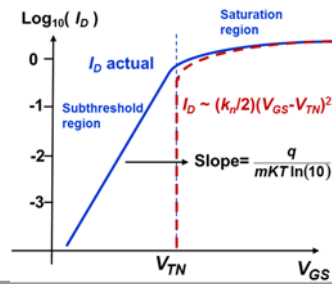


Lecture 25

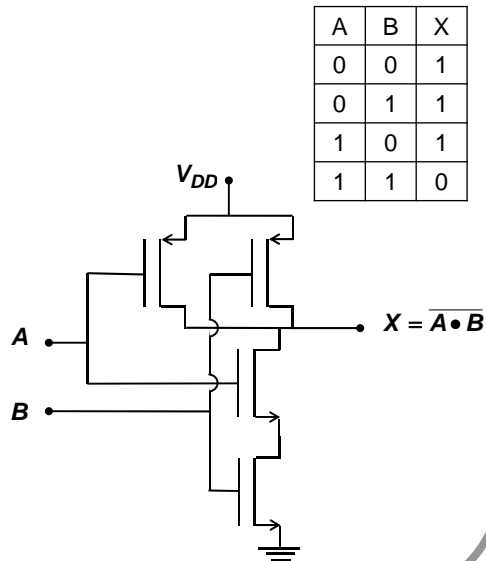
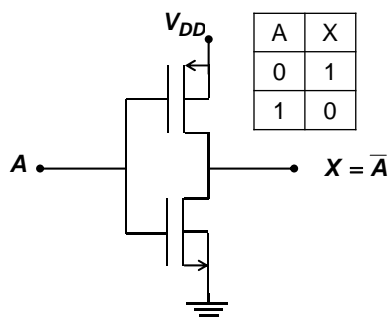
Subthreshold FET Operation and Circuits

In this lecture you will learn:

- FET is not an ideal switch
- Weak inversion and strong inversion
- Leakage current in FETs
- Inverse subthreshold current slope and its implications
- Subthreshold FET circuits

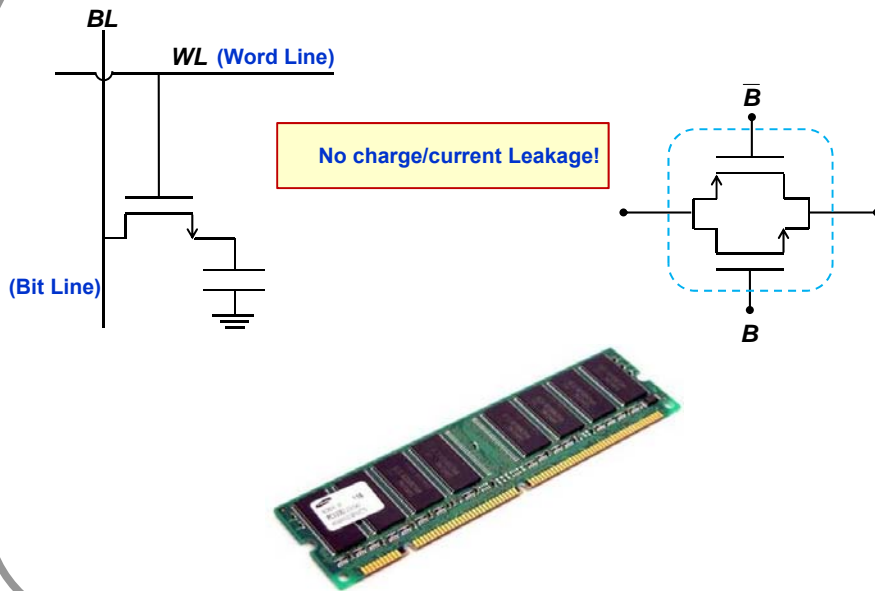


FET as an Ideal Switch

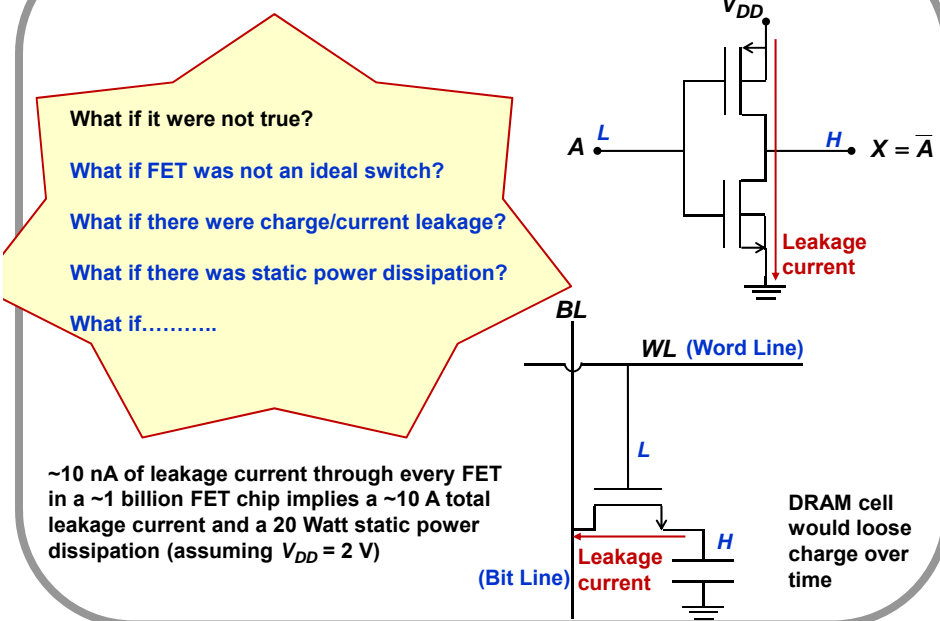


No static power dissipation!

