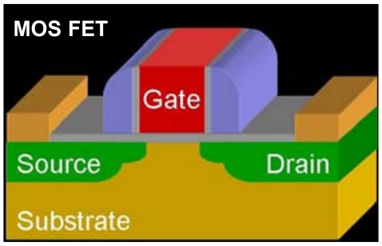
Lecture 8

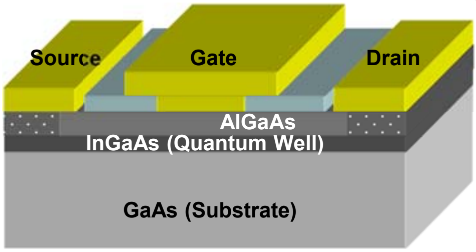
MOS (Metal Oxide Semiconductor) Capacitors

In this lecture you will learn:

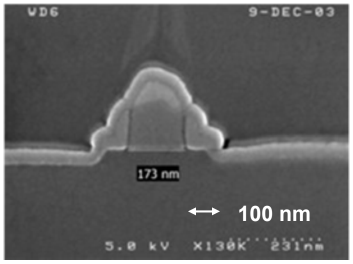
- The fundamental set of equations governing the behavior of PMOS capacitors
- Small signal models of the PMOS capacitor

MOS (Metal Oxide Semiconductor Field Effect Transistors (FETs)

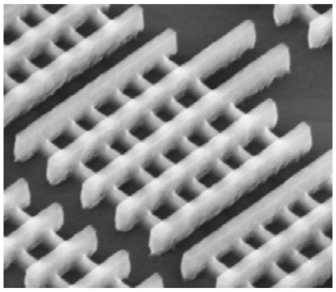




High Electron Mobility FET

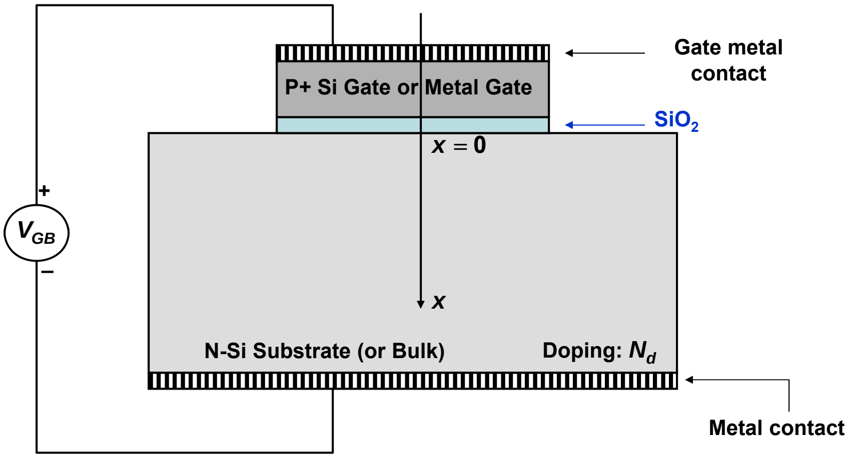


A 173 nm gate length MOS transistor (INTEL)

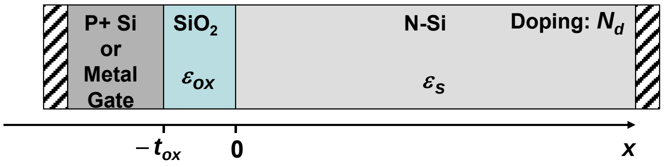


22 nm gate length MOS transistors (INTEL)

A P-MOS (or PMOS) Capacitor



A PMOS Capacitor



Assumptions:

1) The potential in the metal gate is ϕ_M
   If the gate is P+ Si then $\phi_M = \phi_p$

2) The potential deep in the p-Si substrate is ϕ_p

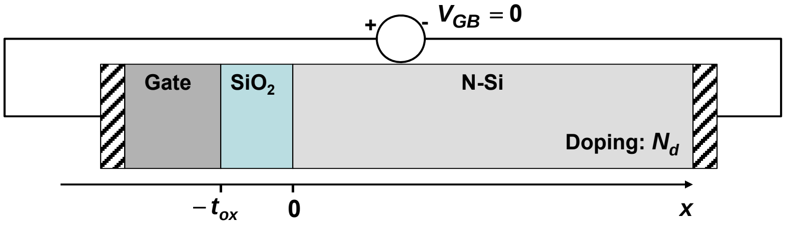
3) The oxide (SiO$_2$) is insulating (near zero conductivity; no free electrons and holes) and is completely free of any charges

4) There cannot be any volume charge density inside the metal gate (it is very conductive). But there can be a surface charge density on the surface of the metal gate

5) Dielectric constants:

$$\varepsilon_{ox} = 3.9\varepsilon_o \qquad \varepsilon_s = 11.7\varepsilon_o$$

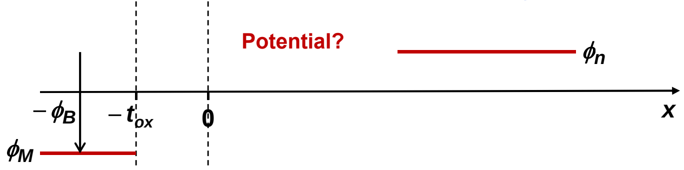
A PMOS Capacitor in Equilibrium



Potential Plot: $\phi(x)$

$\phi_B = \phi_M - \phi_n$

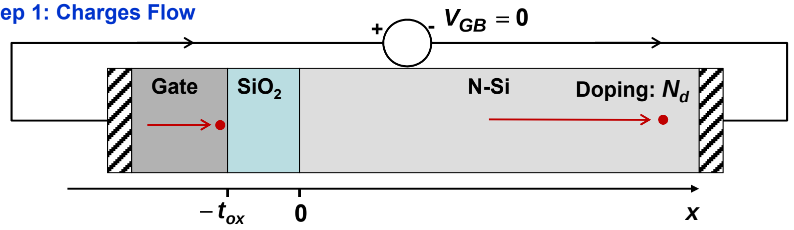
Assume: $\phi_B < 0$



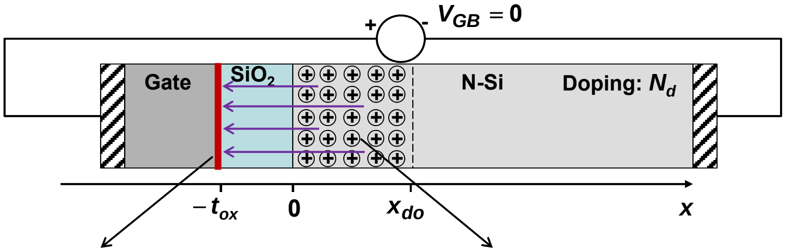
We need to find the potential in equilibrium everywhere

