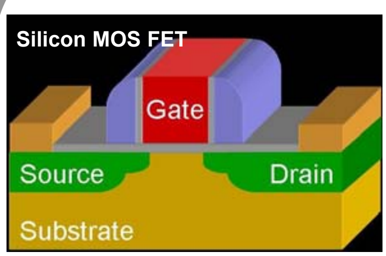
Lecture on

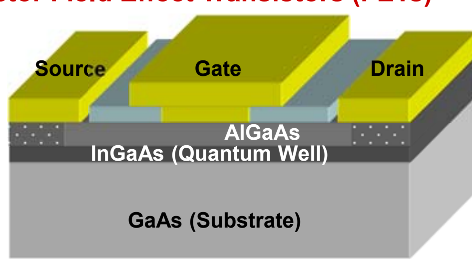
MOS (Metal Oxide Semiconductor) Capacitors

In this lecture you will learn:

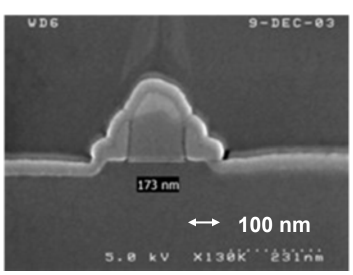
- The fundamental set of equations governing the behavior of NMOS capacitors
- Accumulation, Flatband, Depletion, and Inversion Regimes
- Small signal models of the NMOS capacitor

MOS (Metal Oxide Semiconductor Field Effect Transistors (FETs)

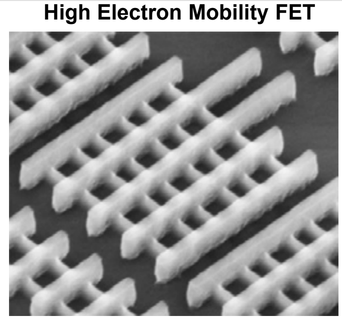




High Electron Mobility FET

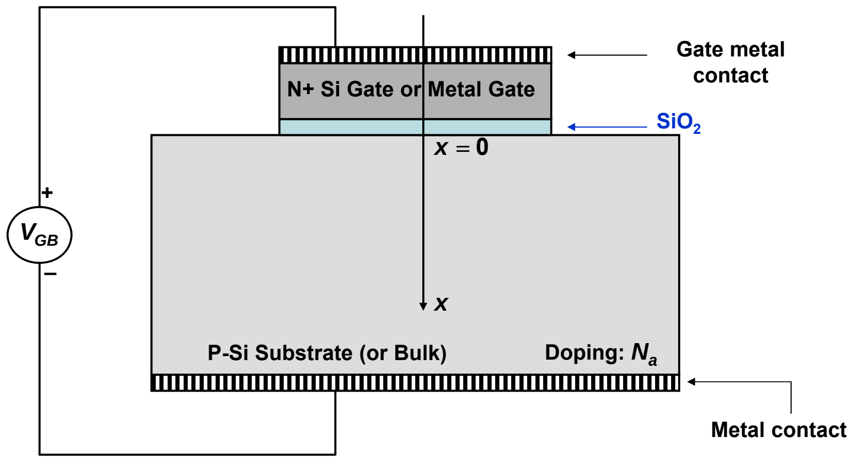


A 173 nm gate length MOS transistor (INTEL)

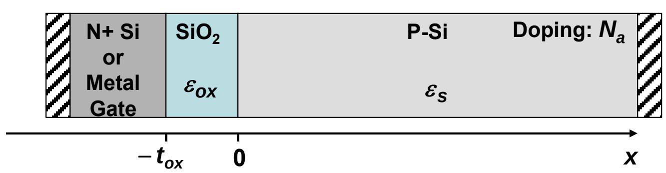


22 nm gate length MOS transistors (INTEL)

A N-MOS (or NMOS) Capacitor



A NMOS Capacitor



Assumptions:

1) The potential in the metal gate is ϕ_M
   If the gate is N+ Si then $\phi_M = \phi_n$

2) The potential deep in the P-Si substrate is ϕ_p

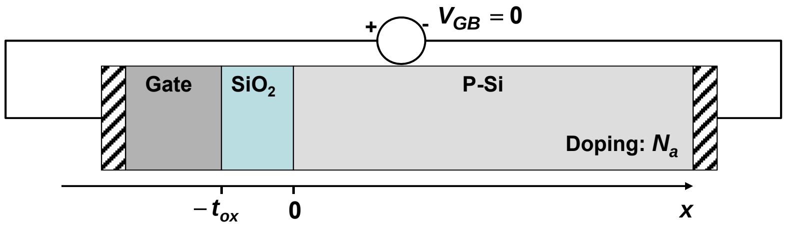
3) The oxide (SiO_2) is insulating (near zero conductivity; no free electrons and holes) and is completely free of any charges

4) There cannot be any volume charge density inside the metal gate (it is very conductive). But there can be a surface charge density on the surface of the metal gate

5) Dielectric constants:

$$\varepsilon_{ox} = 3.9\varepsilon_o \qquad \varepsilon_s = 11.7\varepsilon_o$$

A NMOS Capacitor in Equilibrium



Potential Plot: $\phi(x)$

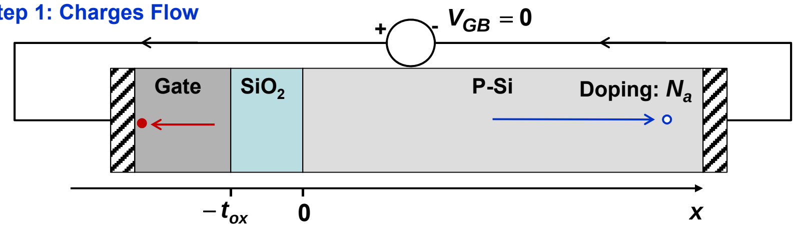
$\phi_B = \phi_M - \phi_p$

Assume: $\phi_B > 0$

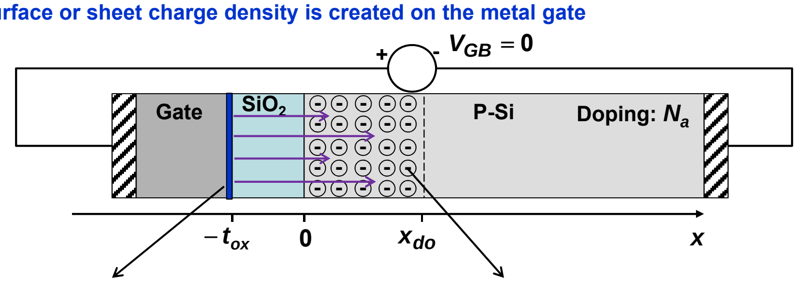
We need to find the potential in equilibrium everywhere

A NMOS Capacitor in Equilibrium: Depletion Region

Step 1: Charges Flow



Step 2: Depletion region is created in the substrate near the oxide interface, and a surface or sheet charge density is created on the metal gate



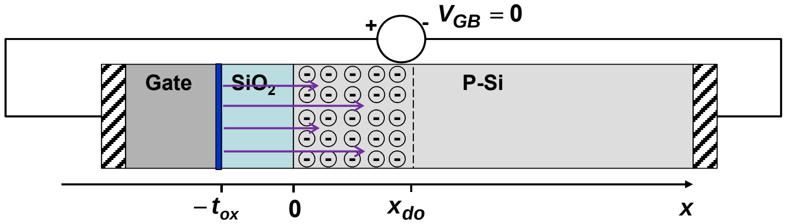
Positive surface charge density (C/cm[2])