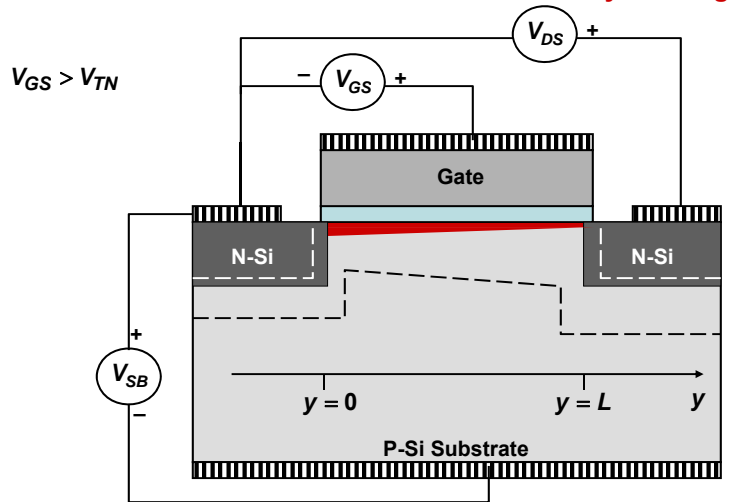
FET as an Ideal Switch



FET is not an Ideal Switch



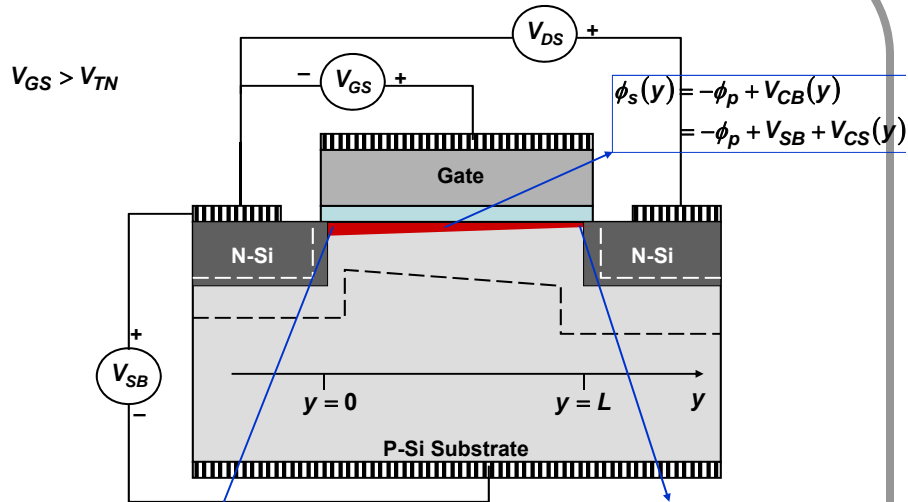
A NFET Above Threshold: Inversion Layer Charge



Inversion charge: $Q_N(y) = -C_{ox}(V_{GS} - V_{TN} - V_{CS}(y))$

Threshold voltage: $V_{TN} = V_{FB} - 2\phi_p + \frac{\sqrt{2\epsilon_s q N_a (-2\phi_p + V_{SB})}}{C_{ox}}$

A NFET Above Threshold: Surface Potential



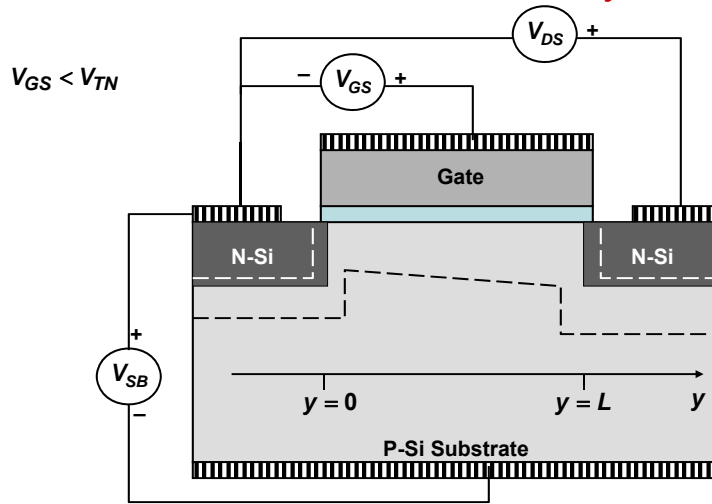
$$\begin{aligned} \phi_s(y) &= -\phi_p + V_{CB}(y) \\ &= -\phi_p + V_{SB} + V_{CS}(y) \end{aligned}$$

$$\begin{aligned} \phi_s(y=0) &= -\phi_p + V_{CB}(y=0) \\ &= -\phi_p + V_{SB} \end{aligned}$$

$$\begin{aligned} \phi_s(y=L) &= -\phi_p + V_{CB}(y=L) \\ &= -\phi_p + V_{DB} = -\phi_p + V_{DS} + V_{SB} \end{aligned}$$