A PMOS Capacitor in Equilibrium: Depletion Region

Step 1: Charges Flow



Step 2: Depletion region is created in the substrate and a surface or sheet charge density on the metal gate



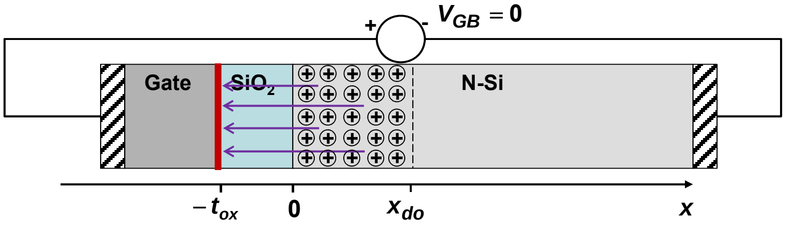
Negative surface charge density (C/cm^2)

$Q_G = -qN_d x_{do}$

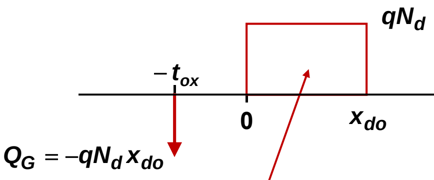
Positive depletion charge density (C/cm^3)

$\rho = +qN_d$

A PMOS Capacitor in Equilibrium: Charge Densities



Charge density plot:

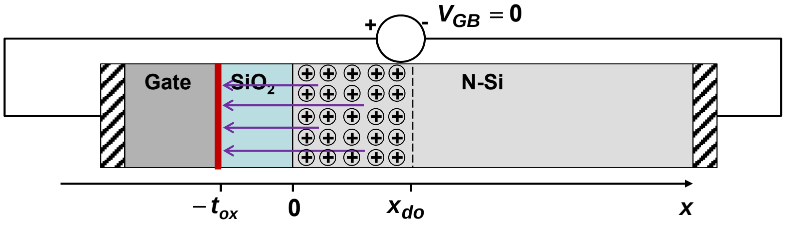


Depletion region charge density (C/cm³)

Total charge per unit area in the semiconductor (C/cm²)

$$Q_B = qN_d x_{do}$$

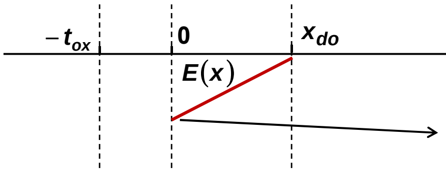
A PMOS Capacitor in Equilibrium: Electric Field



Electric field in the semiconductor:

$$\frac{dE_x}{dx} = \frac{\rho}{\varepsilon_s} = \frac{qN_d}{\varepsilon_s} \qquad \{E_x(x = x_{do}) = 0$$

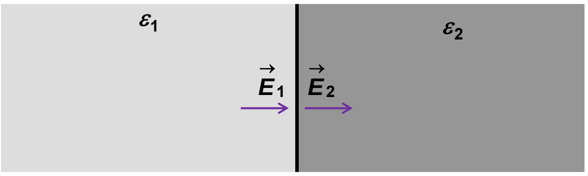
$$\Rightarrow E_x(x) = \frac{qN_d}{\varepsilon_s}(x - x_{do}) \longrightarrow$$ Linearly varying



$$E_x(x = 0) = -\frac{qN_d}{\varepsilon_s} x_{do}$$

Some Electrostatics

Consider an interface between media of different dielectric constants:



Suppose you know $\vec{E}_1$, can you find $\vec{E}_2$???

Use the principle: The product of the dielectric constant and the normal component of the electric field on both sides of an interface are related as follows:

$$\boxed{\varepsilon_2 \vec{E}_2 - \varepsilon_1 \vec{E}_1 = Q_I = \text{Interface sheet charge density}}$$

- Note that $\vec{E}_1$ is the electric field JUST to the left of the interface and $\vec{E}_2$ is the electric field JUST to right of the interface

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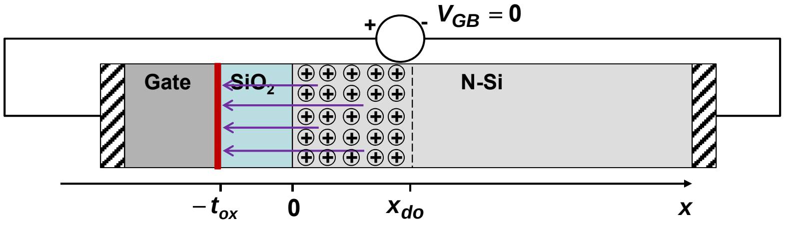
A PMOS Capacitor in Equilibrium: Electric Field



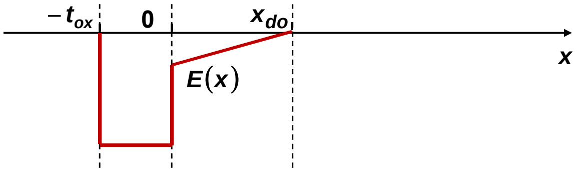
Electric field in the oxide:

$$\frac{dE_x}{dx} = \frac{\rho}{\varepsilon_{ox}} = 0$$

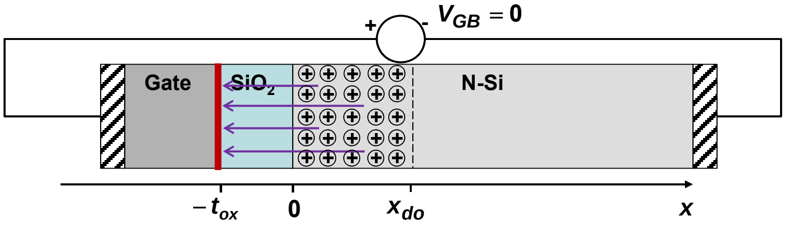
$$\Rightarrow E_x(x) = \text{constant}$$

$$\Rightarrow E_x(x) = -\frac{qN_d x_{do}}{\varepsilon_{ox}}$$

$$\left\{ \begin{array}{l} \varepsilon_{ox} E(x = 0^-) = \varepsilon_s E(x = 0^+) \\ E(x = 0^+) = -\dfrac{qN_d x_{do}}{\varepsilon_s} \\ \Rightarrow E(x = 0^-) = -\dfrac{qN_d x_{do}}{\varepsilon_{ox}} = E_{ox} \end{array} \right.$$