$Q_G = +qN_a x_{do}$

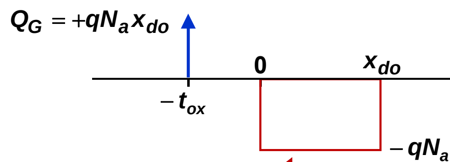
Negative depletion charge density (C/cm[3])

$\rho = -qN_a$

A NMOS Capacitor in Equilibrium: Charge Densities



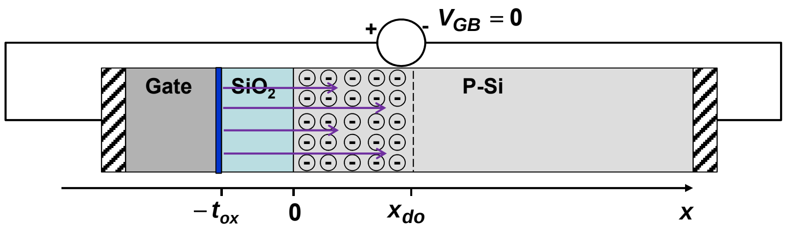
Charge density plot:



Total charge per unit area in the semiconductor (C/cm²)

$$Q_B = -qN_a x_{do}$$

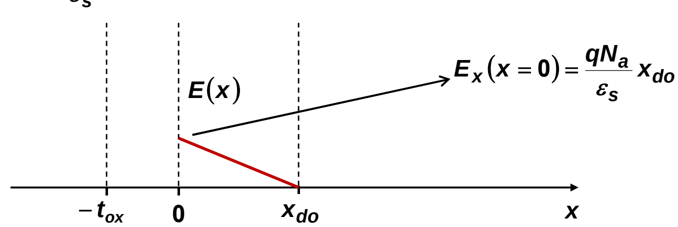
A NMOS Capacitor in Equilibrium: Electric Field



Electric field in the semiconductor:

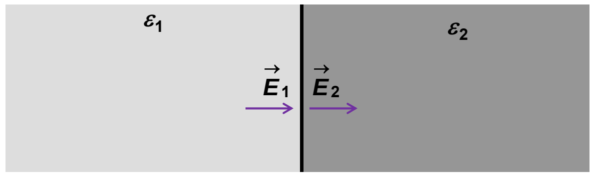
$$\frac{dE_x}{dx} = \frac{\rho}{\varepsilon_s} = -\frac{qN_a}{\varepsilon_s} \qquad \{E_x(x = x_{do}) = 0$$

$$\Rightarrow E_x(x) = \frac{qN_a}{\varepsilon_s}(x_{do} - x) \longrightarrow \text{Linearly varying}$$



Some Electrostatics

Consider an interface between media of different dielectric constants:



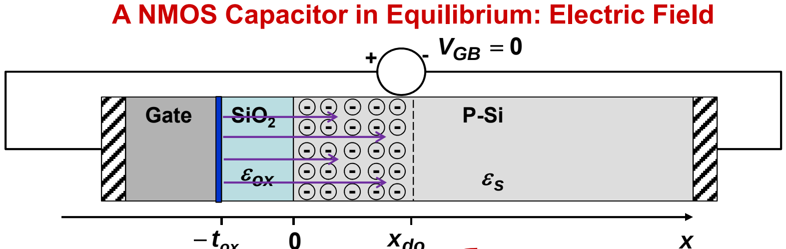
Suppose you know $\vec{E}_1$, can you find $\vec{E}_2$???

Use the principle: The product of the dielectric constant and the normal component of the electric field on both sides of an interface are related as follows:

$$\varepsilon_2 \vec{E}_2 - \varepsilon_1 \vec{E}_1 = Q_I = \text{Interface sheet charge density (C/cm}^2\text{)}$$

- Note that $\vec{E}_1$ is the electric field JUST to the left of the interface and $\vec{E}_2$ is the electric field JUST to right of the interface

A NMOS Capacitor in Equilibrium: Electric Field



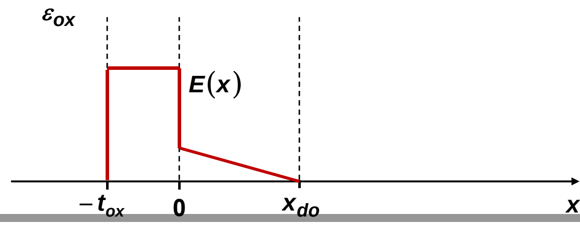
Electric field in the oxide:

$$\frac{dE_x}{dx} = \frac{\rho}{\varepsilon_{ox}} = 0$$

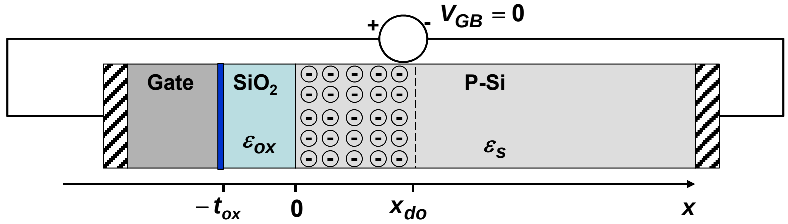
$$\Rightarrow E_x(x) = \text{constant}$$

$$\Rightarrow E_x(x) = \frac{qN_a x_{do}}{\varepsilon_{ox}}$$

$$\begin{cases} \varepsilon_{ox} E(x = 0^-) = \varepsilon_s E(x = 0^+) \\ E(x = 0^+) = \dfrac{qN_a x_{do}}{\varepsilon_s} \\ \Rightarrow E(x = 0^-) = \dfrac{qN_a x_{do}}{\varepsilon_{ox}} \end{cases}$$



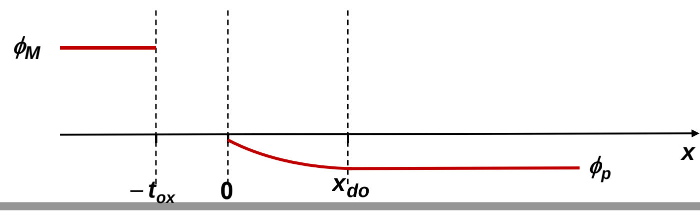
A NMOS Capacitor in Equilibrium: Potential



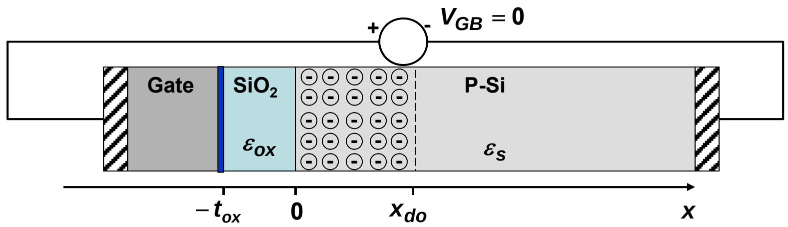
Potential in the semiconductor:

$$\frac{d\phi(x)}{dx} = -E_x(x) = -\frac{qN_a}{\varepsilon_s}(x_{do} - x) \qquad \left\{ \phi(x = x_{do}) = \phi_p \right.$$

$$\phi(x) = \phi_p + \frac{qN_a}{2\varepsilon_s}(x_{do} - x)^2$$



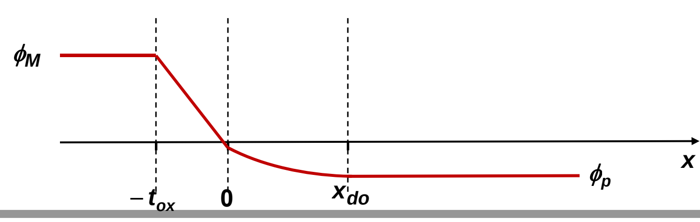
A NMOS Capacitor in Equilibrium: Potential