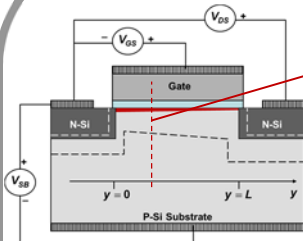
Potential drop in the channel: $\phi_s(y=L) - \phi_s(y=0) = V_{DS}$

A NFET Below Threshold: Inversion Layer Charge



Inversion charge: $Q_N(y) = 0$ → BUT THIS IS NOT TRUE!

NFET Electrostatics



$$V_{GB} - V_{FB} = V_{ox} + V_S$$

$$= E_{ox}(y)t_{ox} + \phi_s(y) - \phi_p$$

$$\begin{cases} \epsilon_{ox} E_{ox}(y) = -Q_N(y) + qN_a x_d(y) \\ x_d(y) = \sqrt{2\epsilon_s q N_a (\phi_s(y) - \phi_p)} \end{cases}$$

$$V_{GB} = V_{FB} - \frac{Q_N(y)}{C_{ox}} + \phi_s(y) - \phi_p + \frac{\sqrt{2\epsilon_s q N_a (\phi_s(y) - \phi_p)}}{C_{ox}}$$

[Holds for all y
Holds above and below threshold]

If we were above threshold we would have assumed:

$$\phi_s(y) \approx -\phi_p + V_{CB}(y) = -\phi_p + V_{SB} + V_{CS}(y)$$

Then:

$$Q_N(y) = -C_{ox}(V_{GS} - V_{TN} - V_{CS}(y))$$

Below threshold, we cannot assume:

$$\phi_s(y) \approx -\phi_p + V_{CB}(y) = -\phi_p + V_{SB} + V_{CS}(y)$$

So how do we find the inversion layer charge below threshold?

Electron Density in the Inversion Layer - I

Recall that when $V_{DS} = V_{SB} = 0$:

$$\begin{aligned} n(x) &= n_i e^{\frac{q\phi(x)}{KT}} \\ &= n_i e^{\frac{q[\phi(x) + \phi_p - \phi_p]}{KT}} \\ &= N_a e^{\frac{q[\phi(x) + \phi_p]}{KT}} \end{aligned}$$

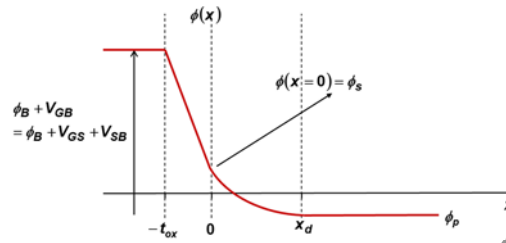
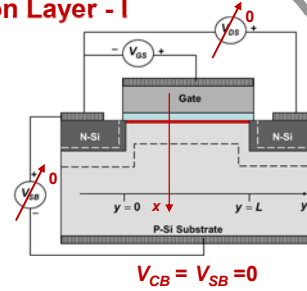
Near the surface:

$$\begin{aligned} \phi(x=0) &= \phi_s \\ \Rightarrow n &= N_a e^{\frac{q(\phi_s + \phi_p)}{KT}} \end{aligned}$$

Below threshold: $\phi_s < -\phi_p$

Above threshold: $\phi_s \approx -\phi_p$

We can generalize the above formula for the case when $V_{CB} \neq 0$



Electron Density in the Inversion Layer - II

Near the surface when $V_{DS} = 0$ but $V_{SB} \neq 0$:

$$n = N_a e^{\frac{q(\phi_s + \phi_p - V_{CB})}{KT}} = N_a e^{\frac{q(\phi_s + \phi_p - V_{SB})}{KT}}$$

This means that above threshold:

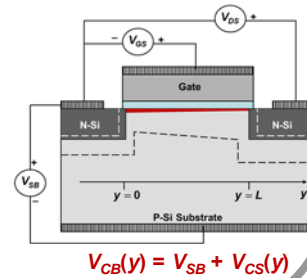
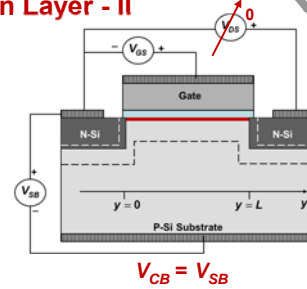
$$\phi_s \approx -\phi_p + V_{CB} = -\phi_p + V_{SB}$$