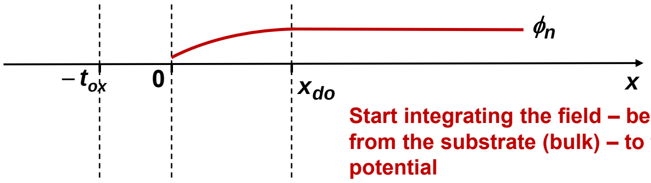
A PMOS Capacitor in Equilibrium: Potential



Potential in the semiconductor:

$$\frac{d\phi(x)}{dx} = -E_x(x) = -\frac{qN_d}{\varepsilon_s}(x - x_{do}) \qquad \{\ \phi(x = x_{do}) = \phi_n$$

$$\phi(x) = \phi_n - \frac{qN_d}{2\varepsilon_s}(x - x_{do})^2$$



Start integrating the field – beginning from the substrate (bulk) – to find the potential

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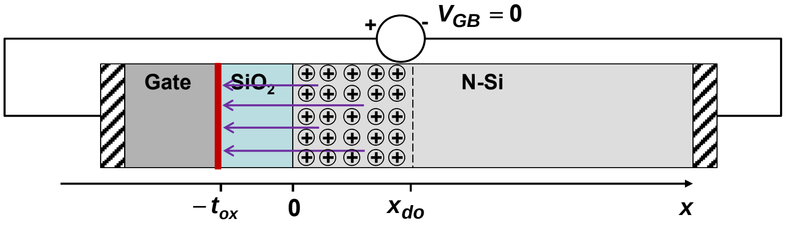
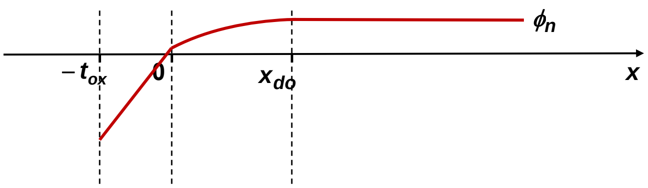
A PMOS Capacitor in Equilibrium: Potential



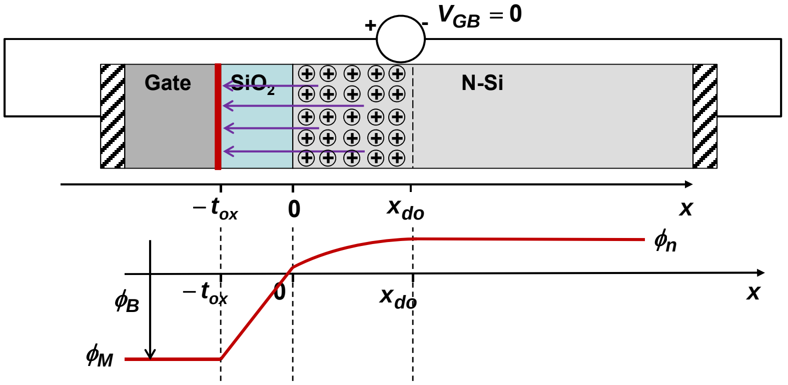
Potential in the oxide:

$$\frac{d\phi(x)}{dx} = -E_x(x) = \frac{qN_d x_{do}}{\varepsilon_{ox}} \qquad \{\ \phi(x = 0) = \phi_n - \frac{qN_d x_{do}^2}{2\varepsilon_s}$$

$$\phi(x) = \phi_n - \frac{qN_d x_{do}^2}{2\varepsilon_s} + \frac{qN_d x_{do}}{\varepsilon_{ox}} x$$



A PMOS Capacitor in Equilibrium: Potential



Must have:

$$\phi(x=-t_{ox}) = \phi_n - \frac{qN_d x_{do}^2}{2\varepsilon_s} - \frac{qN_d x_{do}}{\varepsilon_{ox}} t_{ox} = \phi_M$$

Therefore:

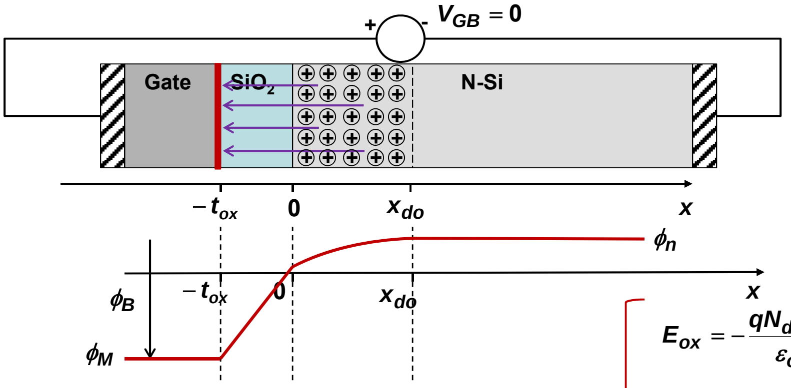
$$x_{do} = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_d}\right)(-\phi_B)}$$

$$\phi_B = \phi_M - \phi_n$$

Oxide capacitance (per unit area)

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

A PMOS Capacitor in Equilibrium: Potential



$$-\phi_B = V_{ox} + V_S = |E_{ox}|\, t_{ox} + V_S$$

$$\Rightarrow -\phi_B = \underbrace{\frac{qN_d x_{do}}{C_{ox}}}_{\text{Potential drop in the oxide}} + \underbrace{\frac{qN_d x_{do}^2}{2\varepsilon_s}}_{\text{Potential drop in the semiconductor}}$$

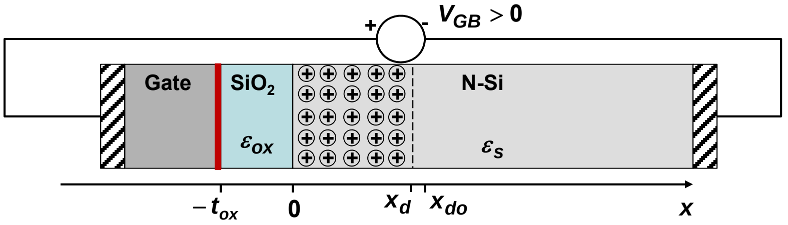
$$E_{ox} = -\frac{qN_d x_{do}}{\varepsilon_{ox}}$$

$$\phi_B = \phi_M - \phi_n$$

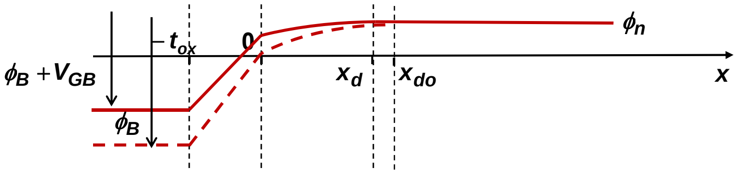
Oxide capacitance (per unit area)