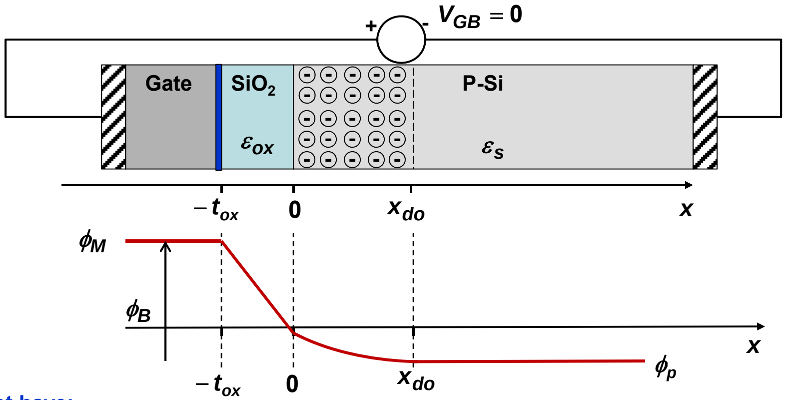
Potential in the oxide:

$$\frac{d\phi(x)}{dx} = -E_x(x) = -\frac{qN_a x_{do}}{\varepsilon_{ox}} \qquad \left\{ \phi(x = 0) = \phi_p + \frac{qN_a x_{do}^2}{2\varepsilon_s} \right.$$

$$\phi(x) = \phi_p + \frac{qN_a x_{do}^2}{2\varepsilon_s} - \frac{qN_a x_{do}}{\varepsilon_{ox}} x$$



A NMOS Capacitor in Equilibrium: Potential



Must have:

$$\phi(x = -t_{ox}) = \phi_p + \frac{qN_a x_{do}^2}{2\varepsilon_s} + \frac{qN_a x_{do}}{\varepsilon_{ox}} t_{ox} = \phi_M$$

Therefore:

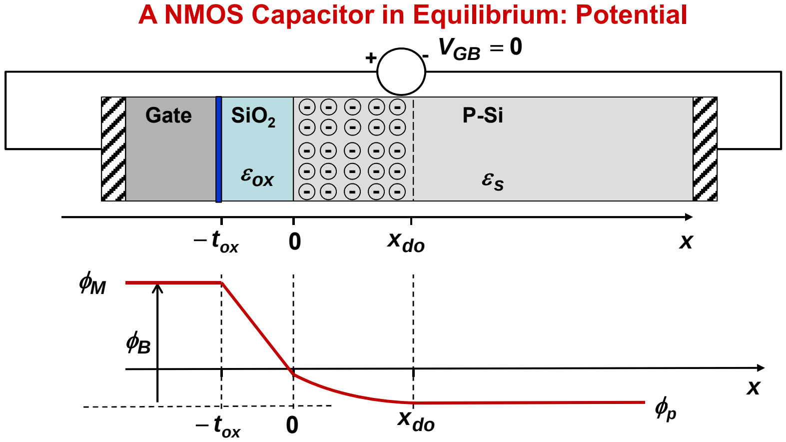
$$x_{do} = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_a}\right)\phi_B}$$

$$\phi_B = \phi_M - \phi_p$$

Oxide capacitance (per unit area):

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

A NMOS Capacitor in Equilibrium: Potential



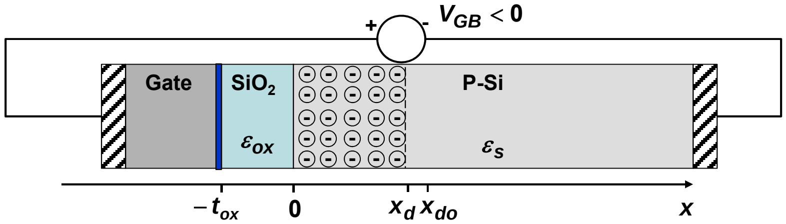
$$\phi_B = V_{ox} + V_S$$

$$= \underbrace{\frac{qN_a x_{do}}{C_{ox}}}_{\text{Potential drop in the oxide}} + \underbrace{\frac{qN_a x_{do}^2}{2\varepsilon_s}}_{\text{Potential drop in the semiconductor}}$$

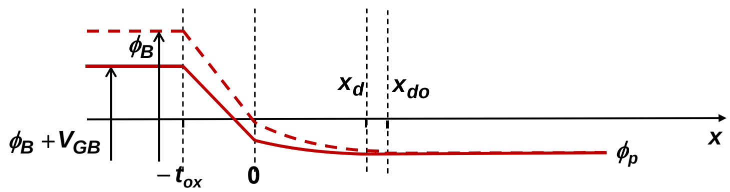
Oxide capacitance (per unit area)

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

A Biased NMOS Capacitor: V_{GB} < 0



All of the applied bias falls across the depletion region and the oxide

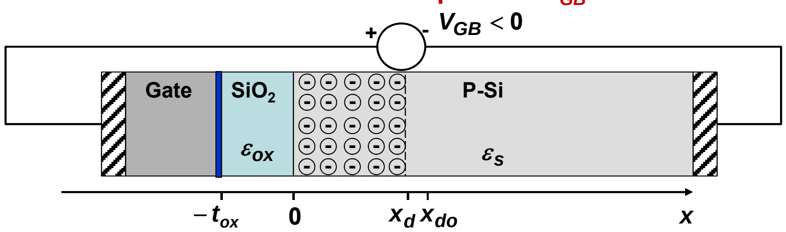


$$\phi_B + V_{GB} = \underbrace{\frac{qN_a x_d}{C_{ox}}}_{\text{Potential drop in the oxide}} + \underbrace{\frac{qN_a x_d^2}{2\varepsilon_s}}_{\text{Potential drop in the semiconductor}}$$

Potential drop in the oxide

Potential drop in the semiconductor

A Biased NMOS Capacitor: V_{GB} < 0



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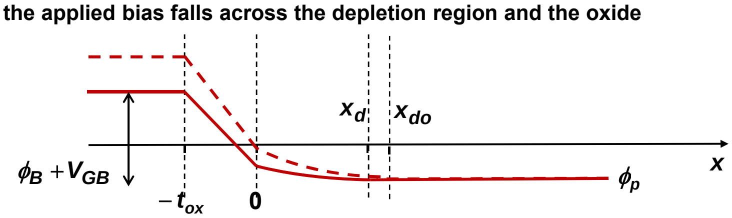
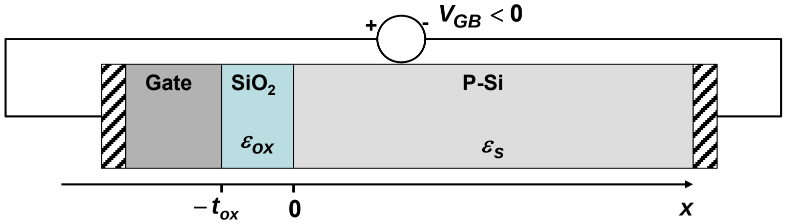