Near the surface when $V_{DS} \neq 0$ and $V_{SB} \neq 0$:

$$n(y) = N_a e^{\frac{q(\phi_s(y) + \phi_p - V_{CB}(y))}{KT}}$$

This means that above threshold:

$$\phi_s(y) \approx -\phi_p + V_{CB}(y) = -\phi_p + V_{SB} + V_{CS}(y)$$



Electron Density in the Inversion Layer - III

Near the surface out of thermal equilibrium ($V_{DS} \neq 0, V_{SB} \neq 0$):

$$n(y) = N_a e^{\frac{q(\phi_s(y) + \phi_p - V_{CB}(y))}{KT}} = N_a e^{\frac{q(\phi_s(y) + \phi_p - V_{SB} - V_{CS}(y))}{KT}}$$

Electron charge density per unit area ($V_{DS} \neq 0, V_{SB} \neq 0$):

$$Q_N(y) = -q\delta n(y) = -q\delta N_a e^{\frac{q(\phi_s(y) + \phi_p - V_{SB} - V_{CS}(y))}{KT}}$$

Thickness of the inversion layer (see the appendix) assumed to be so small that we can ignore any significant potential drop across it in the vertical direction (in order to simplify the analysis)

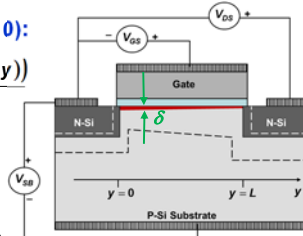
$$\delta \approx \frac{KT}{q^2 N_a} C_b$$

Above threshold, the above can be shown to reduce to the standard expression:

$$Q_N(y) = -C_{ox}(V_{GS} - V_{TN} - V_{CS}(y))$$

But below threshold, we need to use:

$$Q_N(y) = -q\delta n(y) = -q\delta N_a e^{\frac{q(\phi_s(y) + \phi_p - V_{SB} - V_{CS}(y))}{KT}}$$



Surface Potential and Inversion Layer Vs Gate Voltage

Suppose $V_{DS} = 0$:

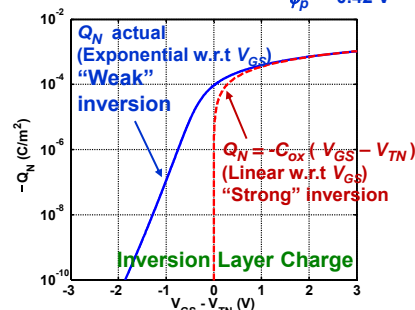
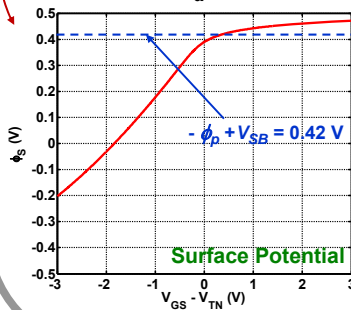
$$V_{GB} = V_{FB} - \frac{Q_N}{C_{ox}} + \phi_s - \phi_p + \sqrt{2 \epsilon_s q N_a (\phi_s - \phi_p)} = F(\phi_s)$$

RHS is a function of ϕ_s only since:

$$Q_N = -q\delta n = -q\delta N_a e^{\frac{q(\phi_s + \phi_p - V_{SB})}{KT}}$$

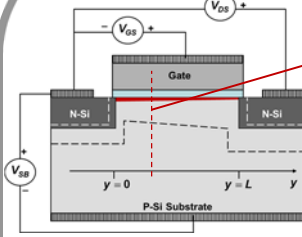
$$\delta \approx \frac{KT}{q^2 N_a} C_b \quad \delta = \text{Thickness of the inversion layer}$$

$$N_a = 10^{17} \text{ cm}^{-3} \\ \phi_p = -0.42 \text{ V}$$



=> The simple assumption of ϕ_s getting stuck at $-\phi_p + V_{SB}$ above threshold is almost true

A NFET Below Threshold: Weak Inversion



$$V_{GB} - V_{FB} = V_{ox} + V_S$$

$$= E_{ox}(y)t_{ox} + \phi_s(y) - \phi_p$$

$$\left\{ \begin{array}{l} \epsilon_{ox} E_{ox}(y) = -Q_N(y) + qN_a x_d(y) \\ x_d(y) = \sqrt{2\epsilon_s q N_a (\phi_s(y) - \phi_p)} \end{array} \right.$$

$$V_{GB} = V_{FB} - \frac{Q_N(y)}{C_{ox}} + \phi_s(y) - \phi_p + \frac{\sqrt{2\epsilon_s q N_a (\phi_s(y) - \phi_p)}}{C_{ox}} = F(\phi_s(y)) \quad \left. \begin{array}{l} \text{Holds for all } y \\ \text{including } y=0 \end{array} \right\}$$