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

A Biased PMOS Capacitor: $V_{GB} > 0$

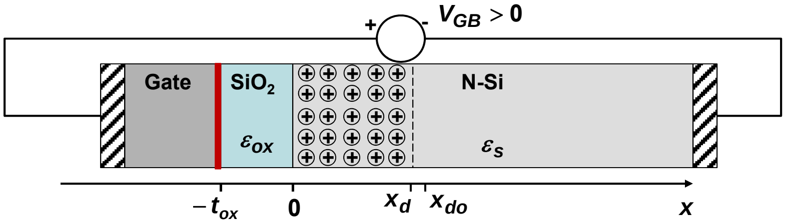


All of the applied bias falls across the depletion region and the oxide

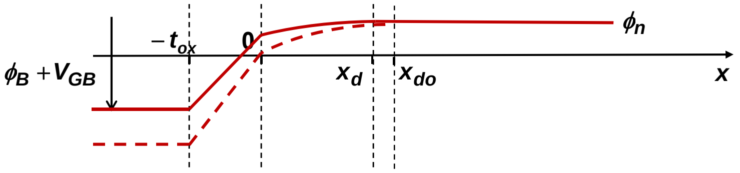


$$-\phi_B - V_{GB} = \underbrace{\frac{qN_d x_d}{C_{ox}}}_{\text{Potential drop in the oxide}} + \underbrace{\frac{qN_d x_d^2}{2\varepsilon_s}}_{\text{Potential drop in the semiconductor}}$$

A Biased PMOS Capacitor: $V_{GB} > 0$



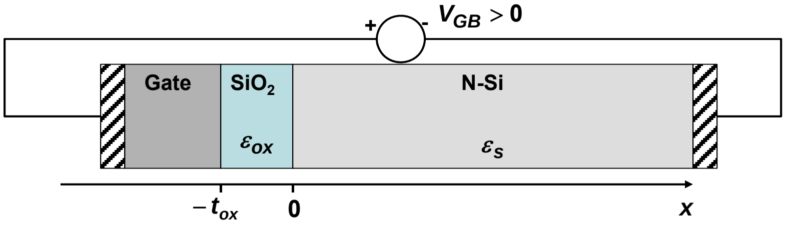
All of the applied bias falls across the depletion region and the oxide



The depletion region shrinks and the oxide field also decreases for $V_{GB} > 0$

$$x_d = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_d}\right)\left(-\phi_B - V_{GB}\right)} \qquad E_{ox} = -\frac{qN_a x_d}{\varepsilon_{ox}}$$

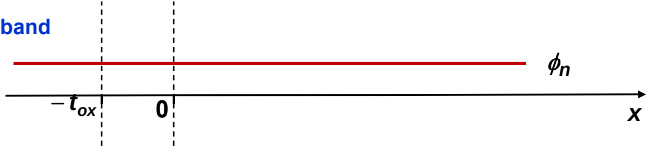
A Biased PMOS Capacitor: Flatband Condition



When V_{GB} is sufficiently positive, the depletion region thickness shrinks to zero

This value of V_{GB} is called the flatband voltage V_{FB}

Potential in flatband condition:

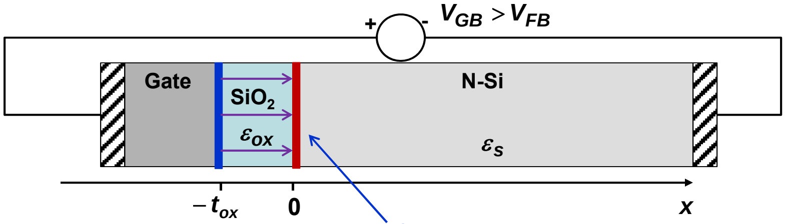


Flatband voltage:

$$x_d = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_d}\right)(-\phi_B - V_{FB})} = 0$$

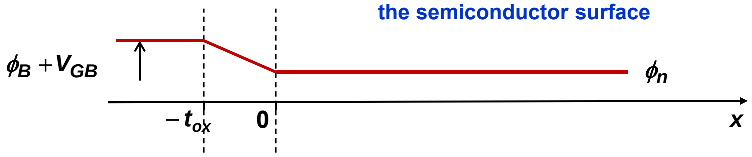
$$\Rightarrow V_{FB} = -\phi_B = -(\phi_M - \phi_n)$$

A Biased PMOS Capacitor: Accumulation ($V_{GB} > V_{FB}$)



Charge accumulation (due to electrons) on the semiconductor surface

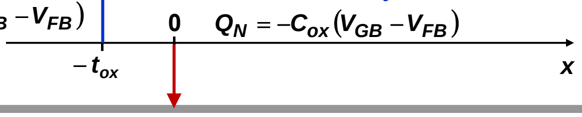
Potential:



The entire potential drop for $V_{GB} > V_{FB}$ falls across the oxide

Charge Density:

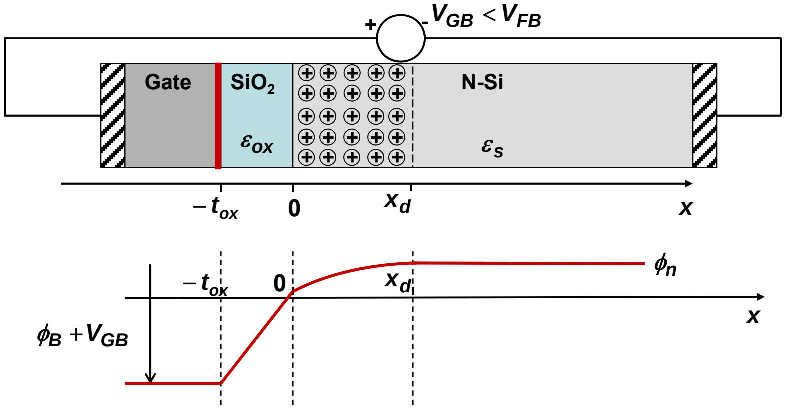
Total charge per unit area in the electron accumulation layer



$Q_G = C_{ox}(V_{GB} - V_{FB})$

$Q_N = -C_{ox}(V_{GB} - V_{FB})$

A Biased PMOS Capacitor: Depletion ($V_{GB} < V_{FB}$)

