Lecture 19

High Frequency Analysis of FET Circuits

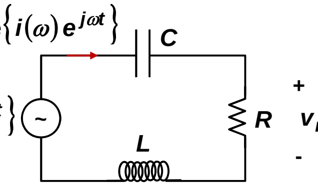
In this lecture you will learn:

- **High Frequency Analysis of FET Circuits**
- **Miller Effect and the Miller Capacitance**
- **The Transition Frequency and the Ultimate Performance limits of FET Devices**

Phasor Analysis: Complex Impedances

$$i(t) = \text{Re}\{i(\omega)\, e^{j\omega t}\}$$

$$v(t) = \text{Re}\{v(\omega)\, e^{j\omega t}\}$$



$$v_r(t) = \text{Re}\{v_r(\omega)\, e^{j\omega t}\}$$

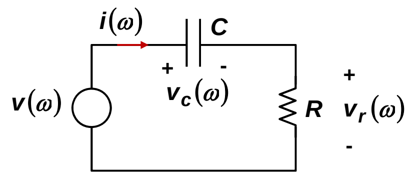
Capacitive Impedance/Admittance:

$$Z(\omega) = \frac{1}{Y(\omega)} = \frac{1}{j\omega C}$$

Inductance Impedance/Admittance:

$$Z(\omega) = \frac{1}{Y(\omega)} = j\omega L$$

Phasor Analysis: Calculations with Impedances

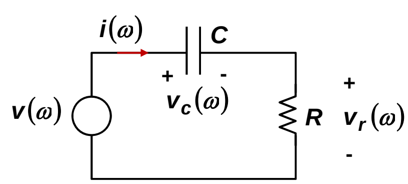


One can compute the voltage phasors using the impedances:

$$v_c(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} v(\omega) = \frac{1}{1 + j\omega RC} v(\omega)$$

$$v_r(\omega) = \frac{R}{R + \frac{1}{j\omega C}} v(\omega) = \frac{j\omega RC}{1 + j\omega RC} v(\omega)$$

Phasor Analysis: Calculations with Impedances

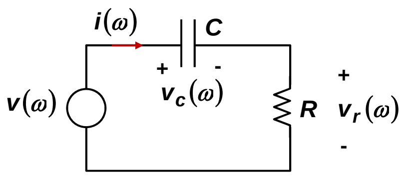


One can compute the voltage phasors using the impedances:

$$v_c(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} v(\omega) = \frac{1}{1 + j\omega RC} v(\omega)$$

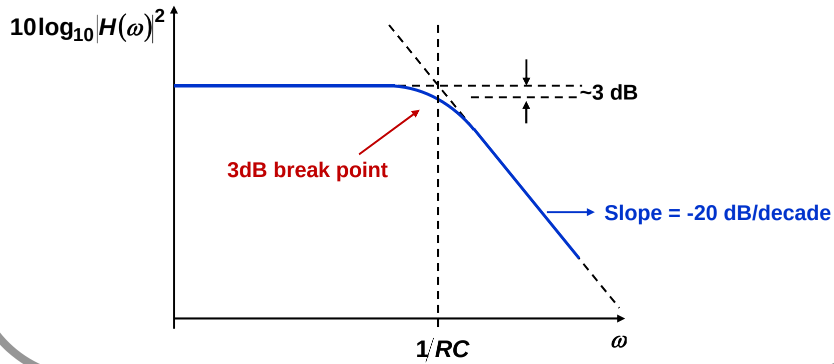
$$v_r(\omega) = \frac{R}{R + \frac{1}{j\omega C}} v(\omega) = \frac{j\omega RC}{1 + j\omega RC} v(\omega)$$