The depletion region shrinks and the oxide field also decreases for V_{GB} < 0

$$x_d = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_a}\right)(\phi_B + V_{GB})}$$

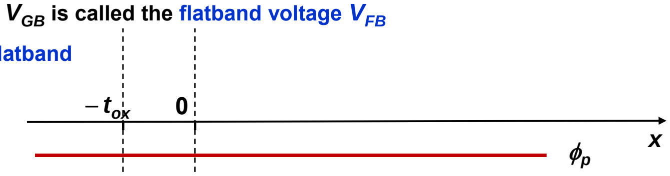
$$E_{ox} = \frac{qN_a x_d}{\varepsilon_{ox}}$$

A Biased NMOS Capacitor: Flatband Condition



When V_{GB} is sufficiently negative, the depletion region thickness shrinks to zero
This value of V_{GB} is called the flatband voltage V_{FB}

Potential in flatband condition:

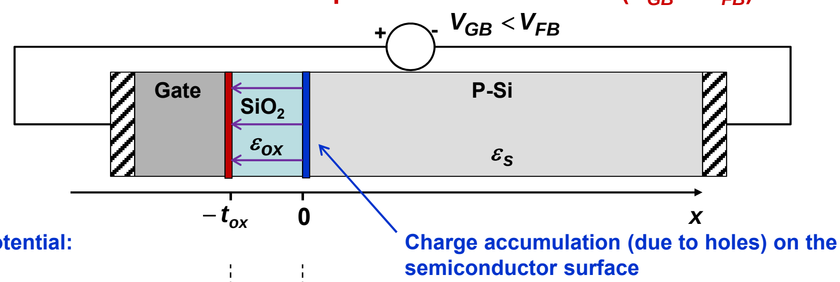


Flatband voltage:

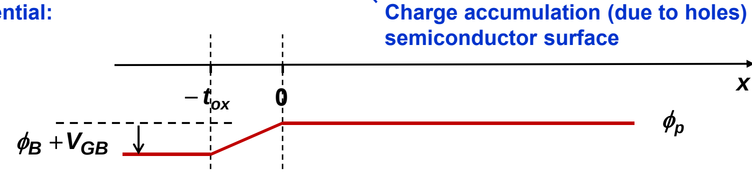
$$x_d = -\frac{\varepsilon_S}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_S}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_S}{qN_a}\right)(\phi_B + V_{FB})} = 0$$

$$\Rightarrow V_{FB} = -\phi_B = -(\phi_M - \phi_p)$$

A Biased NMOS Capacitor: Accumulation ($V_{GB} < V_{FB}$)

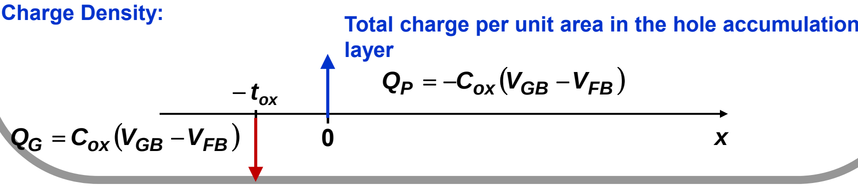


Potential:



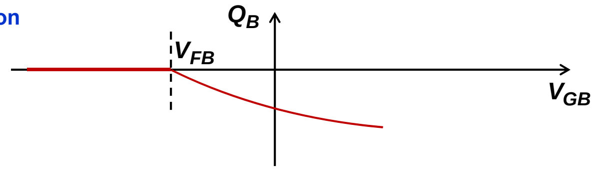
The entire potential drop for $V_{GB} < V_{FB}$ falls across the oxide

Charge Density:



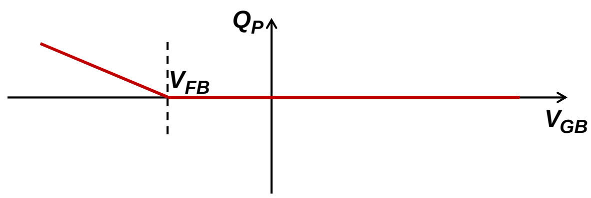
A Biased NMOS Capacitor: Charges

Depletion Region Charge (C/cm²)



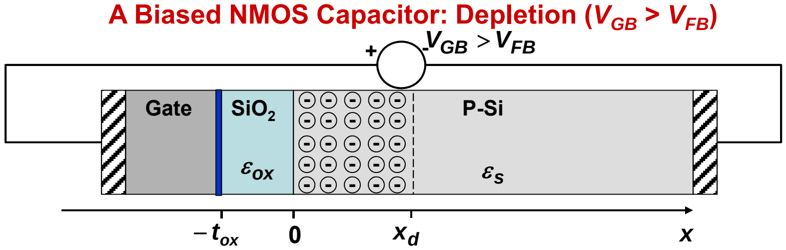
$$Q_B = -qN_a x_d \qquad x_d = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_a}\right)(\phi_B + V_{GB})}$$

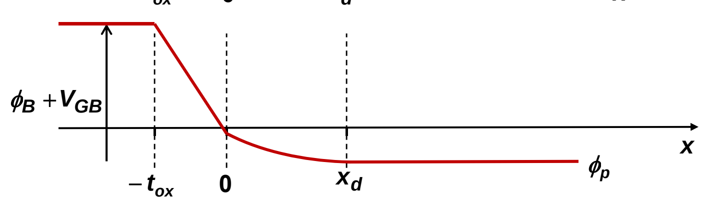
Accumulation Layer Charge (C/cm²)



$$Q_P = -C_{ox}(V_{GB} - V_{FB})$$

A Biased NMOS Capacitor: Depletion ($V_{GB} > V_{FB}$)





$$\phi_B + V_{GB} = V_{ox} + V_S$$

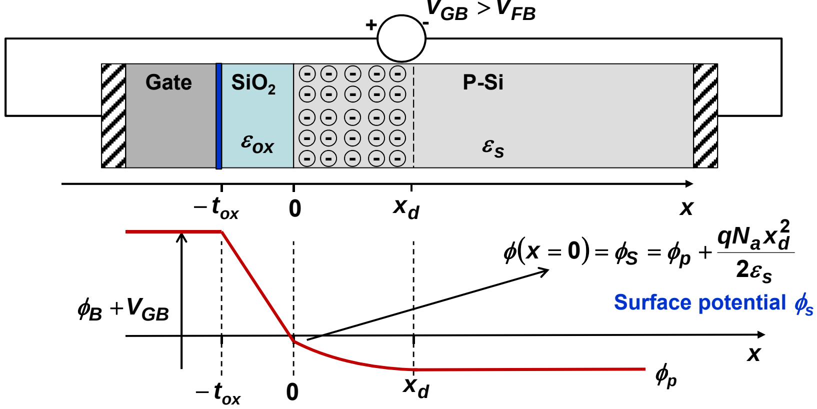
$$V_{GB} - V_{FB} = \underbrace{\frac{qN_a x_d}{C_{ox}}}_{\text{Potential drop in the oxide}} + \underbrace{\frac{qN_a x_d^2}{2\varepsilon_s}}_{\text{Potential drop in the semiconductor}}$$

