Lecture 4

Electrons and Holes in Semiconductors

In this lecture you will learn:

- Generation-recombination in semiconductors in more detail
- The basic set of equations governing the behavior of electrons and holes in semiconductors
- Shockley Equations
- Quasi-neutrality in conductive materials

Majority and Minority Carriers

In N-doped Semiconductors:

Electrons are the majority carriers
Holes are the minority carriers

In P-doped Semiconductors:

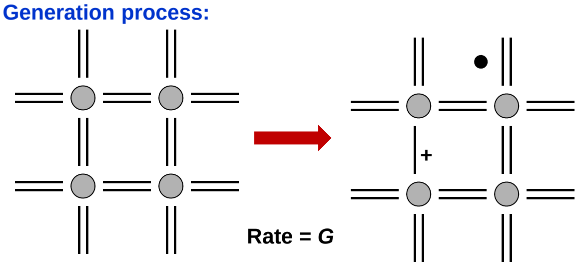
Holes are the majority carriers
Electrons are the minority carriers

Golden Rule of Thumb:

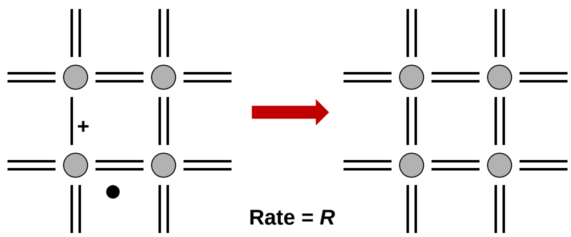
When trying to understand semiconductor devices, always first see what the minority carriers are doing

Generation and Recombination in Semiconductors

Generation process:



Recombination process:



We can write the following equations for the carrier densities:

$$\frac{\partial n(t)}{\partial t} = G - R$$

$$\frac{\partial p(t)}{\partial t} = G - R$$

These equations tell how the electron and hole densities change in time as a result of recombination and generation processes

Generation and Recombination in Thermal Equilibrium

- From the first lecture, in thermal equilibrium:

  The recombination rate $= R_o = k\, n_o\, p_o$

  equals the generation rate $= G_o$

  i.e. $G_o = R_o$

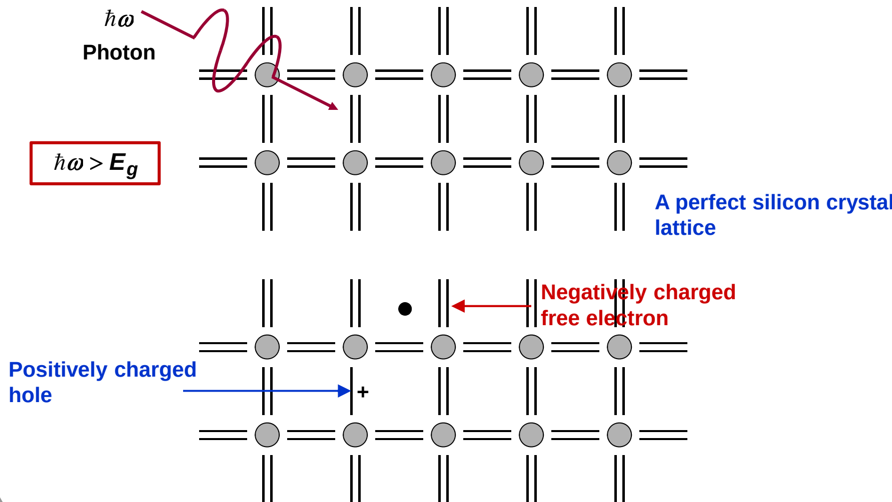
- Then in thermal equilibrium:

$$\frac{\partial n_o(t)}{\partial t} = G_o - R_o = 0$$

$$\frac{\partial p_o(t)}{\partial t} = G_o - R_o = 0$$

Light Absorption in Semiconductors

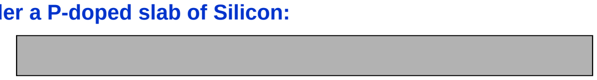
Generation of electrons and holes by photons in semiconductors:



A silicon crystal lattice with one broken bond (one electron and one hole)

