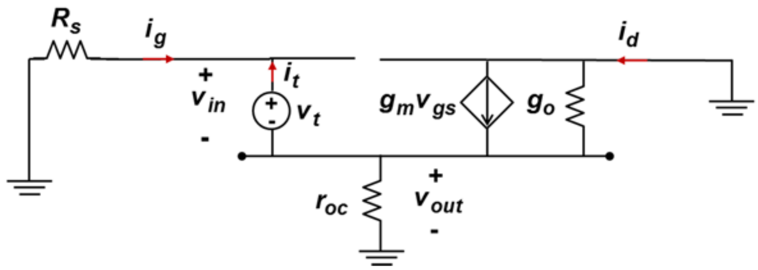
$$\Rightarrow \tau_b = [R_s + (r_o \,||\, r_{oc}) + g_m(r_o \,||\, r_{oc})R_s]C_{gd}$$

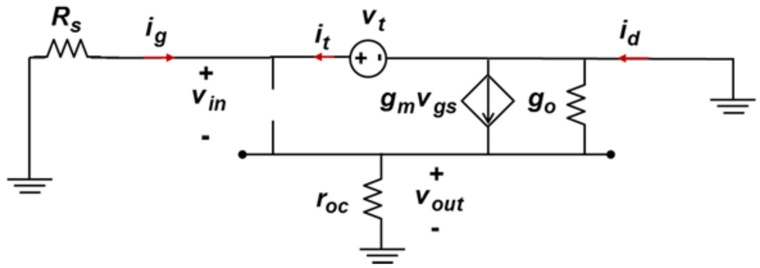
b)

$$\omega_H \sim \frac{1}{\tau_a + \tau_b}$$

The CS stage does suffer from the Miller effect. Note that the capacitor C_{gd} is multiplied by the zero-frequency gain $g_m(r_o \,||\, r_{oc})$ of the CS stage.

c)





For C_{gs} use the top circuit above:

$$\frac{v_{out} + v_t}{R_s} = i_t = -\frac{v_{out}}{(r_o \| r_{oc})} + g_m v_t$$

$$v_{out} = v_t (R_s \| r_o \| r_{oc}) \left[g_m - \frac{1}{R_s} \right]$$

$$\frac{v_t}{i_t} = \frac{R_s}{(R_s \| r_o \| r_{oc}) \left[g_m - \frac{1}{R_s} \right] + 1}$$

$$\Rightarrow \tau_a = \frac{R_s C_{gs}}{(R_s \| r_o \| r_{oc}) \left[g_m - \frac{1}{R_s} \right] + 1}$$

For C_{gd} use the bottom circuit above:

$$\frac{v_t}{i_t} = R_s$$

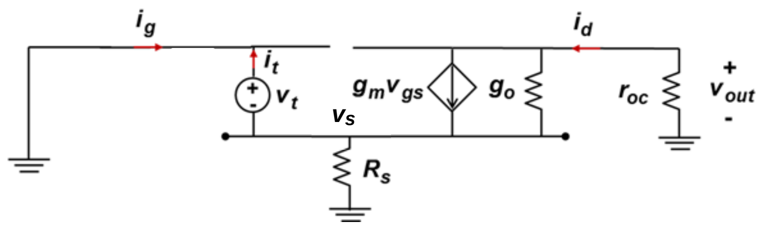
$$\Rightarrow \tau_b = R_s C_{gd}$$

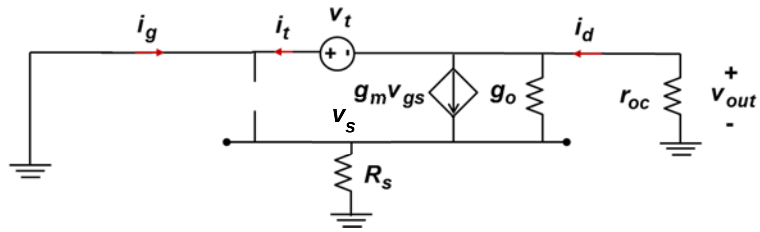
d)

$$\omega_H \sim \frac{1}{\tau_a + \tau_b}$$

The CD stage does have a capacitor sitting in the Miller position. But since the zero-frequency gain of the CD stage is around unity, there is no Miller enhancement.

e)





For C_{gs} use the top circuit above:

$$-\frac{v_s}{R_s} + g_m v_t + \frac{v_{out} - v_s}{r_o} = i_t \qquad g_m v_t + \frac{v_{out} - v_s}{r_o} = -\frac{v_{out}}{r_{oc}} \qquad v_s = -v_t$$

$$\Rightarrow v_{out} = (r_o \parallel r_{oc})\left[\frac{v_s}{r_o} - g_m v_t\right] = -v_t (r_o \parallel r_{oc})\left(\frac{1}{r_o} + g_m\right)$$

$$-\frac{v_s}{R_s} - \frac{v_{out}}{r_{oc}} = i_t$$

$$\Rightarrow \frac{v_t}{R_s} + \frac{v_t}{r_{oc}}(r_o \parallel r_{oc})\left(\frac{1}{r_o} + g_m\right) = i_t$$

$$\Rightarrow \frac{i_t}{v_t} = \frac{1}{R_s} + \frac{1 + g_m r_o}{r_o + r_{oc}}$$

$$\Rightarrow \frac{v_t}{i_t} = \frac{R_s (r_o + r_{oc})}{(1 + g_m r_o) R_s + r_o + r_{oc}}$$

$$\Rightarrow \tau_a = \frac{R_s (r_o + r_{oc}) C_{gs}}{(1 + g_m r_o) R_s + r_o + r_{oc}}$$