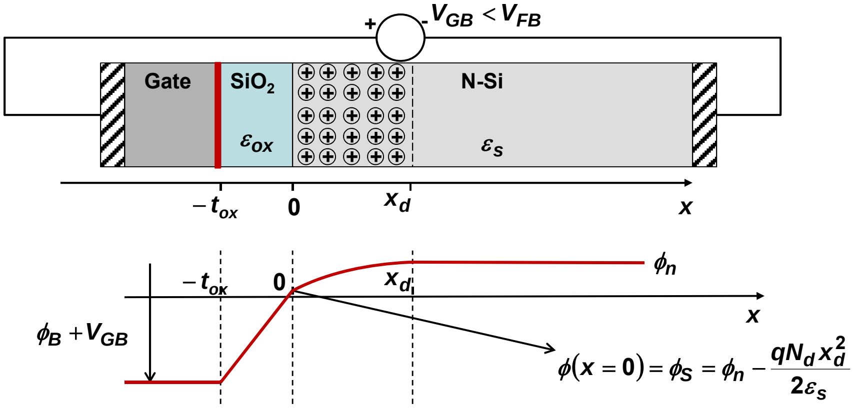


$$-\phi_B - V_{GB} = V_{ox} + V_S$$

$$V_{GB} - V_{FB} = \underbrace{-\frac{qN_d x_d}{C_{ox}}}_{\text{Potential drop in the oxide}} \underbrace{-\frac{qN_d x_d^2}{2\varepsilon_S}}_{\text{Potential drop in the semiconductor}}$$

The depletion region widens and the oxide field also increases for $V_{GB} < V_{FB}$

A Biased PMOS Capacitor: Depletion ($V_{GB} < V_{FB}$)

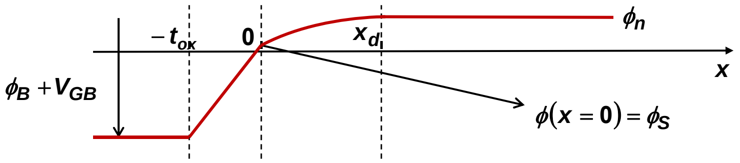


$$\phi(x=0) = \phi_S = \phi_n - \frac{qN_d x_d^2}{2\varepsilon_S}$$

$$V_{GB} - V_{FB} = -\frac{qN_d x_d}{C_{ox}} - \frac{qN_d x_d^2}{2\varepsilon_S}$$

$$\Rightarrow V_{GB} - V_{FB} = \phi_S - \phi_n - \frac{\sqrt{2\varepsilon_S qN_d(-\phi_S + \phi_n)}}{C_{ox}}$$

A Biased PMOS Capacitor: Hole Density



$$V_{GB} - V_{FB} = \phi_S - \phi_n - \frac{\sqrt{2\,\varepsilon_s q N_d\left(-\phi_S + \phi_n\right)}}{C_{ox}}$$

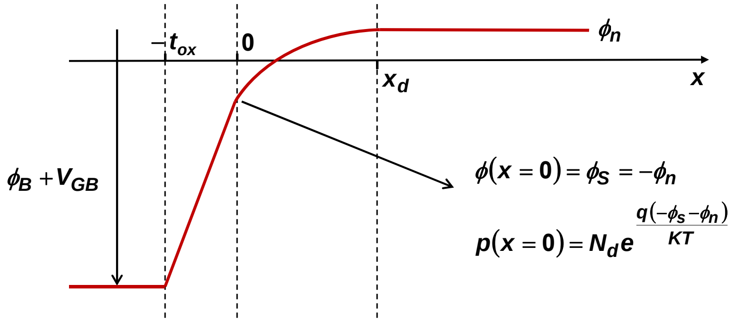
- As V_{GB} is decreased, ϕ_S also decreases
- The hole density in the semiconductor depends on the potential as:

$$p(x) = n_i e^{-\frac{q\phi(x)}{KT}} = n_i e^{\frac{q\phi_n}{KT}} e^{\frac{q(-\phi(x)-\phi_n)}{KT}} = N_d e^{\frac{q(-\phi(x)-\phi_n)}{KT}}$$

Hole density is the largest right at the surface of the semiconductor where the potential is the lowest

$$p(x=0) = N_d e^{\frac{q(-\phi_S-\phi_n)}{KT}}$$

A Biased PMOS Capacitor: Threshold Condition



$$\phi(x=0) = \phi_S = -\phi_n$$

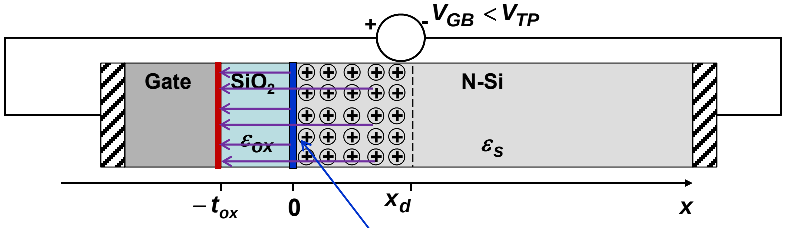
$$p(x=0) = N_d e^{\frac{q(-\phi_S-\phi_n)}{KT}} = N_d$$

- When V_{GB} is decreased and the surface potential ϕ_S reaches -ϕ_n the positive hole charge density at the surface becomes comparable to the positive charge density in the depletion region and cannot be ignored

- The gate voltage at which ϕ_S equals -ϕ_n is called the threshold voltage V_{TP}:

$$V_{TP} - V_{FB} = -2\phi_n - \frac{\sqrt{2\varepsilon_s q N_d\left(2\phi_n\right)}}{C_{ox}}$$

A Biased PMOS Capacitor: Inversion ($V_{GB} < V_{TP}$)

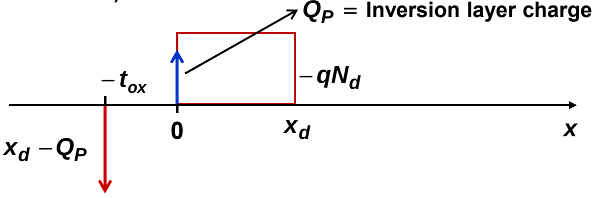


Inversion layer charge (due to holes) on the semiconductor surface

- When the gate voltage V_{GB} is decreased below V_{TP} the hole density right at the surface increases (exponentially with the decrease in the surface potential ϕ_S)

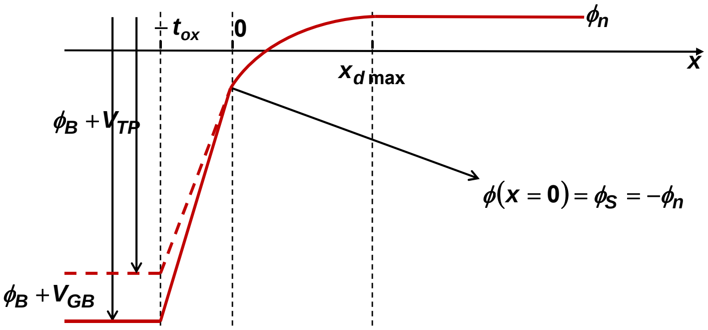
- This surface hole density is called the inversion layer (assumed to be of zero thickness in this course)