Phasor Analysis: Bode Plots

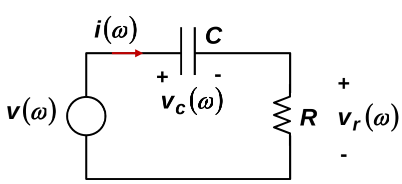


$$\frac{v_c(\omega)}{v(\omega)} = \frac{1}{1 + j\omega RC} = H(\omega)$$

A transfer function with a <u>pole</u> at frequency 1/*RC*

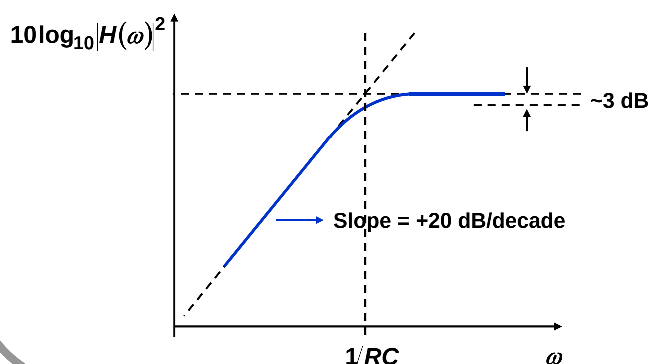


Phasor Analysis: Bode Plots

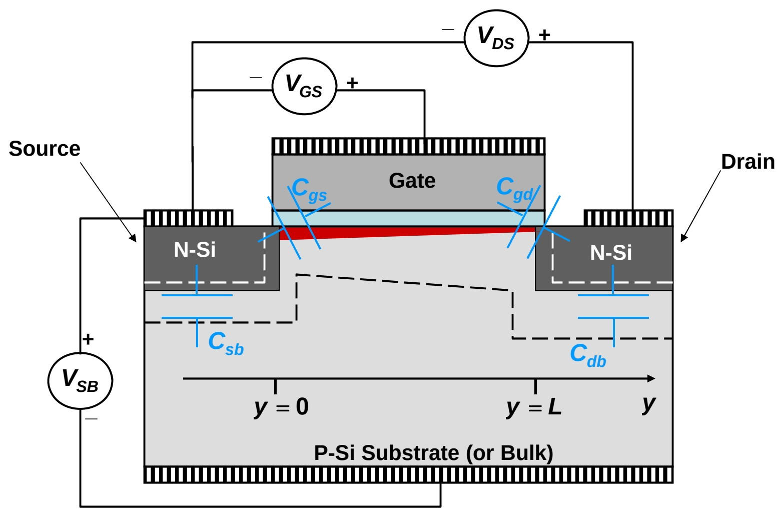


$$\frac{v_r(\omega)}{v(\omega)} = \frac{j\omega RC}{1 + j\omega RC} = H(\omega)$$

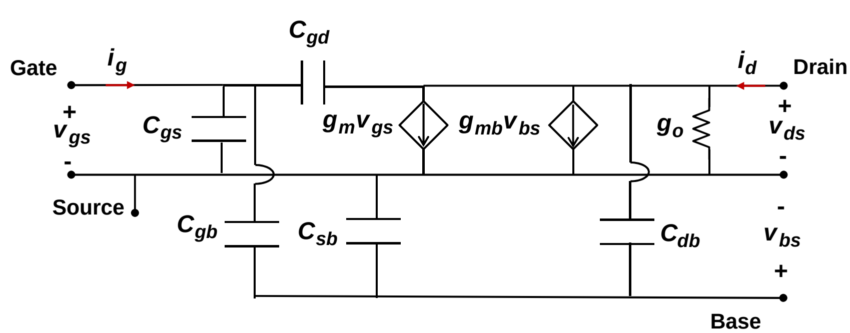
A transfer function with a <u>zero</u> at zero frequency and a <u>pole</u> at frequency 1/*RC*



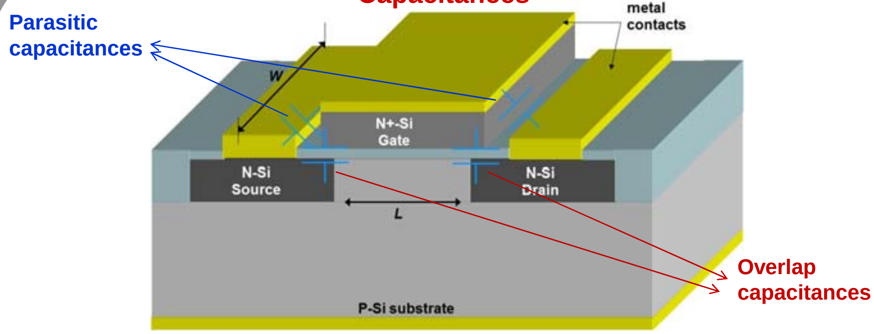
NFET: Capacitances



NFET: High Frequency Small Signal Model



Capacitances

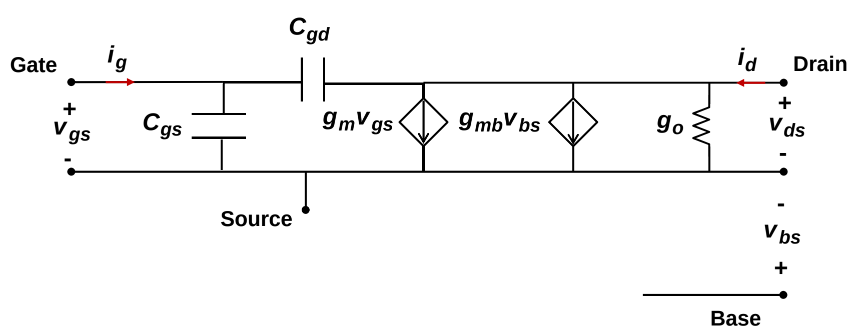


In Saturation:

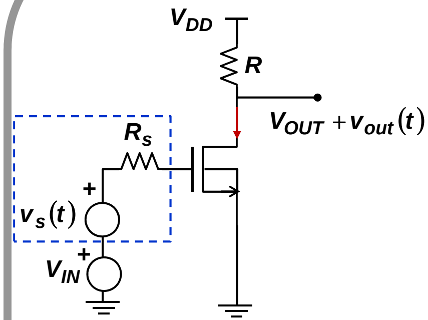
$$C_{gs} = \left.\frac{\partial Q_G}{\partial V_{GS}}\right|_{V_{GD},V_{GB}} = \frac{2}{3}WLC_{ox} + WC_{ov} + WC_p \neq \frac{2}{3}WLC_{ox}$$

$$C_{gd} = \left.\frac{\partial Q_G}{\partial V_{GD}}\right|_{V_{GS},V_{GB}} = WC_{ov} + WC_p \neq 0$$

NFET: Simplified High Frequency Small Signal Model

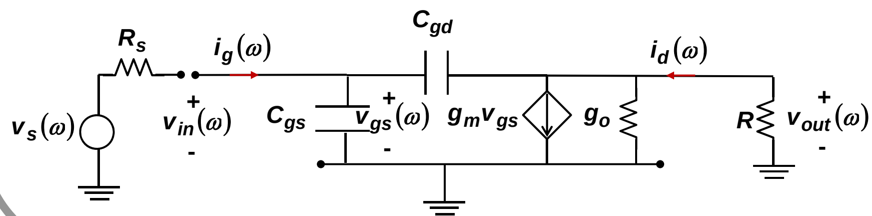


The Common Source Amplifier

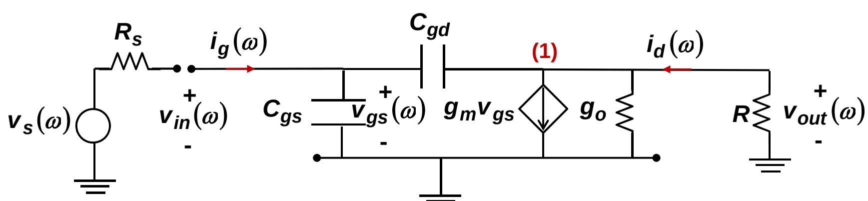


$$v_s(t) = \text{Re}\left\{ v_s(\omega)\, e^{j\omega t} \right\}$$

A high frequency small signal model can be built as follows:



The Common Source Amplifier: Open Circuit Voltage Gain



Need to find:

$$A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} \quad \text{and} \quad H(\omega) = \frac{v_{out}(\omega)}{v_s(\omega)}$$

KCL at (1) gives:

$$i_d(\omega) = g_o v_{out}(\omega) + g_m v_{in}(\omega) + [v_{out}(\omega) - v_{in}(\omega)]\, j\omega C_{gd}$$

Also:

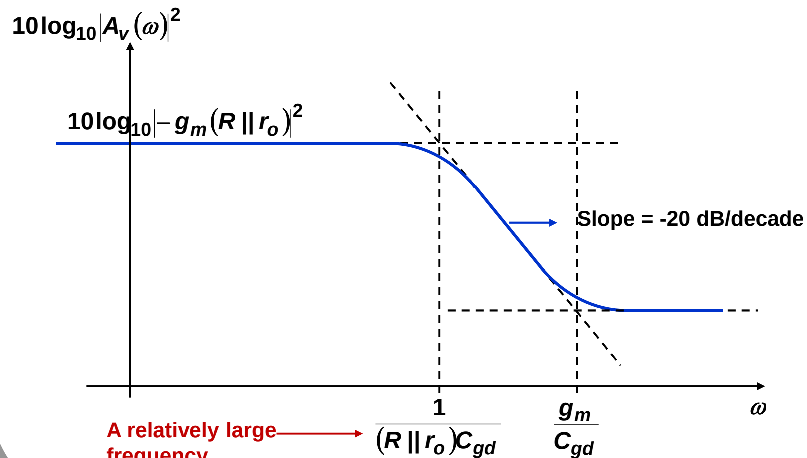
$$v_{out}(\omega) = -i_d(\omega) R$$

The above two give:

$$A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = -\frac{g_m R - j\omega R C_{gd}}{1 + g_o R + j\omega R C_{gd}}$$