Generation and Recombination Out of Thermal Equilibrium

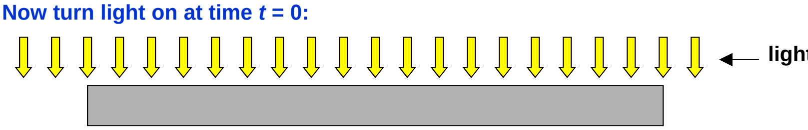
1) Consider a P-doped slab of Silicon:



$(n_o \ll p_o)$

Electron-hole recombination rate in thermal equilibrium $= R_o = k\, n_o\, p_o$
equals the generation rate $= G_o = k\, n_i^2$

2) Now turn light on at time $t = 0$**:**



- **Light breaks the Si-Si covalent bonds and generates excess electron-hole pairs**
- **The net generation rate now becomes:** $G = G_o + G_L$ — **extra part**

3) Mathematical model of the above situation:

$$n = n_o + n'(t)$$
$$p = p_o + p'(t)$$

- $n'(t)$ **and** $p'(t)$ **are the excess electron and hole densities**
- **It must be that:** $n'(t) = p'(t)$
- **We also assume that:** $n'(t), p'(t) \ll p_o$

Generation and Recombination Out of Thermal Equilibrium

- We can use the equations:

$$\frac{\partial n(t)}{\partial t} = G - R \qquad \frac{\partial p(t)}{\partial t} = G - R \quad \Big\} \quad \begin{aligned} n &= n_o + n'(t) \\ p &= p_o + p'(t) \end{aligned}$$

- **Generation rate:**

$$G = G_o + G_L$$

- **Recombination rate:**

$$\begin{aligned} R &= k\, n\, p \\ &= k\,(n_o + n')(p_o + p') \\ &\approx k\,(n_o + n')\,p_o \\ &= k\, n_o\, p_o + k\, n'\, p_o \\ &= R_o + \frac{n'}{\tau_n} \end{aligned}$$

Assumptions: $n', p' << p_o$

$$\frac{1}{\tau_n} = k\, p_o$$

τ_n is the lifetime of the minority carriers (i.e. electrons)

The excess recombination rate is proportional to the excess MINORITY carrier density

- The equation for excess minority carriers (i.e. electrons) becomes:

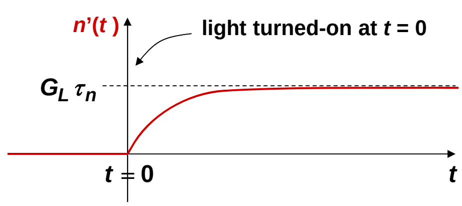
$$\frac{\partial n'(t)}{\partial t} = G_L - \frac{n'(t)}{\tau_n}$$

Generation and Recombination Out of Thermal Equilibrium

$$\frac{\partial n'(t)}{\partial t} = G_L - \frac{n'(t)}{\tau_n}$$

- Solution with the boundary condition, $n'(t=0) = 0$, is:

$$n'(t) = G_L\, \tau_n \left(1 - e^{-\frac{t}{\tau_n}} \right)$$



- Excess hole density is, of course :

$$p'(t) = n'(t)$$

- As $t \to \infty$ the excess electron and hole densities reach a steady state value

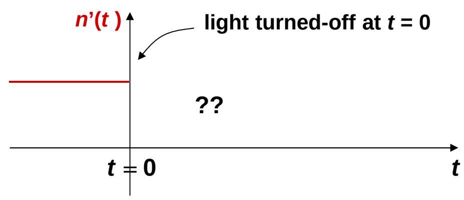
$$\begin{aligned} n'(t \to \infty) &= G_L\, \tau_n \\ p'(t \to \infty) &= G_L\, \tau_n \end{aligned} \qquad \text{and} \qquad \begin{aligned} n(t \to \infty) &= n_o + G_L\, \tau_n \\ p(t \to \infty) &= p_o + G_L\, \tau_n \end{aligned}$$

Generation and Recombination Out of Thermal Equilibrium

Now suppose that light had been turned-on for a very very long time and it was turned-off at time $t = 0$

At time $t = 0$: $n' = G_L\ \tau_n$ and $n = n_o + G_L\ \tau_n$

$p' = G_L\ \tau_n$ $\qquad$ $p = p_o + G_L\ \tau_n$



• Since $n \neq n_o$, and $p \neq p_o$, the carrier densities are not equal to their thermal equilibrium values. Thermal equilibrium must get restored since the light has been turned-off

Question: How does thermal equilibrium gets restored??