Q_P = Inversion layer charge density (C/cm^2)



$Q_G = -qN_d x_d - Q_P$

A Biased PMOS Capacitor: Inversion ($V_{GB} < V_{TP}$)

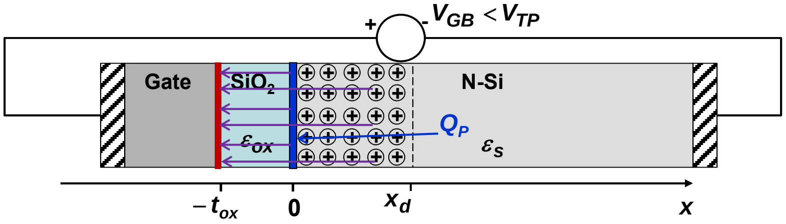


$\phi(x=0) = \phi_S = -\phi_n$

- When the gate voltage V_{GB} is decreased below V_{TP} the inversion layer charge increases so rapidly that the extra applied potential drops entirely across the oxide, and the surface potential ϕ_S remains close to -ϕ_n
- Consequently, the depletion region thickness (and the depletion region charge) does not increase when the gate voltage V_{GB} is decreased below V_{TP}

$$\phi_S - \phi_n = -\frac{qN_d x_d^2}{2\varepsilon_s} \Rightarrow 2\phi_n = \frac{qN_d x_{d\max}^2}{2\varepsilon_s} \left\{ \begin{array}{l} V_{TP} = V_{FB} - 2\phi_n - \dfrac{\sqrt{2\varepsilon_s qN_d (2\phi_n)}}{C_{ox}} \\ \Rightarrow V_{TP} = V_{FB} - \dfrac{qN_d x_{d\max}}{C_{ox}} - \dfrac{qN_d x_{d\max}^2}{2\varepsilon_s} \end{array} \right.$$

A Biased NMOS Capacitor: Inversion (V_{GB} < V_{TP})



How to calculate the inversion layer charge Q_P when $V_{GB} < V_{TP}$?

Start from: $V_{FB} - V_{GB} = V_{ox} + V_S$

$$= -E_{ox} t_{ox} + \frac{qN_a x_{d\max}^2}{2\varepsilon_S} \qquad \left\{ V_S = \frac{qN_d x_{d\max}^2}{2\varepsilon_S} \right.$$

By Gauss' law: $-\varepsilon_{ox} E_{ox} = Q_P + qN_d x_{d\max}$

Therefore:

$$V_{FB} - V_{GB} = +\frac{Q_P}{C_{ox}} + \frac{qN_d x_{d\max}}{C_{ox}} + \frac{qN_d x_{d\max}^2}{2\varepsilon_S}$$

$$\Rightarrow V_{GB} = -\frac{Q_P}{C_{ox}} + V_{TP}$$

$$\boxed{\Rightarrow Q_P = -C_{ox}\left(V_{GB} - V_{TP}\right)}$$

A Biased PMOS Capacitor: Summary of Different Regimes

Flatband (V_{GB} = V_{FB}):

No depletion region in the semiconductor and no accumulation charge

Accumulation (V_{GB} > V_{FB}):

No depletion region in the semiconductor but majority carrier accumulation charge on the surface of the semiconductor

Depletion (V_{TP} < V_{GB} < V_{FB}):

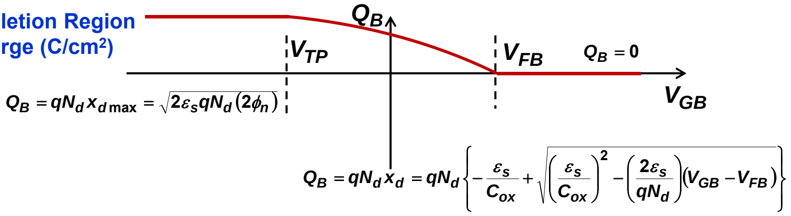
Depletion region in the semiconductor but no majority carrier accumulation charge or minority carrier inversion charge on the surface of the semiconductor

Inversion (V_{GB} < V_{TP}):

Depletion region in the semiconductor and minority carrier inversion charge on the surface of the semiconductor

A Biased PMOS Capacitor: Charges

Depletion Region Charge (C/cm²)

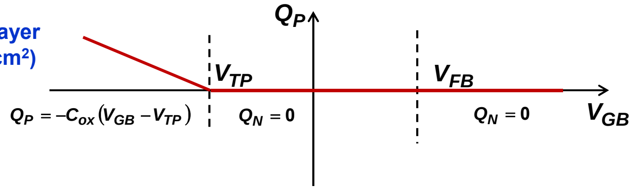


$Q_B = 0$

$Q_B = qN_d x_{d\max} = \sqrt{2\varepsilon_s qN_d(2\phi_n)}$

$$Q_B = qN_d x_d = qN_d\left\{-\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 - \left(\frac{2\varepsilon_s}{qN_d}\right)(V_{GB} - V_{FB})}\right\}$$

Inversion Layer Charge (C/cm²)



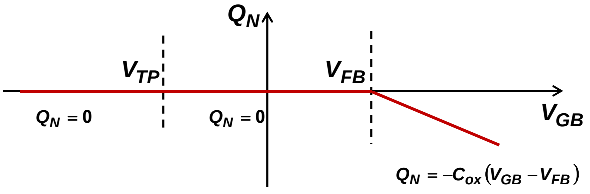
$Q_P = -C_{ox}(V_{GB} - V_{TP})$

$Q_N = 0$

$Q_N = 0$

A Biased PMOS Capacitor: Charges

Accumulation Layer Charge (C/cm²)

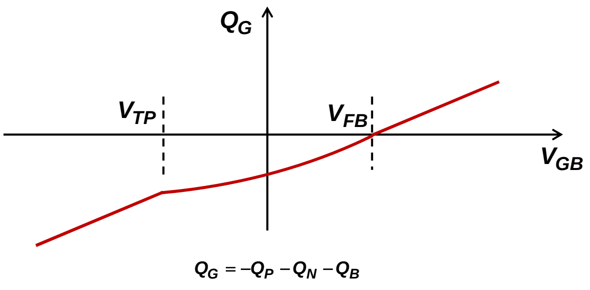


$Q_N = 0$

$Q_N = 0$

$Q_N = -C_{ox}(V_{GB} - V_{FB})$

Gate Charge (C/cm²)