In particular at $y=0$:

$$V_{GB} = V_{FB} - \frac{Q_N(y=0)}{C_{ox}} + \phi_s(y=0) - \phi_p + \frac{\sqrt{2\epsilon_s q N_a (\phi_s(y=0) - \phi_p)}}{C_{ox}} = F(\phi_s(y=0))$$

The above expression holds above threshold as well as below threshold
Now we evaluate it below threshold

A NFET Below Threshold: Weak Inversion

At $y=0$:

$$V_{GB} = V_{FB} - \frac{Q_N(y=0)}{C_{ox}} + \phi_s(y=0) - \phi_p + \frac{\sqrt{2\epsilon_s q N_a (\phi_s(y=0) - \phi_p)}}{C_{ox}} = F(\phi_s(y=0))$$

Small below threshold

We know that at threshold:

$$\phi_s(y=0) \approx -\phi_p + V_{SB}$$

We Taylor expand w.r.t. $\phi_s(y=0)$ around $\phi_s(y=0) = -\phi_p + V_{SB}$ for the below threshold situation when $\phi_s(y=0) < -\phi_p + V_{SB}$:

$$V_{GB} = F \Big|_{\phi_s(y=0) = -\phi_p + V_{SB}} + \frac{\partial F}{\partial \phi_s(y=0)} \Big|_{\phi_s(y=0) = -\phi_p + V_{SB}} [\phi_s(y=0) + \phi_p - V_{SB}]$$

$$\Rightarrow V_{GB} = V_{TN} + V_{SB} + \frac{\partial F}{\partial \phi_s(y)} \Big|_{\phi_s(y=0) = -\phi_p + V_{SB}} [\phi_s(y=0) + \phi_p - V_{SB}]$$

$$\Rightarrow V_{GS} - V_{TN} = \frac{\partial F}{\partial \phi_s(y)} \Big|_{\phi_s(y=0) = -\phi_p + V_{SB}} [\phi_s(y=0) + \phi_p - V_{SB}]$$

A NFET Below Threshold: Weak Inversion

$$V_{GS} - V_{TN} = \frac{\partial F}{\partial \phi_s(y)} \Big|_{\phi_s(y=0) = -\phi_p + V_{SB}} [\phi_s(y=0) + \phi_p - V_{SB}]$$

$$\Rightarrow V_{GS} - V_{TN} = \left(1 + \frac{\sqrt{q\epsilon_s N_a / 2} (-2\phi_p + V_{SB})}{C_{ox}} \right) [\phi_s(y=0) + \phi_p - V_{SB}]$$

$$= \left(1 + \frac{C_b}{C_{ox}} \right) [\phi_s(y=0) + \phi_p - V_{SB}]$$

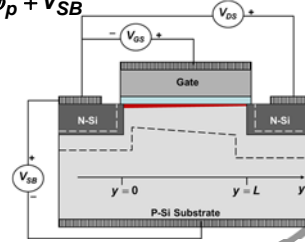
$$= m [\phi_s(y=0) + \phi_p - V_{SB}]$$

$$\Rightarrow \phi_s(y=0) + \phi_p - V_{SB} = \frac{V_{GS} - V_{TN}}{m} \begin{cases} \phi_s(y=0) < -\phi_p + V_{SB} \\ V_{GS} < V_{TN} \end{cases}$$

$$\left[m = 1 + \frac{C_b}{C_{ox}} > 1 \right]$$

Depletion capacitance at the source end

Oxide capacitance



Electron Density in Weak Inversion

Suppose $V_{DS} = 0$:

$$n(y) = N_a e^{\frac{q(\phi_s(y) + \phi_p - V_{SB} - V_{GS}(y))}{KT}} = N_a e^{\frac{q(\phi_s(y=0) + \phi_p - V_{SB})}{KT}}$$

Electron density can be written as:

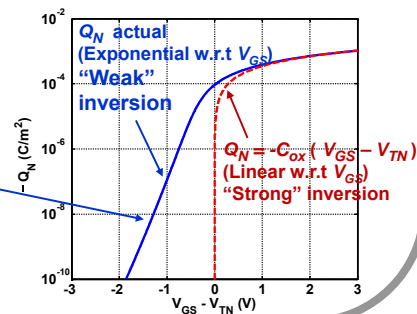
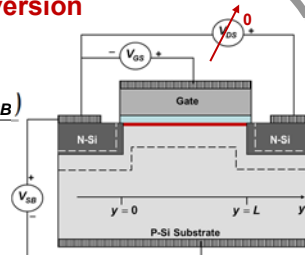
$$n(y) = N_a e^{\frac{q(\phi_s(y=0) + \phi_p - V_{SB})}{KT}} \begin{cases} \phi_s(y=0) < -\phi_p + V_{SB} \\ V_{GS} < V_{TN} \end{cases}$$

$$= N_a e^{\frac{q(V_{GS} - V_{TN})}{mKT}}$$

Exponential w.r.t V_{GS}
Agrees with:

Recall from previous slide:

$$\phi_s(y=0) + \phi_p - V_{SB} = \frac{V_{GS} - V_{TN}}{m}$$



Electron Current Density in Weak Inversion

Now suppose $V_{DS} \neq 0$:

Channel current can be written as:

$$I_D = -W\delta J_{ny} = -W\delta \left[\underbrace{q\mu_n n(y)}_{\text{Small}} E_y(y) + \underbrace{qD_n \frac{dn(y)}{dy}}_{\text{Large}} \right]$$

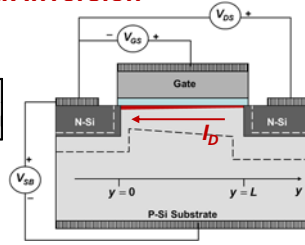
$$\left\{ \begin{aligned} n(y) &= N_a e^{\frac{q(\phi_s(y) + \phi_p - V_{SB} - V_{CS}(y))}{KT}} \\ E_y(y) &= -\frac{d\phi_s(y)}{dy} \end{aligned} \right.$$

$$I_D = -W\delta \left[-q\mu_n n(y) \frac{d\phi_s(y)}{dy} + \frac{q}{KT} qD_n n(y) \frac{d}{dy} [\phi_s(y) - V_{CS}(y)] \right]$$

Cancel

$$I_D = W \left[\frac{q}{KT} qD_n \delta n(y) \frac{dV_{CS}(y)}{dy} \right] = -Wq\mu_n Q_N(y) \frac{dV_{CS}(y)}{dy} \rightarrow \text{We have used this expression before in strong inversion!}$$

$$I_D = W \left[\frac{q}{KT} qD_n \delta n(y) \frac{dV_{CS}(y)}{dy} \right] \rightarrow \text{But now we are going to use this one! Current flow is dominated by DIFFUSION}$$



Electron Current Density in Weak Inversion

$$I_D = WqD_n \delta N_a e^{\frac{q(\phi_s(y) + \phi_p - V_{SB} - V_{CS}(y))}{KT}} \frac{q}{KT} \frac{dV_{CS}(y)}{dy}$$

$$I_D \approx WqD_n \delta N_a e^{\frac{q(\phi_s(y=0) + \phi_p - V_{SB} - V_{CS}(y))}{KT}} \frac{q}{KT} \frac{dV_{CS}(y)}{dy}$$

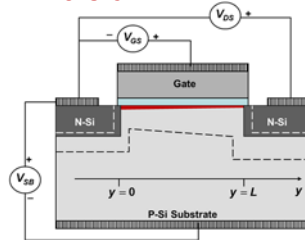
$$= WqD_n \delta N_a e^{\frac{q(V_{GS} - V_{TN})}{mKT}} \frac{1}{e} \frac{qV_{CS}(y)}{KT} \frac{q}{KT} \frac{dV_{CS}(y)}{dy}$$

$$\left\{ n(y) = N_a e^{\frac{q(\phi_s(y) + \phi_p - V_{SB} - V_{CS}(y))}{KT}} \right.$$

Integrate from $y=0$ to $y=L$ on both sides:

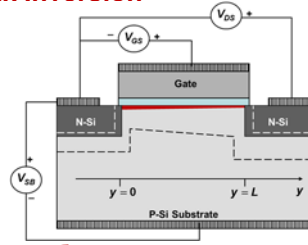
$$I_D L \approx WqD_n \delta N_a e^{\frac{q(V_{GS} - V_{TN})}{mKT}} \int_0^L dy e^{-\frac{qV_{CS}(y)}{KT}} \frac{q}{KT} \frac{dV_{CS}(y)}{dy}$$

$$\Rightarrow I_D = q \frac{W}{L} D_n \delta N_a e^{\frac{q(V_{GS} - V_{TN})}{mKT}} \left[1 - e^{-\frac{qV_{DS}}{KT}} \right]$$



Electron Current Density in Weak Inversion

$$\begin{aligned}
 I_D &= q \frac{W}{L} D_n \delta N_a e^{\frac{q(V_{GS}-V_{TN})}{mKT}} \left[1 - e^{-\frac{qV_{DS}}{KT}} \right] \\
 &= q \frac{W}{L} D_n \frac{KT}{q^2} C_b e^{\frac{q(V_{GS}-V_{TN})}{mKT}} \left[1 - e^{-\frac{qV_{DS}}{KT}} \right] \\
 &= \frac{W}{L} D_n \frac{KT}{q} (m-1) C_{ox} e^{\frac{q(V_{GS}-V_{TN})}{mKT}} \left[1 - e^{-\frac{qV_{DS}}{KT}} \right] \\
 &= \frac{W}{L} \mu_n C_{ox} \left(\frac{KT}{q} \right)^2 (m-1) e^{\frac{q(V_{GS}-V_{TN})}{mKT}} \left[1 - e^{-\frac{qV_{DS}}{KT}} \right] \\
 &= k_n \left(\frac{KT}{q} \right)^2 (m-1) e^{\frac{q(V_{GS}-V_{TN})}{mKT}} \left[1 - e^{-\frac{qV_{DS}}{KT}} \right]
 \end{aligned}$$



$$\begin{aligned}
 \delta &\approx \frac{KT}{q^2 N_a} C_b \\
 m &= 1 + \frac{C_b}{C_{ox}} > 1
 \end{aligned}$$