Generation and Recombination Out of Thermal Equilibrium

• We can use the equations:

$$\frac{\partial n(t)}{\partial t} = G - R \qquad \frac{\partial p(t)}{\partial t} = G - R \qquad \left.\right\} \qquad \begin{aligned} n &= n_o + n'(t) \\ p &= p_o + p'(t) \end{aligned}$$

• Generation rate:

$$G = G_o$$

• Recombination rate:

$$\begin{aligned} R &= k\ n\ p \\ &= k\ (n_o + n')(p_o + p') \\ &\approx k\ (n_o + n')\ p_o \\ &= k\ n_o\ p_o + k\ n'\ p_o \\ &= R_o + \frac{n'}{\tau_n} \end{aligned}$$

Assumptions: $n', p' << p_o$

The excess recombination rate is proportional to the excess MINORITY carrier density

• The equation for excess minority carriers (i.e. electrons) becomes:

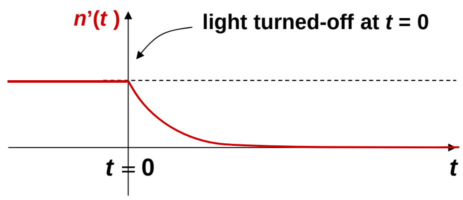
$$\frac{\partial n'(t)}{\partial t} = -\frac{n'(t)}{\tau_n}$$

Generation and Recombination Out of Thermal Equilibrium

$$\frac{\partial n'(t)}{\partial t} = -\frac{n'(t)}{\tau_n}$$

• Solution is:

$$n'(t) = n'(t=0)\, e^{-\frac{t}{\tau_n}} \quad \Rightarrow$$ Excess electron density decays exponentially to zero from its initial value



The excess carrier densities decay with time and thermal equilibrium values for carrier densities are restored

• The excess hole density will also decay in the same way: $p'(t) = p'(t=0)\, e^{-\frac{t}{\tau_n}}$

• As $t \to \infty$ the electron and hole densities reach their equilibrium values:

$$n'(t \to \infty) = 0 \qquad n(t \to \infty) = n_o$$

and

$$p'(t \to \infty) = 0 \qquad p(t \to \infty) = p_o$$

Generation and Recombination in Doped Semiconductors

Whenever you have to find an expression for R use the following recipe:

• If it is a p-doped semiconductor:

$$R = R_o + \frac{n'(x,t)}{\tau_n}$$

τ_n is the minority carrier lifetime

• If it is a n-doped semiconductor:

$$R = R_o + \frac{p'(x,t)}{\tau_p}$$

τ_p is the minority carrier lifetime

The excess recombination rate (i.e. R - R_o) is always proportional to the excess MINORITY carrier density

Electron and Hole Current Density Equations

From last lecture…………

$$J_n(x) = q\, n(x)\, \mu_n\, E(x) + q\, D_n \frac{d\, n(x)}{dx} \qquad (1)$$

$$J_p(x) = q\, p(x)\, \mu_p\, E(x) - q\, D_p \frac{d\, p(x)}{dx} \qquad (2)$$

These are two of Shockley's equations !


William Shockley


John Bardeen


Walter Brattain

Shockley, Bardeen, and Brattain from Bell Labs were awarded the Nobel Prize for inventing the semiconductor transistor

Electron and Hole Current Continuity Equations

• You have already seen the equations:

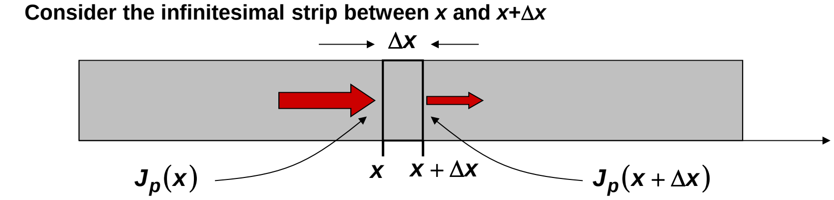
$$\frac{\partial n(x,t)}{\partial t} = G - R$$
$$\frac{\partial p(x,t)}{\partial t} = G - R$$

These equations tell how the electron and hole densities change in time as a result of recombination and generation processes.

