Lecture 20

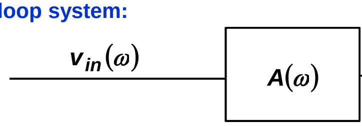
Negative Feedback, Stability, Gain Margins, Phase Margins

In this lecture you will learn:

- Negative Feedback and Stability
- High Frequency Behavior of Amplifier Circuits
- Gain Margin, Phase Margin, and Stability
- Frequency Compensation

Negative Feedback and Stability

Open loop system:

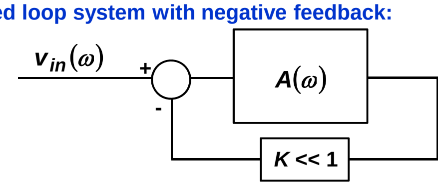


$$v_{out}(\omega) = A(\omega) v_{in}(\omega)$$

Open loop gain: $A(\omega)$

Stability problems: suppose the open loop gain is sensitive to the temperature or the power supply voltage. As the temperature or the power supply fluctuates, the output is going to fluctuate.

Closed loop system with negative feedback:



$$v_{out}(\omega) = \frac{A(\omega)}{1+KA(\omega)} v_{in}(\omega)$$

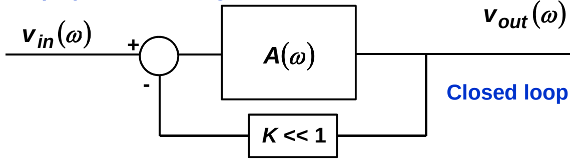
$$[v_{in}(\omega) - K v_{out}(\omega)] A(\omega) = v_{out}(\omega)$$

$$\Rightarrow v_{out}(\omega) = \frac{A(\omega)}{1+KA(\omega)} v_{in}(\omega)$$

Closed loop gain: $\dfrac{A(\omega)}{1+KA(\omega)}$

Negative Feedback and Stability

Closed loop system with negative feedback:



$$v_{out}(\omega) = \frac{A(\omega)}{1+KA(\omega)} v_{in}(\omega)$$

Closed loop gain: $\frac{A(\omega)}{1+KA(\omega)}$

If for small frequencies, $A(\omega \sim 0)$ >>>> 1 and $KA(\omega \sim 0)$ >> 1 :

Closed loop gain: $\frac{A(\omega \sim 0)}{1+KA(\omega \sim 0)} \approx \frac{1}{K} >> 1$

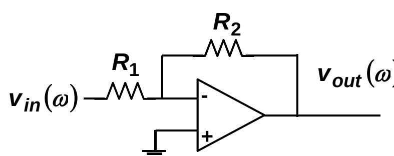
Stability problems resolved: Now as the temperature or the power supply fluctuates, the output is going to be much more stable (because it is almost independent of $A(\omega)$!!

Negative feedback improves stability at the expense of gain

A positive feedback can lead to instability and/or oscillations!

Differential Amplifiers, Negative Feedback, and Stability

A differential amplifier is almost always operated using a negative feedback:



$$v_{out}(\omega) = A(\omega)[v_+(\omega) - v_-(\omega)]$$

$$v_{out}(\omega) = -v_{in}(\omega)\frac{R_2}{R_1}\frac{A(\omega)}{1+A(\omega)+\frac{R_2}{R_1}} = -v_{in}(\omega)\frac{\frac{R_2}{R_1}}{1+\frac{R_2}{R_1}}\frac{A(\omega)}{1+KA(\omega)}$$

$$K = \frac{1}{1+\frac{R_2}{R_1}}$$

If $A(\omega \sim 0)$ >>>> 1, then:

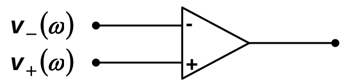
$$v_{out}(\omega) \approx -v_{in}(\omega)\frac{R_2}{R_1}$$

Negative feedback improves stability at the expense of gain

A positive feedback can lead to instability and/or oscillations!