$Q_G = -Q_P - Q_N - Q_B$

The Small Signal Capacitance of a PMOS Capacitor

- The small signal capacitance (per unit area) of the MOS capacitor is defined as:

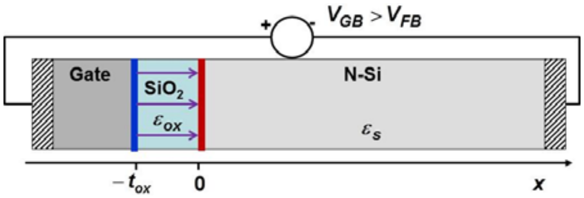
$$C = \frac{dQ_G}{dV_{GB}}$$

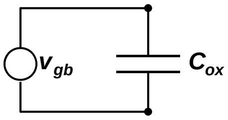
where Q_G is the charge density (units: C/cm^2) on the gate

(1) Accumulation ($V_{GB} > V_{FB}$):

$$Q_G = C_{ox}\left(V_{GB} - V_{FB}\right)$$

$$\Rightarrow \quad C = C_{ox}$$



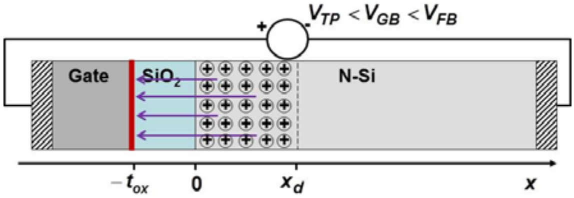


The Small Signal Capacitance of a PMOS Capacitor

(2) Depletion ($V_{TP} < V_{GB} < V_{FB}$):

$$Q_G = -qN_d x_d$$

$$C = \frac{dQ_G}{dV_{GB}} = -qN_d \frac{dx_d}{dV_{GB}}$$

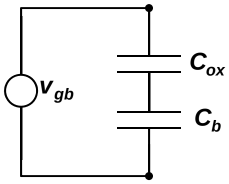


Differentiate the equation (derived earlier):

$$\frac{qN_d x_d^2}{2\,\varepsilon_s} + \frac{qN_d x_d}{C_{ox}} = -V_{GB} + V_{FB}$$

To get: $\left[\frac{x_d}{\varepsilon_s} + \frac{1}{C_{ox}}\right] qN_d \, dx_d = -dV_{GB}$

Define: $C_b = \frac{\varepsilon_s}{x_d}$



Finally: $\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_b}$

The Small Signal Capacitance of a PMOS Capacitor

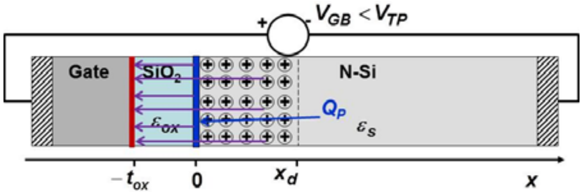
(3) Inversion ($V_{GB} < V_{TP}$):

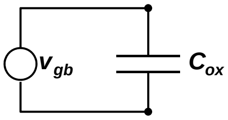
$$Q_G = -qN_d x_{d\max} - Q_P$$

$$C = \frac{dQ_G}{dV_{GB}} = -\frac{dQ_P}{dV_{GB}} = C_{ox}$$

$$Q_P = -C_{ox}(V_{GB} - V_{TP})$$

x_{dmax} does not change with V_{GB} in inversion

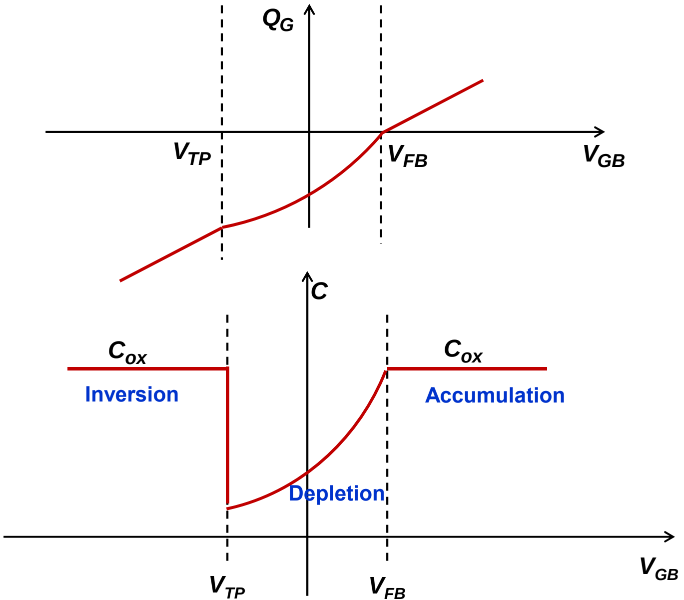




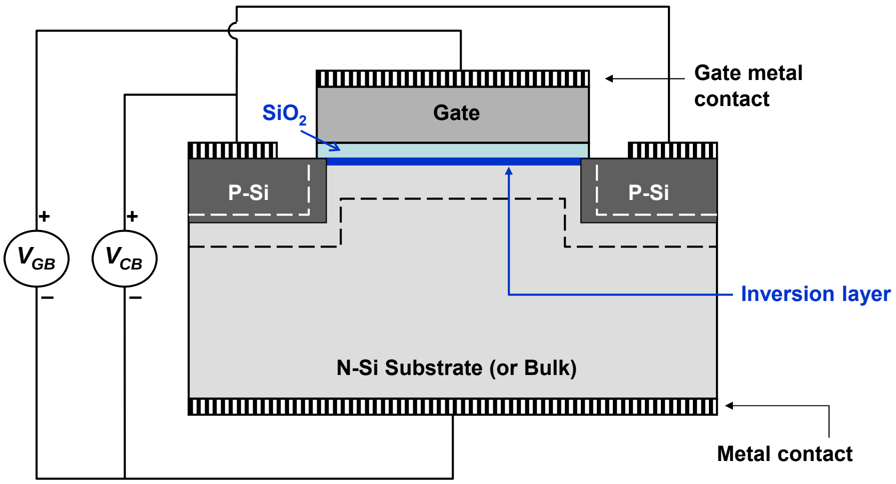
The Small Signal Capacitance of a PMOS Capacitor