Potential drop in the oxide

Potential drop in the semiconductor

The depletion region widens and the oxide field increases with V_{GB} for $V_{GB} > V_{FB}$

A Biased NMOS Capacitor: Depletion ($V_{GB} > V_{FB}$)



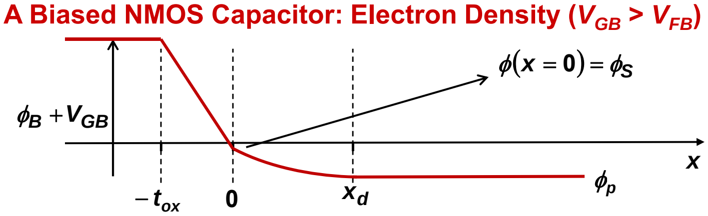
$$V_{GB} - V_{FB} = \frac{qN_a x_d}{C_{ox}} + \frac{qN_a x_d^2}{2\varepsilon_s}$$

$$\Rightarrow V_{GB} - V_{FB} = \phi_s - \phi_p + \frac{\sqrt{2\varepsilon_s q N_a (\phi_s - \phi_p)}}{C_{ox}}$$

Potential drop in the semiconductor

Potential drop in the oxide

A Biased NMOS Capacitor: Electron Density ($V_{GB} > V_{FB}$)



$$V_{GB} - V_{FB} = \phi_s - \phi_p + \frac{\sqrt{2\,\varepsilon_s q N_a (\phi_s - \phi_p)}}{C_{ox}}$$

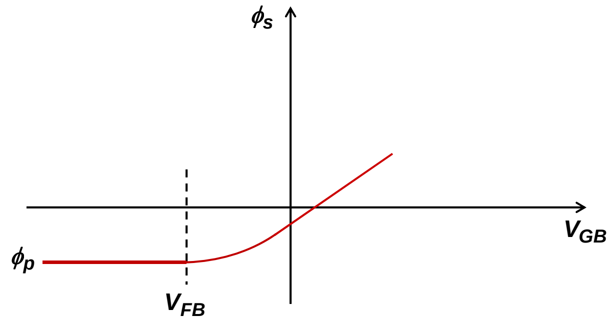
- As V_{GB} is increased, ϕ_S also increases
- The electron density in the semiconductor depends on the potential as:

$$n(x) = n_i e^{\frac{q\phi(x)}{KT}} = n_i e^{-\frac{q\phi_p}{KT}} e^{\frac{q(\phi(x)+\phi_p)}{KT}} \approx N_a e^{\frac{q(\phi(x)+\phi_p)}{KT}}$$

Electron density is the largest right at the surface of the semiconductor where the potential is the highest

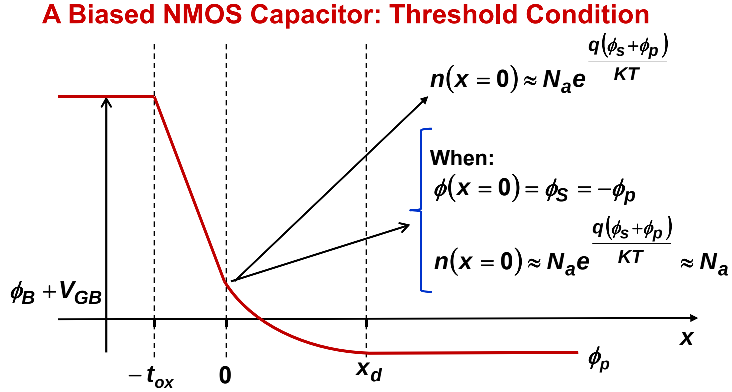
$$n(x=0) \approx N_a e^{\frac{q(\phi_S+\phi_p)}{KT}}$$

A Biased NMOS Capacitor: Surface Potential



$$
\begin{cases}
\phi_s = +\phi_p & V_{GB} \le V_{FB} \\
V_{GB} - V_{FB} = \phi_s - \phi_p + \dfrac{\sqrt{2\,\varepsilon_s q N_a (\phi_s - \phi_p)}}{C_{ox}} & V_{FB} \le V_{GB}
\end{cases}
$$

A Biased NMOS Capacitor: Threshold Condition

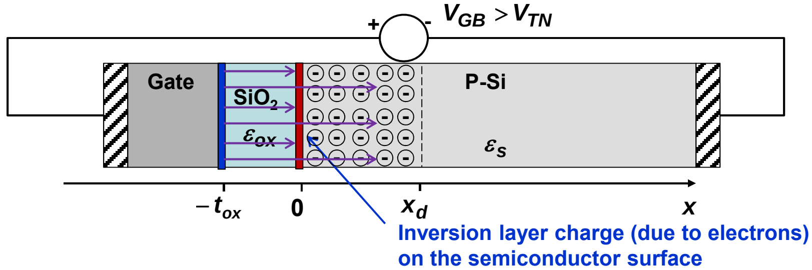


- When V_{GB} is increased and the surface potential ϕ_S reaches $-\phi_p$ the electron density at the surface becomes comparable to the hole density in the substrate and cannot be ignored

- The gate voltage V_{GB} at which ϕ_S equals $-\phi_p$ is called the threshold voltage V_{TN}:

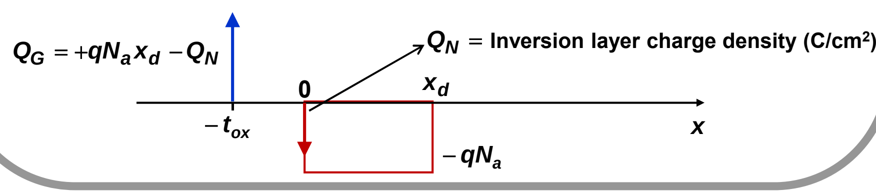
$$V_{TN} - V_{FB} = -2\phi_p + \frac{\sqrt{2\varepsilon_s q N_a (-2\phi_p)}}{C_{ox}}$$

A Biased NMOS Capacitor: Inversion (V_{GB} > V_{TN})

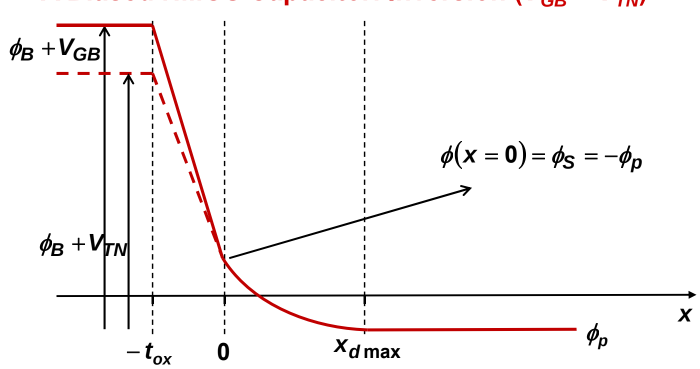


- When the gate voltage V_{GB} is increased above V_{TN} the electron density right at the surface increases (exponentially with the surface potential ϕ_S)