Amplifier Gain: Frequency Response

Consider a differential amplifier:



$$v_{out}(\omega) = A(\omega)[v_+(\omega) - v_-(\omega)]$$

The amplifier gain can be expressed (most generally) as:

$$A(\omega) = A_o \frac{(1 + j\omega t_1)(1 + j\omega t_2)(1 + j\omega t_3)\ldots\ldots}{(1 + j\omega \tau_1)(1 + j\omega \tau_2)(1 + j\omega \tau_3)\ldots\ldots}$$

→ Multiple zeros

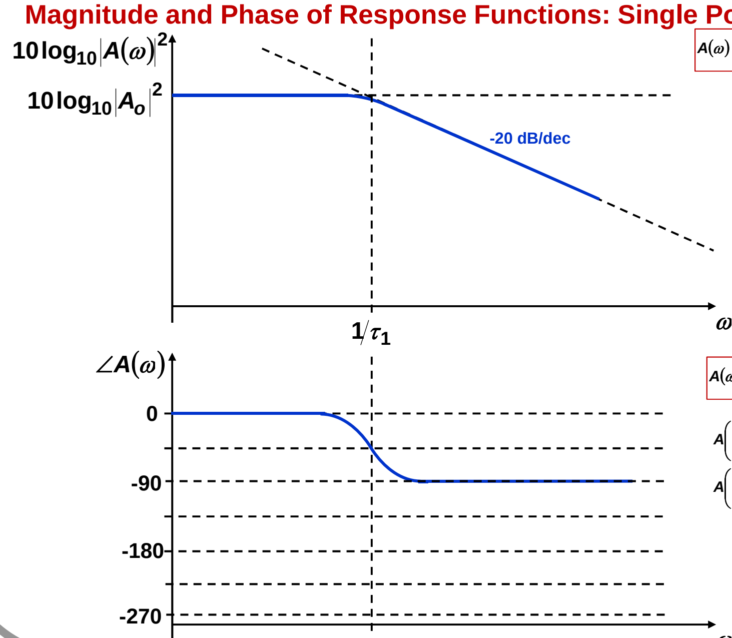
→ Multiple poles

Suppose, for simplicity, the amplifier gain can be expressed as:

$$A(\omega) = \frac{A_o}{(1 + j\omega \tau_1)(1 + j\omega \tau_2)(1 + j\omega \tau_3)\ldots\ldots}$$

→ Multiple poles

Magnitude and Phase of Response Functions: Single Pole Case



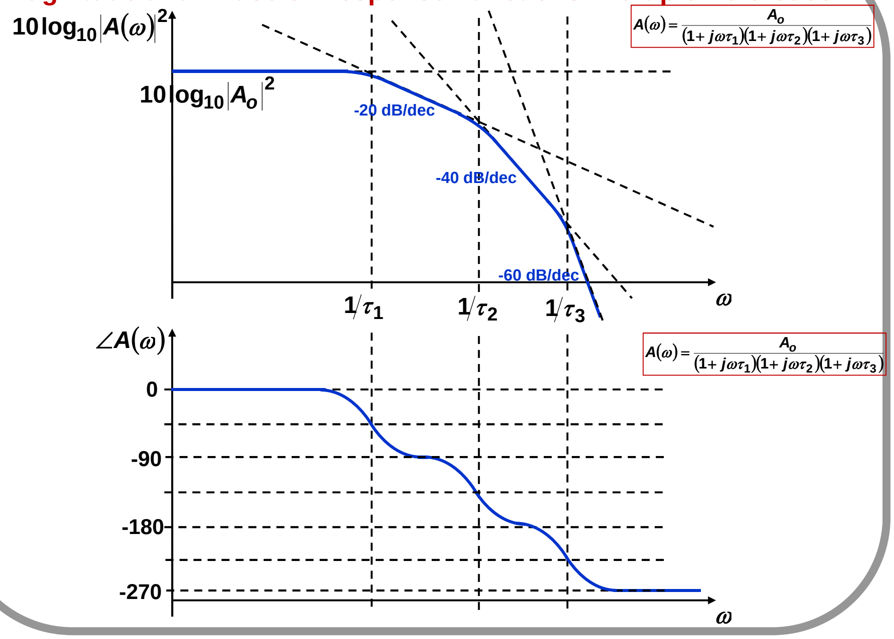
$$A(\omega) = \frac{A_o}{(1 + j\omega \tau_1)}$$

$$A(\omega) = \frac{A_o}{(1 + j\omega \tau_1)}$$

$$A\left(\omega << \frac{1}{\tau_1}\right) = A_o$$

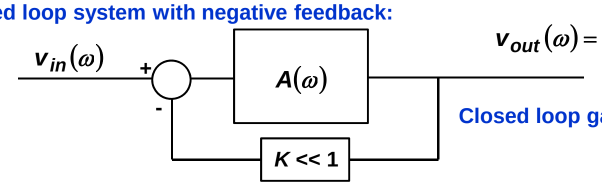
$$A\left(\omega >> \frac{1}{\tau_1}\right) = -j\frac{A_o}{\omega \tau_1}$$