Gate Charge (C/cm^2)

$$C = \frac{dQ_G}{dV_{GB}}$$

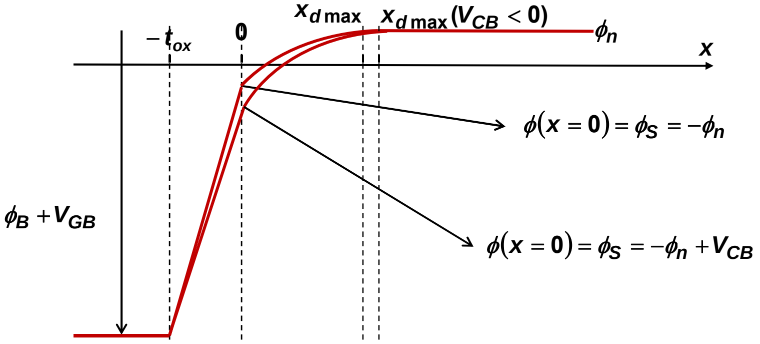


A PMOS Capacitor with a Channel Contact



- In the presence of an inversion layer, the additional contacts allow one to directly change the potential of the inversion layer channel w.r.t. to the bulk (substrate)

A Biased PMOS Capacitor: Inversion with V_{CB} ≠0

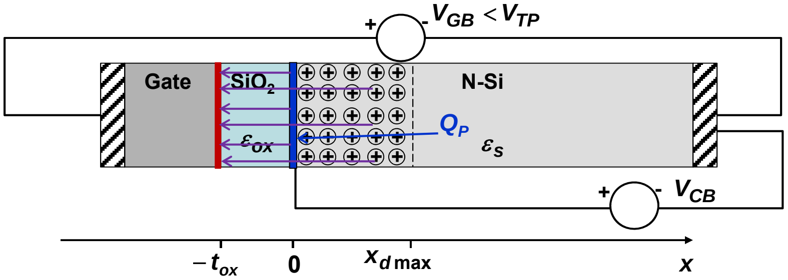


- We had said that the surface potential ϕ_S remains fixed at $-\phi_n$ when V_{GB} is decreased below V_{TP}
- But with a non-zero V_{CB}, the surface potential ϕ_S in inversion can be changed to ($-\phi_n + V_{CB}$)
- The new value of the depletion region width is:

$$\phi_s - \phi_n = -\frac{qN_d x_d^2}{2\varepsilon_s} \Rightarrow -2\phi_n + V_{CB} = -\frac{qN_d x_{d\,\max}^2}{2\varepsilon_s}$$

Question: How do we now find the inversion layer charge Q_P when V_{CB} is not zero?

A Biased NMOS Capacitor: Inversion with $V_{CB} \neq 0$



How to calculate the inversion layer charge Q_P? Same way as before……..

Start from: $V_{FB} - V_{GB} = V_{ox} + V_S$

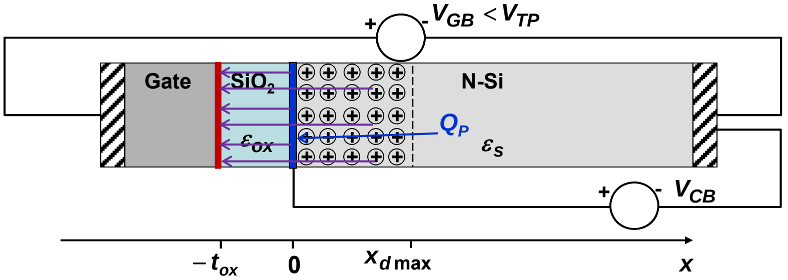
$$= -E_{ox}t_{ox} + \frac{qN_d x_{d\max}^2}{2\varepsilon_S} \qquad \left\{ V_S = \frac{qN_a x_{d\max}^2}{2\varepsilon_S} \right.$$

By Gauss' law: $-\varepsilon_{ox}E_{ox} = Q_P + qN_d x_{d\max}$

Therefore: $V_{FB} - V_{GB} = \frac{Q_P}{C_{ox}} + \frac{qN_d x_{d\max}}{C_{ox}} + \frac{qN_d x_{d\max}^2}{2\varepsilon_S}$

$$V_{GB} = -\frac{Q_P}{C_{ox}} + V_{FB} - \frac{qN_d x_{d\max}}{C_{ox}} - \frac{qN_d x_{d\max}^2}{2\varepsilon_S}$$

A Biased NMOS Capacitor: Inversion with $V_{CB} \neq 0$



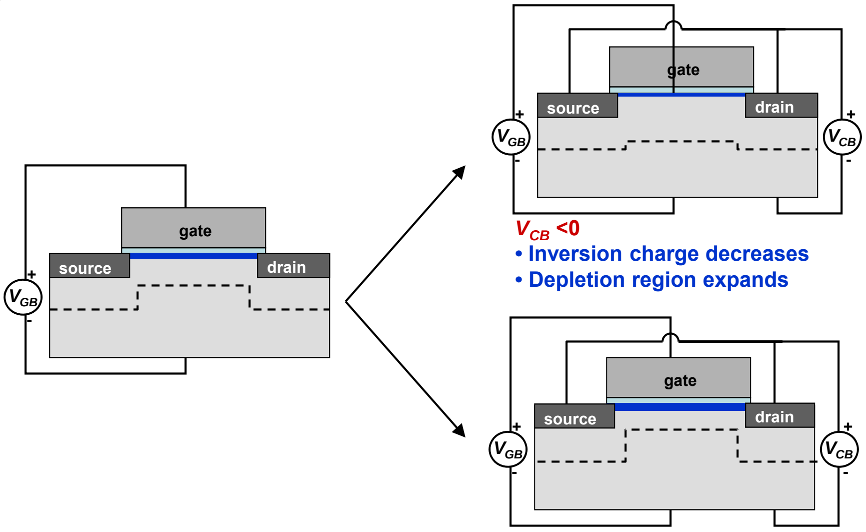
$$V_{GB} = -\frac{Q_P}{C_{ox}} \underbrace{+ V_{FB} - \frac{qN_a x_{d\max}}{C_{ox}} - \frac{qN_a x_{d\max}^2}{2\varepsilon_S}}$$

$$V_{TP} = V_{FB} - \frac{qN_d x_{d\max}}{C_{ox}} - \frac{qN_d x_{d\max}^2}{2\varepsilon_S}$$

$$= V_{FB} - 2\phi_n + V_{CB} - \frac{\sqrt{2\varepsilon_S qN_d(2\phi_n - V_{CB})}}{C_{ox}}$$

$\Rightarrow Q_P = -C_{ox}(V_{GB} - V_{TP})$ ⟶ Same as before but now V_{TP} depends on V_{CB}

PMOS Capacitor: Effect of V_{CB} ($V_{GB} < V_{TP}$)



V_{CB} <0

- Inversion charge decreases
- Depletion region expands

V_{CB} >0

- Inversion charge increases
- Depletion region shrinks