- This surface electron density is called the inversion layer (assumed to be of zero thickness in this course)



A Biased NMOS Capacitor: Inversion (V_{GB} > V_{TN})



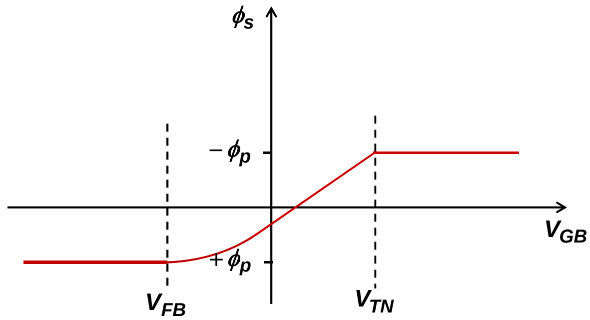
- When the gate voltage V_{GB} is increased above V_{TN} the inversion layer charge increases so rapidly that the extra applied potential drops entirely across the oxide, and the surface potential ϕ_S remains close to $-\phi_p$
- Consequently, the depletion region thickness (and the depletion region charge) does not increase when the gate voltage V_{GB} is increased above V_{TN}

$$\phi_S - \phi_p = \frac{qN_a x_d^2}{2\varepsilon_s} \Rightarrow -2\phi_p = \frac{qN_a x_{d\max}^2}{2\varepsilon_s}$$

$$V_{TN} = V_{FB} - 2\phi_p + \frac{\sqrt{2\varepsilon_s qN_a(-2\phi_p)}}{C_{ox}}$$

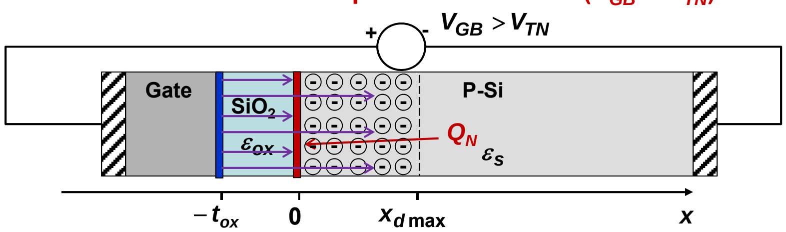
$$\Rightarrow V_{TN} = V_{FB} + \frac{qN_a x_{d\max}}{C_{ox}} + \frac{qN_a x_{d\max}^2}{2\varepsilon_s}$$

A Biased NMOS Capacitor: Surface Potential



$$
\begin{cases}
\phi_s = +\phi_p & V_{GB} \le V_{FB} \\
V_{GB} - V_{FB} = \phi_s - \phi_p + \dfrac{\sqrt{2\,\varepsilon_s q N_a (\phi_s - \phi_p)}}{C_{ox}} & V_{FB} \le V_{GB} \le V_{TN} \\
\phi_s = -\phi_p & V_{GB} \ge V_{TN}
\end{cases}
$$

A Biased NMOS Capacitor: Inversion (V_{GB} > V_{TN})



How to calculate the inversion layer charge Q_N when $V_{GB} > V_{TN}$?

Start from: $V_{GB} - V_{FB} = V_{ox} + V_S$

$$= E_{ox} t_{ox} + \frac{qN_a x_{d\max}^2}{2\varepsilon_S} \qquad \left\{ V_S = \frac{qN_a x_{d\max}^2}{2\varepsilon_S} \right.$$

By Gauss' law: $-\varepsilon_{ox} E_{ox} = Q_N - qN_a x_{d\max}$

Therefore:

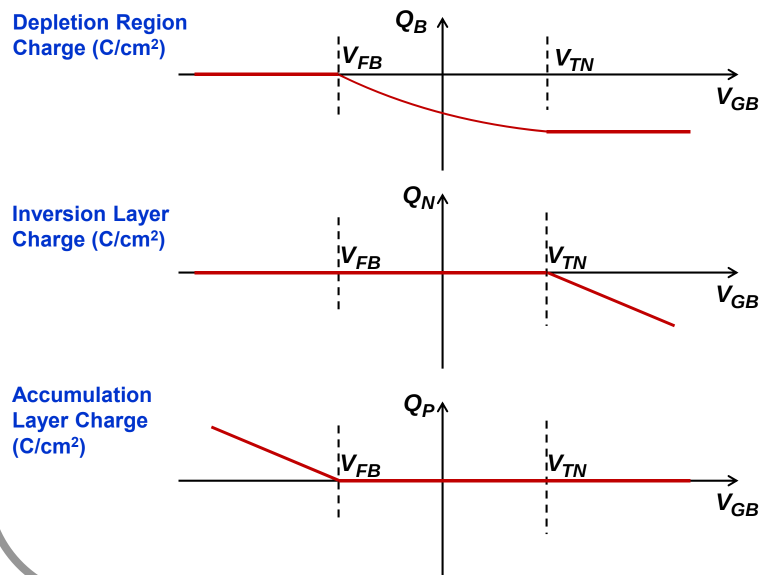
$$V_{GB} - V_{FB} = -\frac{Q_N}{C_{ox}} + \frac{qN_a x_{d\max}}{C_{ox}} + \frac{qN_a x_{d\max}^2}{2\varepsilon_S}$$

$$\Rightarrow V_{GB} = -\frac{Q_N}{C_{ox}} + V_{TN}$$

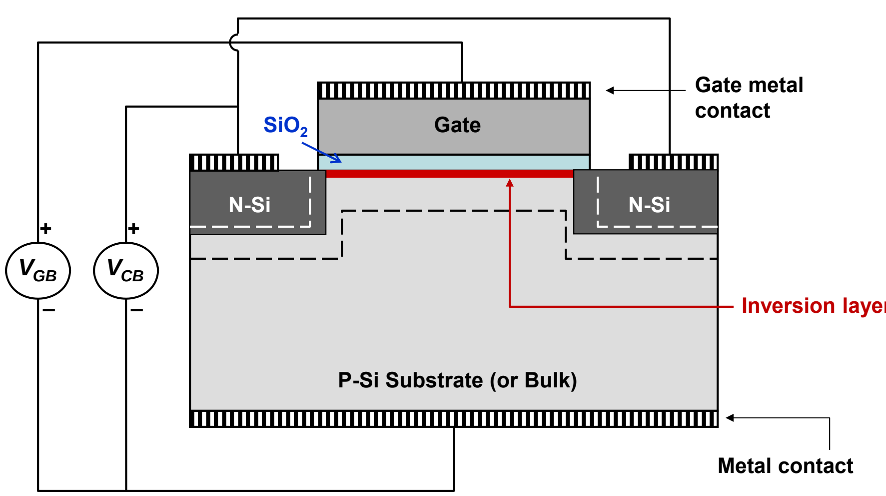
$$\Rightarrow Q_N = -C_{ox}(V_{GB} - V_{TN})$$