• Carrier densities can also change in time if the current densities change in space !!!

Electron and Hole Current Continuity Equations

Consider the infinitesimal strip between x and $x+\Delta x$



The difference in hole fluxes at x and $x+\Delta x$ must result in piling up of holes in the infinitesimal strip …….

$$J_p(x,t) - J_p(x+\Delta x,t) = q\frac{\partial p(x,t)\Delta x}{\partial t}$$

Note that $q\,p(x,t)$ is the hole charge density

$$\Rightarrow -\frac{J_p(x+\Delta x,t) - J_p(x,t)}{\Delta x} = q\frac{\partial p(x,t)}{\partial t}$$

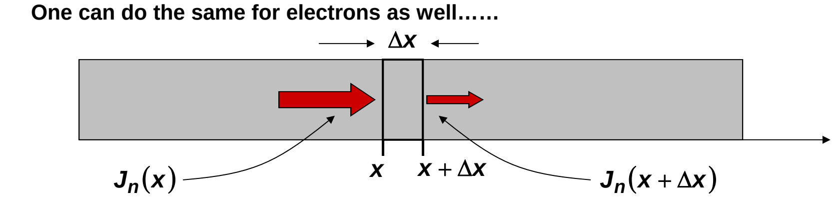
$$\Rightarrow -\frac{\partial J_p(x,t)}{\partial x} = q\frac{\partial p(x,t)}{\partial t} \longrightarrow \frac{\partial p(x,t)}{\partial t} = -\frac{1}{q}\frac{\partial J_p(x,t)}{\partial x}$$

Now add recombination and generation to the above equation:

$$\frac{\partial p(x,t)}{\partial t} = G - R - \frac{1}{q}\frac{\partial J_p(x,t)}{\partial x}$$

Electron and Hole Current Continuity Equations - III

One can do the same for electrons as well……



$$\frac{\partial n(x,t)}{\partial t} = G - R + \frac{1}{q}\frac{\partial J_n(x,t)}{\partial x}$$

So now we have two new equations,

$$\frac{\partial p(x,t)}{\partial t} = G - R - \frac{1}{q}\frac{\partial J_p(x,t)}{\partial x} \qquad (3)$$

$$\frac{\partial n(x,t)}{\partial t} = G - R + \frac{1}{q}\frac{\partial J_n(x,t)}{\partial x} \qquad (4)$$

These are two more of Shockley's equations !

Gauss's Law and Electrostatics

The net charge density in a semiconductor is,

$$\rho(x,t) = q\left[+ N_d(x) - N_a(x) + p(x,t) - n(x,t) \right]$$

Gauss's Law in differential form:

$$\frac{\partial E(x,t)}{\partial x} = \frac{\rho(x,t)}{\varepsilon_s}$$

$$\frac{\partial E(x,t)}{\partial x} = \frac{q\left[+ N_d(x) - N_a(x) + p(x,t) - n(x,t) \right]}{\varepsilon_s} \qquad (5)$$

This is the fifth and the last of the Shockley's equations !

$$\varepsilon_o = 8.85 \times 10^{-12} \text{ Farads/m}$$
$$= 8.85 \times 10^{-14} \text{ Farads/cm}$$

For Silicon: $\varepsilon_s = 11.7\varepsilon_o$

The Five Shockley Equations

$$J_n(x,t) = q\, n(x,t)\, \mu_n\, E(x,t) + q\, D_n \frac{d\, n(x,t)}{dx} \qquad (1)$$

$$J_p(x,t) = q\, p(x,t)\, \mu_p\, E(x,t) - q\, D_p \frac{d\, p(x,t)}{dx} \qquad (2)$$

$$\frac{\partial p(x,t)}{\partial t} = G - R - \frac{1}{q} \frac{\partial J_p(x,t)}{\partial x} \qquad (3)$$

$$\frac{\partial n(x,t)}{\partial t} = G - R + \frac{1}{q} \frac{\partial J_n(x,t)}{\partial x} \qquad (4)$$

$$\frac{\partial E(x,t)}{\partial x} = \frac{q\left[+ N_d(x) - N_a(x) + p(x,t) - n(x,t) \right]}{\varepsilon_s} \qquad (5)$$

Using these equations one can understand the behavior of semiconductor microelectronic devices !!