Example: suppose $V_{GS} = V_{TN}$ and $V_{DS} \gg KT/q$ then:

$$I_D \sim k_n \left(\frac{KT}{q} \right)^2 (m-1) \quad \left\{ \begin{array}{l} k_n = 200 \mu A/V^2 \\ m \sim 1.8 \\ I_D \sim 100 \text{ nA} \end{array} \right. \quad \text{EXTREMELY LARGE!!!}$$

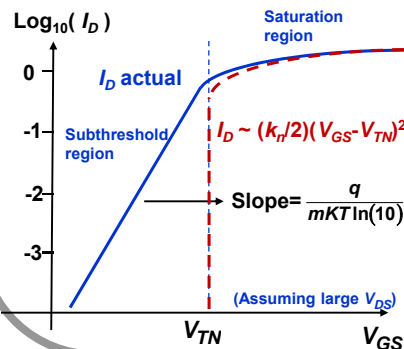
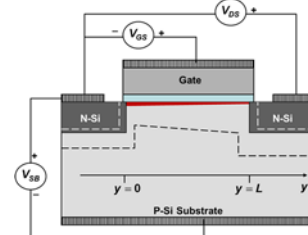
Inverse Subthreshold Slope of Current in Weak Inversion

When $V_{GS} > V_{TN}$ (assuming saturation):

$$I_D \sim \frac{k_n}{2} (V_{GS} - V_{TN})^2$$

When $V_{GS} < V_{TN}$:

$$I_D \sim (m-1) k_n \left(\frac{KT}{q} \right)^2 e^{\frac{q}{mKT} (V_{GS} - V_{TN})} \left(1 - e^{-\frac{qV_{DS}}{KT}} \right)$$



Amount of gate voltage reduction needed to change the current by one order of magnitude in the subthreshold region:

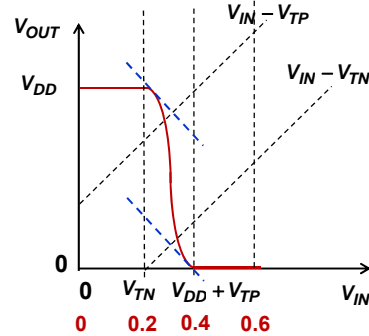
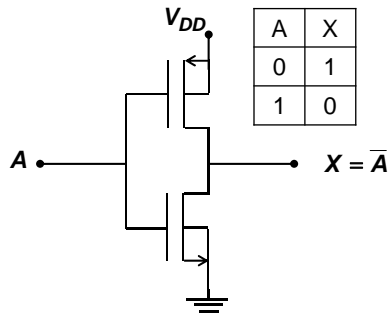
$$\frac{mKT \ln(10)}{q} \approx 60 \text{ mV/decade}$$

Assuming $T = 300 \text{ K}$
 $m = 1$ (but m is typically > 1)

The smaller the inverse subthreshold slope the smaller the leakage current

Typically the inverse slope $> 100 \text{ mV/decade}$

Implications of the Inverse Subthreshold Slope and Power Dissipation



Suppose one would like to have V_{DD} of 0.6 V in order to improve power dissipation

Inverse subthreshold slope of FETs is 100 mV/decade, and $k_n = k_p = 200 \mu\text{A/V}^2$

This means V_{GS} needs to be below V_{TN} for NFETs (or above $V_{DD} + V_{TP}$ for PFETs) by 0.2 V in order to keep the leakage current not much larger than $\sim 1 \text{ nA}$

But this would be impossible unless $V_{DD} \gg 0.6 \text{ V}$
Subthreshold slope limits the minimum possible value of V_{DD}

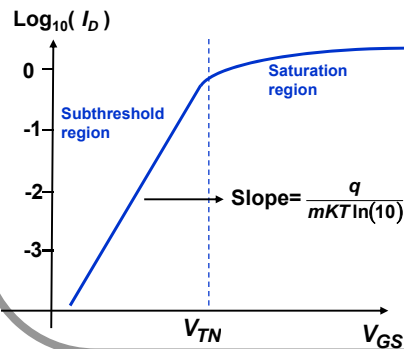
Subthreshold Operation of FETs

When $V_{GS} > V_{TN}$ (assuming saturation):

$$I_D \sim \frac{k_n}{2} (V_{GS} - V_{TN})^2 \longrightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = k_n (V_{GS} - V_{TN})$$

When $V_{GS} < V_{TN}$:

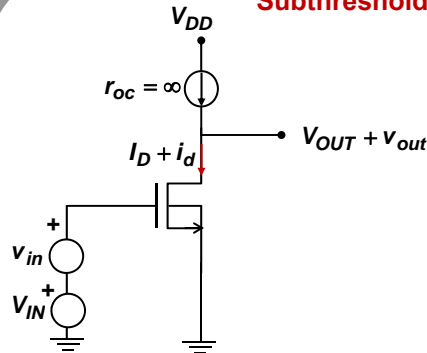
$$I_D \sim (m-1)k_n \left(\frac{KT}{q}\right)^2 e^{\frac{q}{mKT}(V_{GS}-V_{TN})} \left(1 - e^{-\frac{qV_{DS}}{KT}}\right)$$