For C_{gd} use the bottom circuit above:

$$-\frac{v_{out}}{r_{oc}} = i_t - g_m v_s + \frac{v_{out} - v_s}{r_o} \qquad v_{out} = -v_t \qquad -g_m v_s + \frac{v_{out} - v_s}{r_o} = \frac{v_s}{R_s}$$

$$\Rightarrow v_s = v_{out} \frac{1}{r_o\left(g_m + \frac{1}{R_s}\right)}$$

$$\Rightarrow -\frac{v_{out}}{r_{oc}} = i_t + v_{out} \frac{1}{r_o (g_m R_s + 1)} \Rightarrow \frac{v_t}{r_{oc}} = i_t - v_t \frac{1}{r_o (g_m R_s + 1)}$$

$$\Rightarrow$$

$$\Rightarrow \tau_b = (r_{oc} \parallel r_o (g_m R_s + 1)) C_{gd}$$

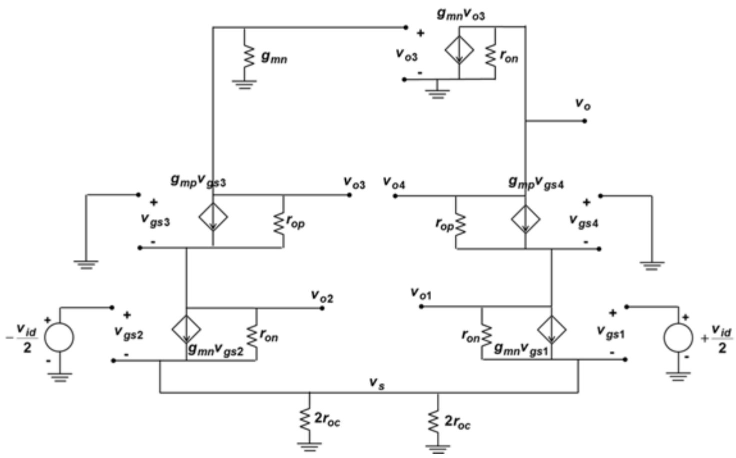
f)

$$\omega_H \sim \frac{1}{\tau_a + \tau_b}$$

The CG stage does not have a capacitor sitting in the Miller position.

Problem 11.2: (Folded Cascode Differential Amplifier)

a)

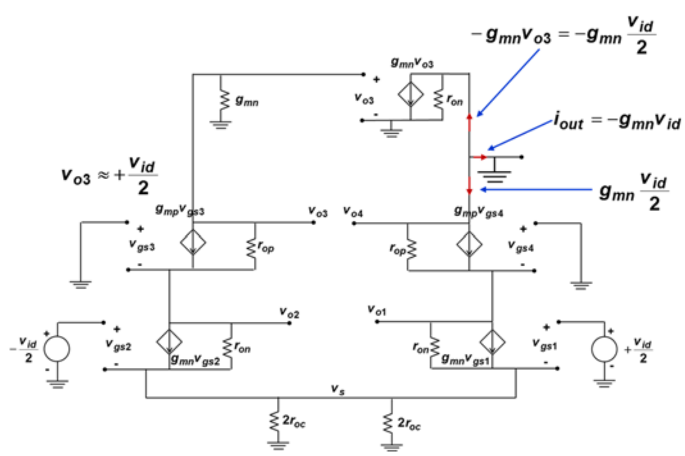


The small signal circuit is shown above. Since each arm of the diff amp contains a cascode, the standard cascode analysis will apply. We first find v_{o3} as a function of $-v_{id}/2$ input. Note that the cascode is loaded on top with a very small resistance $1/g_{mn}$. So the current pulled in by the cascode (drain current of M3) stage on the left side will be $-v_{id}\, g_{mn}/2$ and the voltage v_{o3} will be $v_{id}/2$. Similarly, the drain current of M4 will be $+v_{id}\, g_{mn}/2$. The cascode on the right side as well as the current source on the top on the right side pull current through the resistance r_{on} (on the top right side).

$$v_o = v_{o4} \approx -g_{mn} v_{o3} r_{on} - g_{mn} r_{on} \frac{v_{id}}{2} = -g_{mn} r_{on} v_{id}$$

$$\Rightarrow A_{vd} = -g_{mn} r_{on}$$

b)



Now the output is shorted to the ground. v_{o3} will equal $v_{id}/2$, as in part (a) above. But on the right hand side, the current pulled in by the cascode and the current pulled in by the current source on the top come not through the resistance resistance r_{on} (on the top right) but from the shorted output. So,

$$i_{out} \approx -g_{mn}v_{o3} - g_{mn}\frac{v_{id}}{2} = -g_{mn}v_{id}$$

c) From parts (a) and (b), the output resistance (ratio of open circuit output voltage to the short circuit output current) is just resistance r_{on}.