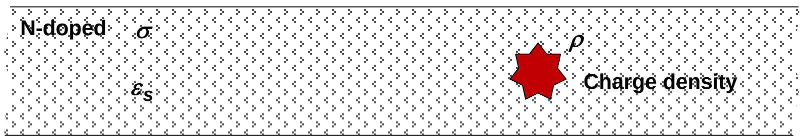
Quasi-Neutrality

Materials with large conductivities are "quasi-neutral"

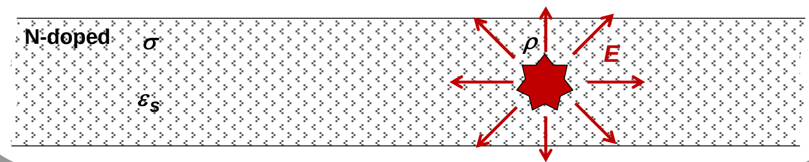
"Quasi-neutrality" implies that there cannot be large charge densities or electric fields inside a conductive material

Lets see why this is true…..and how deviations from quasi-neutrality disappear…….

Consider an infinite and conductive N-doped semiconductor with a net charge density at time t=0:



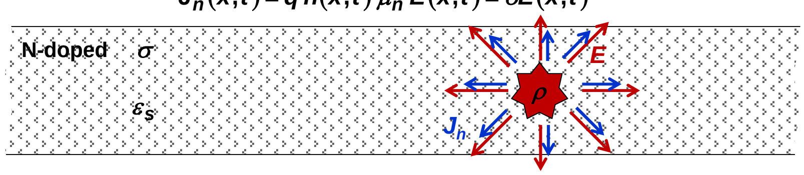
The charge density will generate electric fields (by Gauss' law):



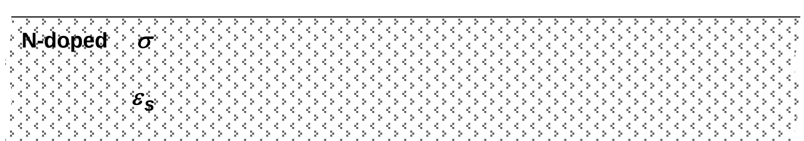
Quasi-Neutrality

The electric field will generate electrical currents:

$$J_n(x,t) = q\, n(x,t)\, \mu_n\, E(x,t) = \sigma E(x,t)$$



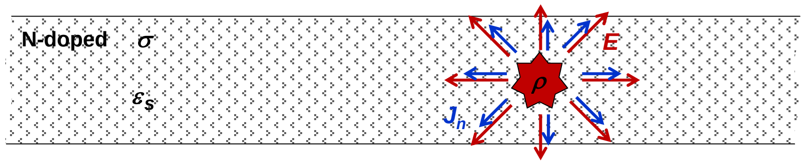
The electrical currents will pile electrons on top of the charge density and neutralize it and then there is no charge density left in the medium…………



This whole process takes a time of the order of the dielectric relaxation time τ_d :

$$\tau_d = \frac{\varepsilon_s}{\sigma} \sim 10^{-15} - 10^{-13} \quad \text{Seconds}$$

Appendix: Restoration of Quasi-Neutrality



From Gauss' law:

$$\nabla.\vec{E}(\vec{r},t)=\frac{\rho(\vec{r},t)}{\varepsilon_S}$$

Current equation:

$$\vec{J}(\vec{r},t)=\sigma\vec{E}(\vec{r},t)$$

Use the continuity equation for charge:

$$\frac{\partial\rho(\vec{r},t)}{\partial t}=-\nabla.\vec{J}(\vec{r},t)=-\sigma\nabla.\vec{E}(\vec{r},t)=-\frac{\sigma}{\varepsilon_S}\rho(\vec{r},t)$$

$$\Rightarrow\frac{\partial\rho(\vec{r},t)}{\partial t}+\frac{\rho(\vec{r},t)}{\tau_d}=0 \qquad \left\{\tau_d=\frac{\varepsilon_S}{\sigma}\right.$$

Solution:

$$\rho(\vec{r},t)=\rho(\vec{r},t=0)e^{-\frac{t}{\tau_d}}$$

→ Charge density in a conductive medium disappears on a time scale of τ_d