Inversion layer charge increases linearly with the gate voltage above threshold

A Biased NMOS Capacitor: Charges

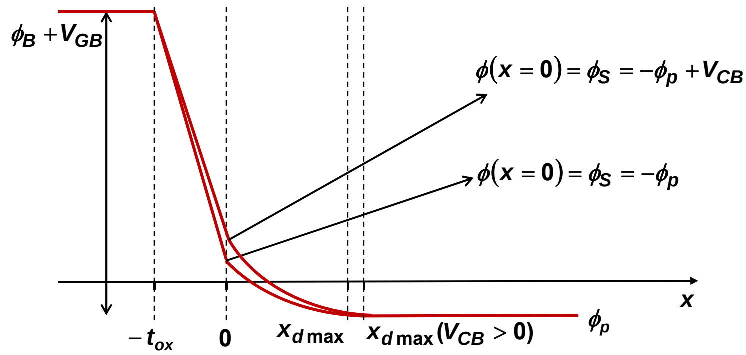


A NMOS Capacitor with a Channel Contact



- In the presence of an inversion layer, the additional contacts allow one to directly change the potential of the inversion layer channel w.r.t. to the bulk (substrate)

A Biased NMOS Capacitor: Inversion with $V_{CB} \neq 0$

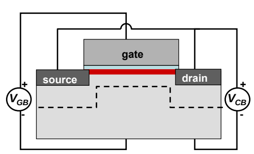


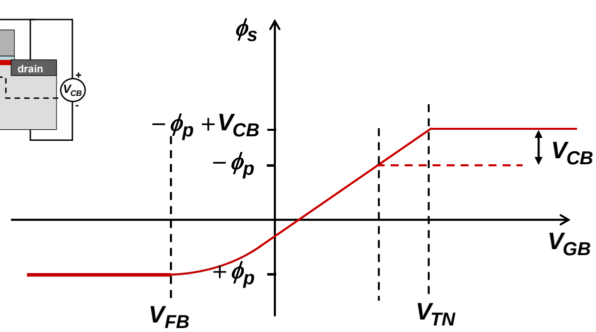
- We had said that the surface potential ϕ_S remains fixed at $-\phi_p$ when V_{GB} is increased beyond V_{TN}
- But with a non-zero V_{CB}, the surface potential ϕ_S in inversion can be changed to $(-\phi_p + V_{CB})$
- The new value of the depletion region width is:

$$\phi_S - \phi_p = \frac{qN_a x_d^2}{2\varepsilon_s} \Rightarrow -2\phi_p + V_{CB} = \frac{qN_a x_{d\,max}^2}{2\varepsilon_s}$$

Question: How do we now find the inversion layer charge Q_N when V_{CB} is not zero?

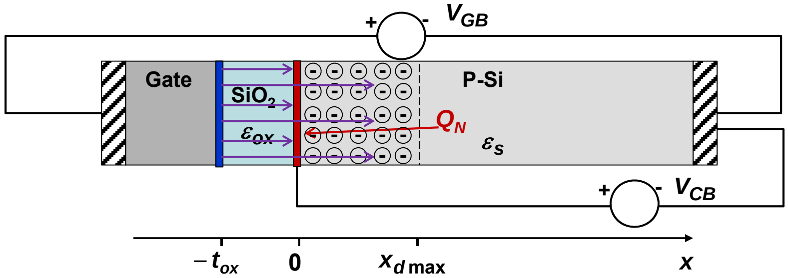
A Biased NMOS Capacitor: Surface Potential





$$\begin{cases} \phi_s = +\phi_p & V_{GB} \le V_{FB} \\ V_{GB} - V_{FB} = \phi_s - \phi_p + \dfrac{\sqrt{2\,\varepsilon_s q N_a (\phi_s - \phi_p)}}{C_{ox}} & V_{FB} \le V_{GB} \le V_{TN} \\ \phi_s = -\phi_p + V_{CB} & V_{GB} \ge V_{TN} \end{cases}$$

A Biased NMOS Capacitor: Inversion with $V_{CB} \neq 0$



How to calculate the inversion layer charge Q_N? Same way as before……..

Start from: $V_{GB} - V_{FB} = V_{ox} + V_S$

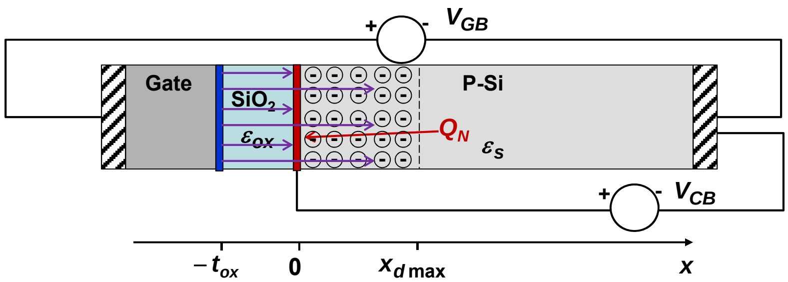
$$= E_{ox} t_{ox} + \frac{qN_a x_{d\max}^2}{2\varepsilon_s} \qquad V_S = \frac{qN_a x_{d\max}^2}{2\varepsilon_s}$$

By Gauss' law: $-\varepsilon_{ox} E_{ox} = Q_N - qN_a x_{d\max}$

Therefore: $V_{GB} - V_{FB} = -\frac{Q_N}{C_{ox}} + \frac{qN_a x_{d\max}}{C_{ox}} + \frac{qN_a x_{d\max}^2}{2\varepsilon_s}$

$$\Rightarrow V_{GB} = -\frac{Q_N}{C_{ox}} + V_{FB} + \frac{qN_a x_{d\max}}{C_{ox}} + \frac{qN_a x_{d\max}^2}{2\varepsilon_s}$$

A Biased NMOS Capacitor: Inversion with $V_{CB} \neq 0$



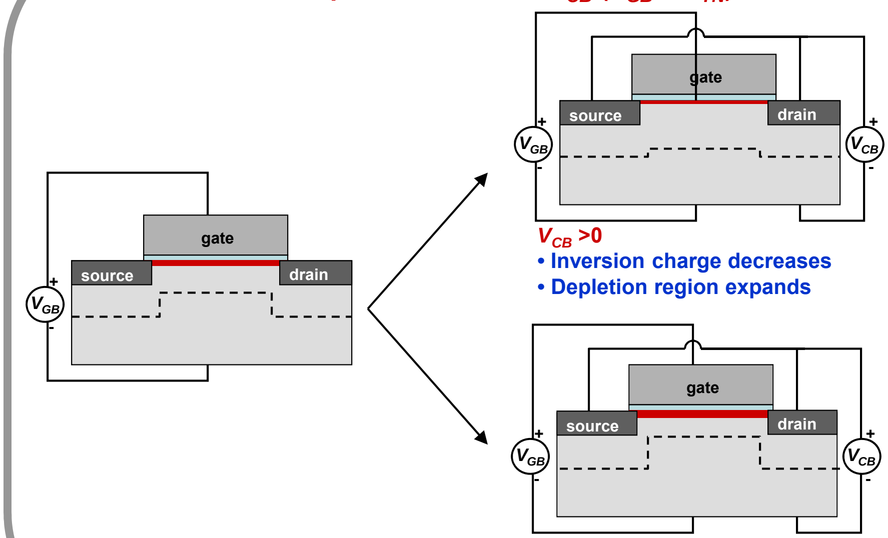
$$V_{GB} = -\frac{Q_N}{C_{ox}} + V_{FB} + \frac{qN_a x_{d\max}}{C_{ox}} + \frac{qN_a x_{d\max}^2}{2\varepsilon_s}$$

$$V_{TN} = V_{FB} + \frac{qN_a x_{d\max}}{C_{ox}} + \frac{qN_a x_{d\max}^2}{2\varepsilon_s}$$

$$= V_{FB} - 2\phi_p + V_{CB} + \frac{\sqrt{2\varepsilon_s q N_a \left(-2\phi_p + V_{CB}\right)}}{C_{ox}}$$