Magnitude and Phase of Response Functions: Multiple Pole Case



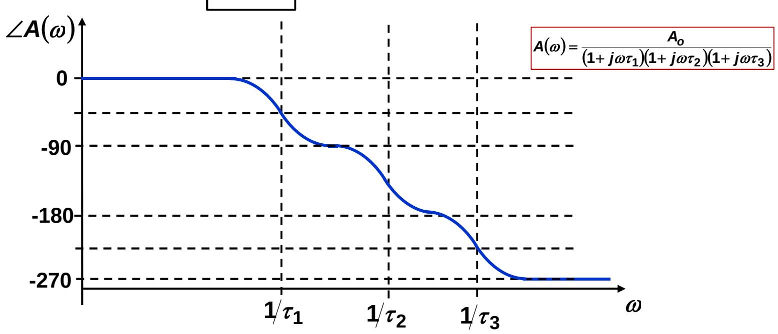
Negative Feedback and Stability

Closed loop system with negative feedback:



$$v_{out}(\omega)=\frac{A(\omega)}{1+KA(\omega)}v_{in}(\omega)$$

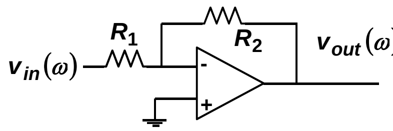
Closed loop gain: $\frac{A(\omega)}{1+KA(\omega)}$



At frequencies between $1/\tau_2$ and $1/\tau_3$, $\angle A(\omega)$ is 180-degrees
=> The feedback is positive, not negative!!!

Phase Response and Amplifier Stability

Consider a differential amplifier operated using a negative feedback:



$$v_{out}(\omega) = A(\omega)[v_+(\omega) - v_-(\omega)]$$

$$v_{out}(\omega) = -v_{in}(\omega)\frac{R_2}{R_1}\frac{A(\omega)}{1 + A(\omega) + \frac{R_2}{R_1}} = -v_{in}(\omega)\frac{\frac{R_2}{R_1}}{1+\frac{R_2}{R_1}}\frac{A(\omega)}{1+KA(\omega)}$$

$$K = \frac{1}{1+\frac{R_2}{R_1}}$$

A positive feedback can happen at high frequencies when:

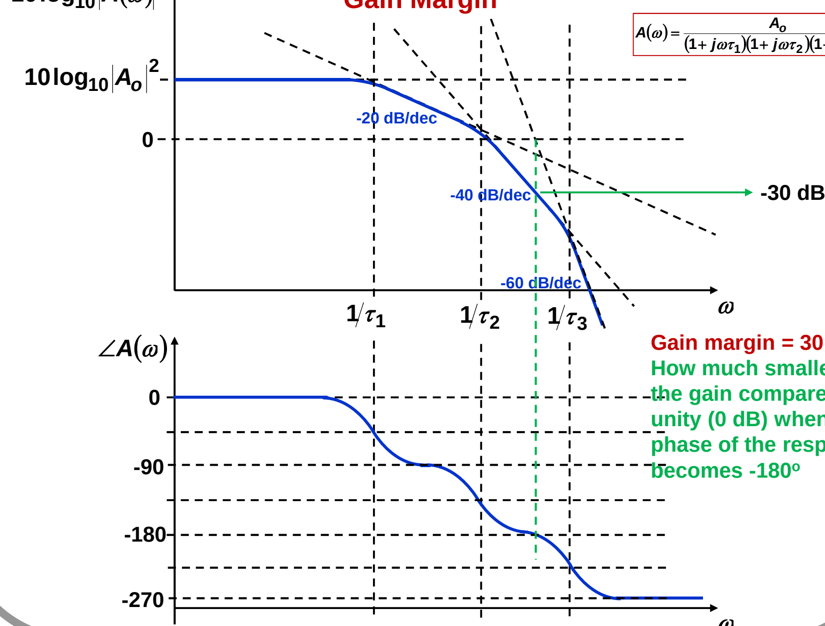
$$\angle A(\omega) \rightarrow -180^o$$

Denominator can become very small or zero!