$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{qI_D}{mKT}$$

$$g_o = \frac{1}{r_o} = \frac{\partial I_D}{\partial V_{DS}} = \frac{qI_D}{KT} \left(\frac{1}{e^{\frac{qV_{DS}}{KT}} + 1} \right)$$

Subthreshold FET Amplifiers



Advantages of subthreshold FET amplifiers:

Can get decent gain with very small DC bias currents and voltages

And with very small DC power dissipation

Disadvantages:
Slow speed

Open circuit voltage gain:

$$A_v = \frac{v_{out}}{v_{in}} = g_m r_o = \frac{g_m}{g_o}$$

In subthreshold operation:

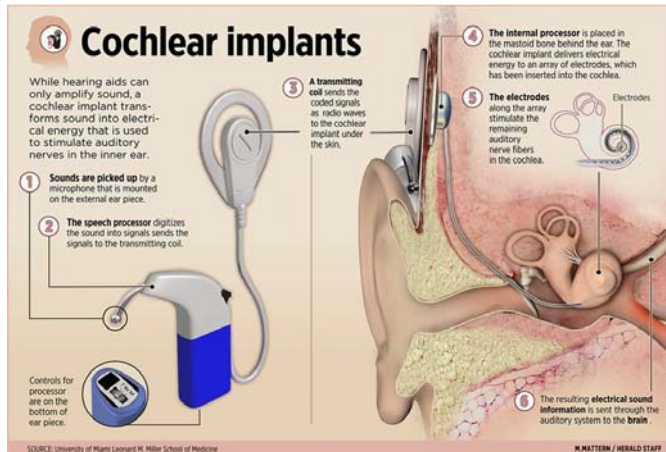
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{qI_D}{mKT}$$

$$g_o = \frac{1}{r_o} = \frac{\partial I_D}{\partial V_{DS}} = \frac{qI_D}{KT} \left(\frac{1}{e^{\frac{qV_{DS}}{KT}} + 1} \right)$$

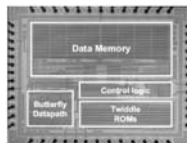
Subthreshold open circuit voltage gain:

$$A_v = \frac{v_{out}}{v_{in}} = g_m r_o = \frac{g_m}{g_o} = \left(\frac{e^{\frac{qV_{DS}}{KT}} + 1}{m} \right) \gg 1$$

Subthreshold FET Circuits: Applications Space



Micro sensors, pacemakers, medical implants



Digital CMOS circuits: DLMS filter, sensor processor, FFT processor, μ controller since 2000s



Wrist watches



Energy constrained circuits, portable devices, gyroscope

Appendix: Thickness of the Inversion Layer Below Threshold

Electron density is given by:

$$n(x) = N_a e^{\frac{q(\phi(x) + \phi_p - V_{SB})}{KT}}$$

The vertical electric field in the oxide is:

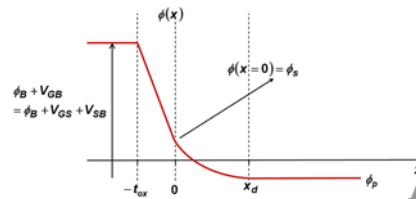
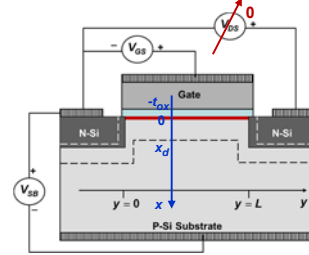
$$\epsilon_{ox} E_{ox} = -Q_N + qN_a x_d$$

The vertical electric field in the semiconductor is:

$$\begin{aligned} \epsilon_s E_s(x=0) &= \epsilon_{ox} E_{ox}(x=0) = qN_a x_d \\ \Rightarrow E_s(x=0) &= \frac{qN_a x_d}{\epsilon_s} \end{aligned}$$

The potential in the semiconductor near the surface is:

$$\phi(x) \approx \phi(x=0) - E_s(x=0)x$$



Appendix: Thickness of the Inversion Layer Below Threshold

Electron density is given by:

$$\begin{aligned} n(x) &= N_a e^{\frac{q(\phi(x) + \phi_p - V_{SB})}{KT}} \\ &= N_a e^{\frac{q(\phi(x=0) - E_s(x=0)x + \phi_p - V_{SB})}{KT}} \\ &= n(x=0) e^{-\frac{qE_s(x=0)x}{KT}} = n(x=0) e^{-\frac{x}{\delta}} \end{aligned}$$

The length scale δ is:

$$\delta = \frac{KT}{qE_s(x=0)} = \frac{KT\epsilon_s}{q^2 N_a x_d} = \frac{KT}{q^2 N_a} C_b$$