$\Rightarrow Q_N = -C_{ox}(V_{GB} - V_{TN})$ ⟶ Same as before but now V_{TN} depends on V_{CB}

NMOS Capacitor: Effect of V_{CB} ($V_{GB} > V_{TN}$)



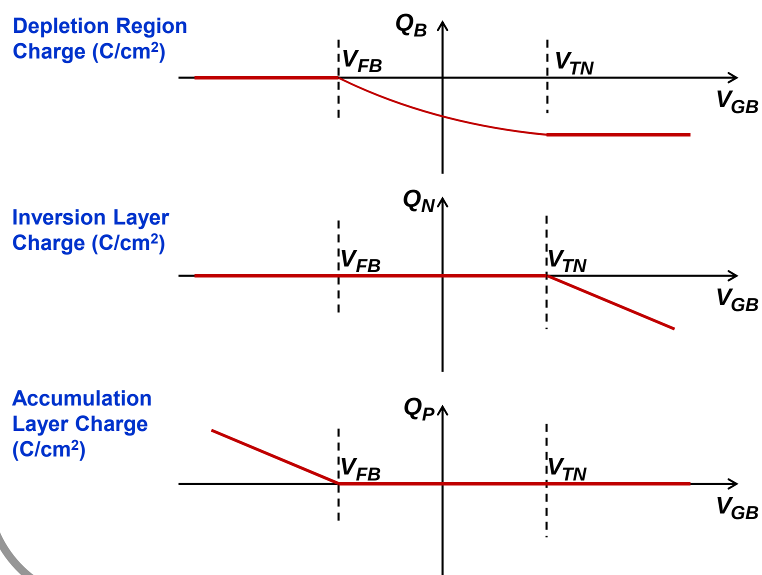
$V_{CB} > 0$

- Inversion charge decreases
- Depletion region expands

$V_{CB} < 0$

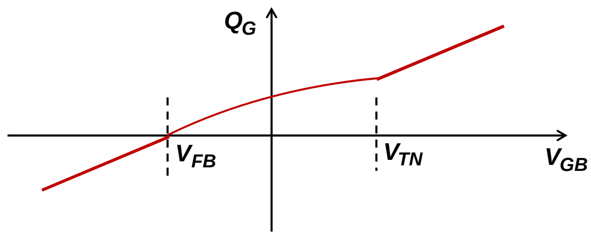
- Inversion charge increases
- Depletion region shrinks

A Biased NMOS Capacitor: Charges



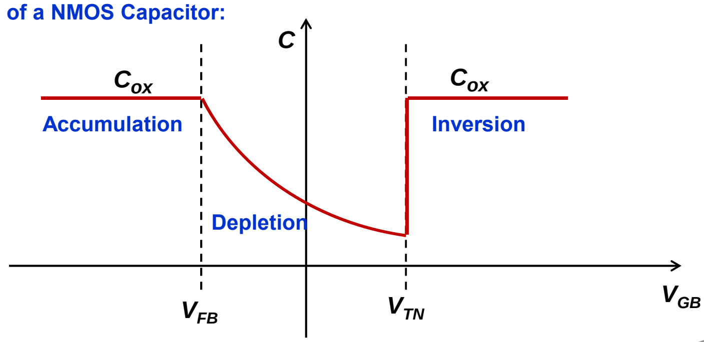
A Biased NMOS Capacitor: Charges

Gate Charge (C/cm^2)



Capacitance of a NMOS Capacitor:

$$C = \frac{dQ_G}{dV_{GB}}$$



The Small Signal Capacitance of a NMOS Capacitor

- The small signal capacitance (per unit area) of the MOS capacitor is defined as:

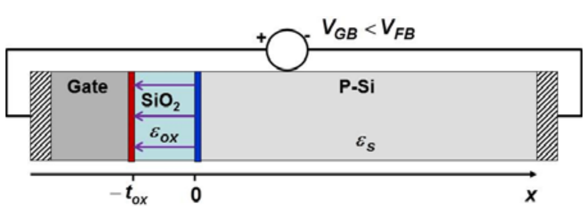
$$C = \frac{dQ_G}{dV_{GB}}$$

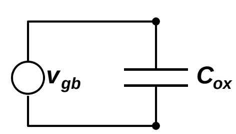
where Q_G is the charge density (units: C/cm^2) on the gate

(1) Accumulation ($V_{GB} < V_{FB}$):

$$Q_G = C_{ox}(V_{GB} - V_{FB})$$

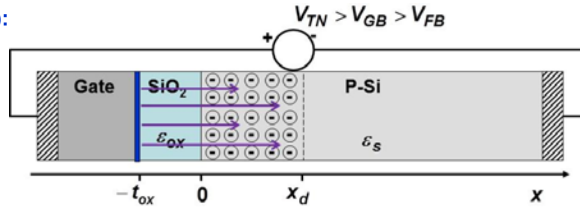
$$\Rightarrow \quad C = C_{ox}$$





The Small Signal Capacitance of a NMOS Capacitor

(2) Depletion ($V_{TN} > V_{GB} > V_{FB}$):



$$Q_G = qN_a x_d$$

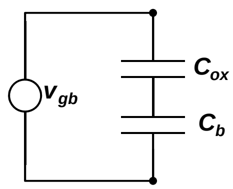
$$C = \frac{dQ_G}{dV_{GB}} = qN_a \frac{dx_d}{dV_{GB}}$$

Differentiate the equation (derived earlier):

$$\frac{qN_a x_d^2}{2\,\varepsilon_s} + \frac{qN_a x_d}{C_{ox}} = V_{GB} - V_{FB} \qquad \left\{ x_d = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_a}\right)(\phi_B + V_{GB})} \right.$$

To get: $\left[\frac{x_d}{\varepsilon_s} + \frac{1}{C_{ox}}\right] qN_a \, dx_d = dV_{GB}$

Define: $C_b = \frac{\varepsilon_s}{x_d}$



Finally: $\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_b}$

The Small Signal Capacitance of a NMOS Capacitor

(3) Inversion ($V_{GB} > V_{TN}$):

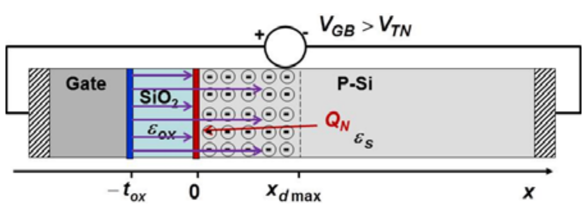
$$Q_G = qN_a x_{d\max} - Q_N$$

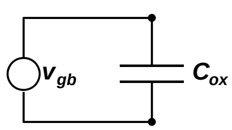
$$C = \frac{dQ_G}{dV_{GB}} = -\frac{dQ_N}{dV_{GB}}$$

$$= C_{ox}$$

$Q_N = -C_{ox}(V_{GB} - V_{TN})$

x_{dmax} does not change with V_{GB} above threshold





The Small Signal Capacitance of a NMOS Capacitor