Solution:
To avoid this positive feedback from happening, the magnitude $|A(\omega)|$ of the gain must get much less than unity before $\angle A(\omega) = -180^o$

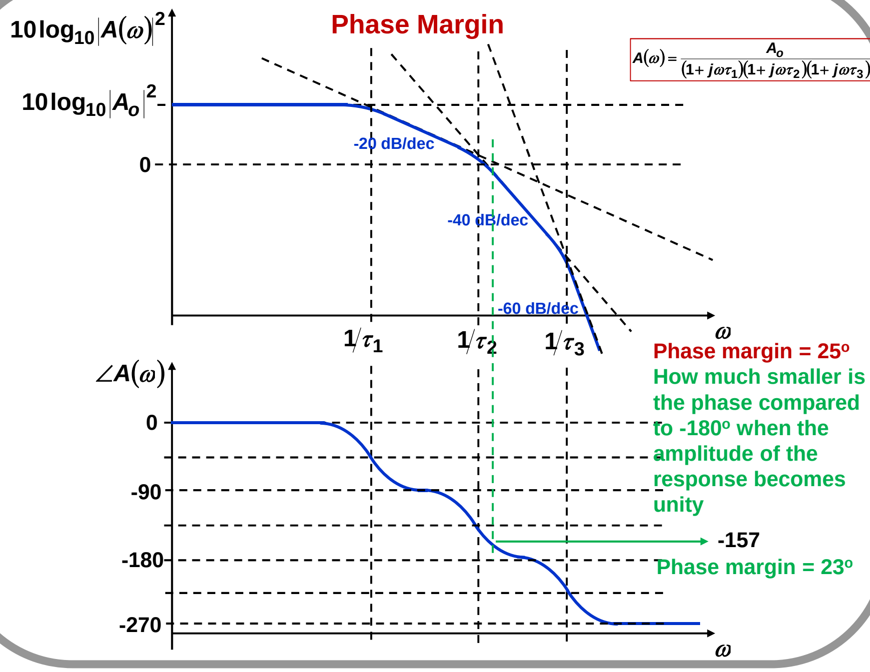
Gain Margin

$$A(\omega) = \frac{A_o}{(1+j\omega\tau_1)(1+j\omega\tau_2)(1+j\omega\tau_3)}$$



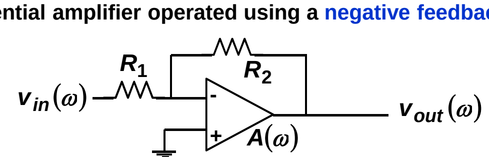
Gain margin = 30 dB
How much smaller is the gain compared to unity (0 dB) when the phase of the response becomes -180°

Phase Margin



Frequency Compensation

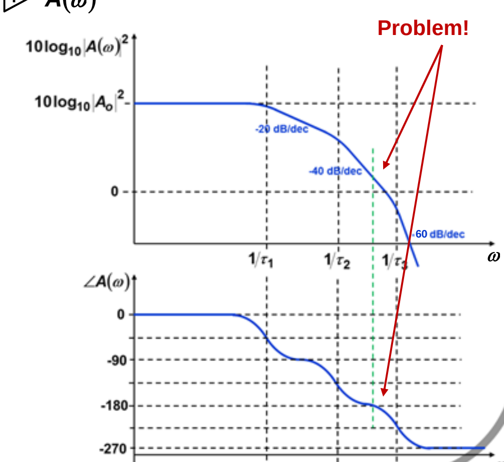
Consider a differential amplifier operated using a negative feedback:



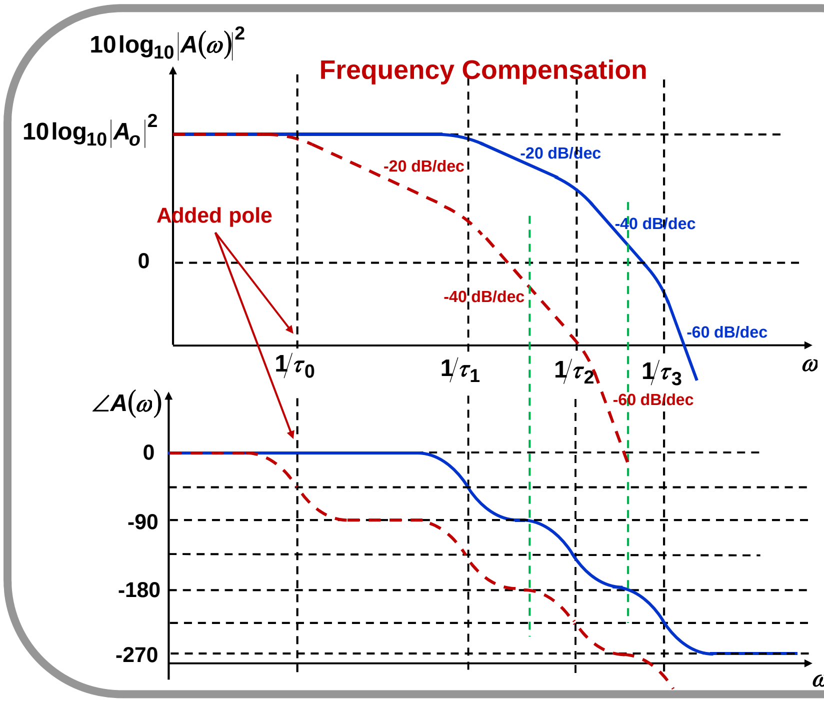
Very often, almost always in fact, when you are done designing the amplifier you figure out that you don't have enough gain and phase margins!!

How to solve this problem?

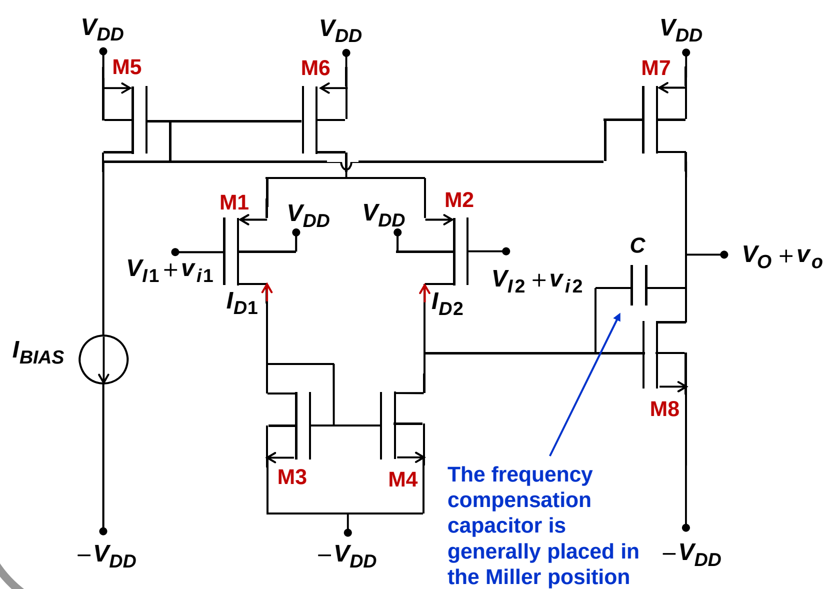
Frequency Compensation:
Add a low frequency pole inside $A(\omega)$ (by adding extra capacitors, for example), sacrifice bandwidth, but regain stability (PTO…)



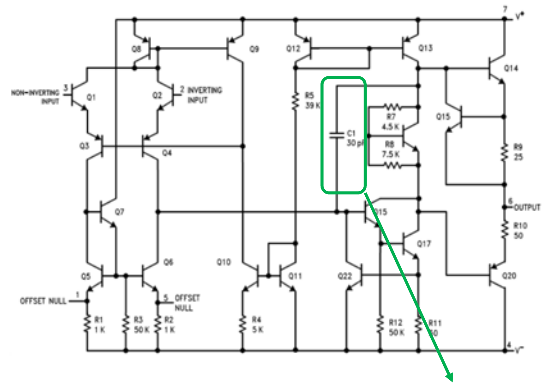
Frequency Compensation



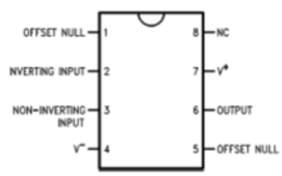
Frequency Compensation



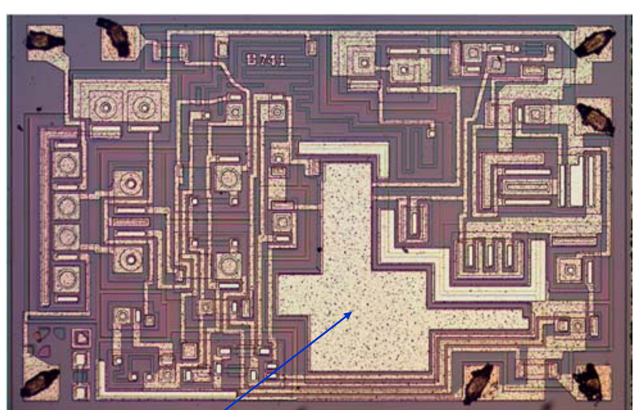
Texas Instruments LM 741 Operational Amplifier



Frequency Compensation capacitor



Texas Instruments LM 741 Operational Amplifier



Frequency compensation capacitor