The Common Source Amplifier: Open Circuit Voltage Gain

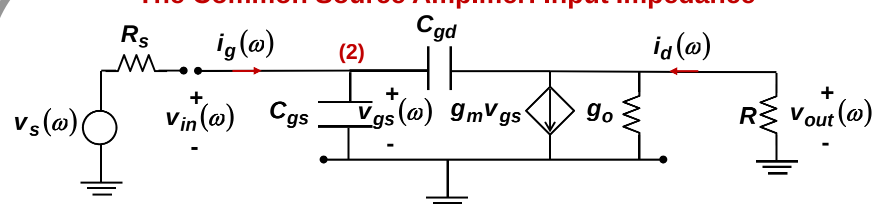
$$A_v(\omega)=\frac{v_{out}(\omega)}{v_{in}(\omega)}=-\frac{g_m R-j\omega RC_{gd}}{1+g_o R+j\omega RC_{gd}}=-g_m(R\,||\,r_o)\frac{1-j\omega\dfrac{C_{gd}}{g_m}}{1+j\omega C_{gd}(R\,||\,r_o)}$$



A relatively large frequency

These poles might not limit the frequency performance…….PTO

The Common Source Amplifier: Input Impedance



KCL at (2) gives:

$$i_g(\omega)=\frac{v_s(\omega)-v_{in}(\omega)}{R_s}$$

$$=j\omega C_{gs}v_{in}(\omega)+j\omega C_{gd}[v_{in}(\omega)-v_{out}(\omega)]$$

$$=\left(j\omega C_{gs}+j\omega C_{gd}[1-A_v(\omega)]\right)v_{in}(\omega)$$

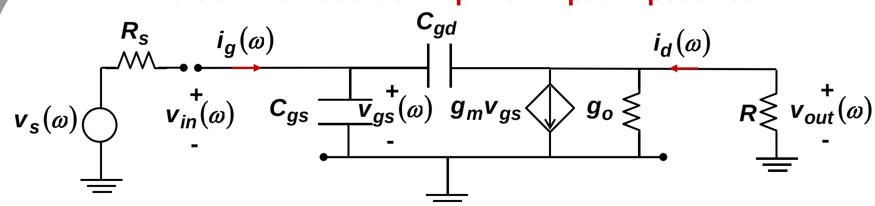
$$\frac{v_{out}(\omega)}{v_{in}(\omega)}=A_v(\omega)$$

$$\frac{v_{in}(\omega)}{i_g(\omega)}=Z_{in}(\omega)=\frac{1}{j\omega C_{gs}+j\omega C_{gd}[1-A_v(\omega)]}$$

The input impedance is:

$$Z_{in}(\omega)\approx\frac{1}{j\omega[C_{gs}+C_{gd}(1-A_v(\omega))]}$$

The Common Source Amplifier: Input Impedance

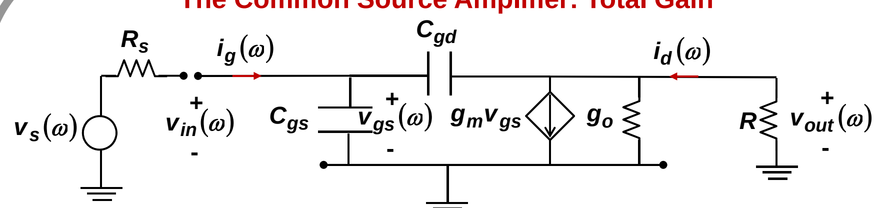


Looking in from the input terminal the capacitance C_{gd} seems to be magnified by a factor proportional to the open circuit voltage gain of the amplifier!!

$$\frac{v_{in}(\omega)}{v_s(\omega)} \approx \frac{Z_{in}(\omega)}{R_s + Z_{in}(\omega)} = \frac{1}{1 + j\omega\left[C_{gs} + \underbrace{C_{gd}(1 - A_v(\omega))}\right]R_s}$$

Looking in from the input terminal the capacitance C_{gd} seems to be magnified by a factor proportional to the open circuit voltage gain of the amplifier!!

The Common Source Amplifier: Total Gain



Finally:

$$H(\omega) = \frac{v_{out}(\omega)}{v_s(\omega)} = \frac{v_{out}(\omega)}{v_{in}(\omega)}\frac{v_{in}(\omega)}{v_s(\omega)} = A_v(\omega)\frac{Z_{in}(\omega)}{R_s + Z_{in}(\omega)}$$

$$= \frac{A_v(\omega)}{1 + j\omega\left[C_{gs} + C_{gd}(1 - A_v(\omega))\right]R_s}$$

Where:

$$A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = -g_m(R \,||\, r_o)\left[\frac{1 - j\omega\frac{C_{gd}}{g_m}}{1 + j\omega C_{gd}(R \,||\, r_o)}\right]$$

The Common Source Amplifier: Poles and Zeros of the Total Gain

$$H(\omega) = \frac{A_v(\omega)}{1 + j\omega\left[C_{gs} + C_{gd}\left(1 - A_v(\omega)\right)\right]R_s} = \frac{H(0)(1 - j\omega\tau_3)}{(1 + j\omega\tau_1)(1 + j\omega\tau_2)}$$

A little tedious algebra can show that the transfer function above has two poles and a zero

$$H(0) \approx -g_m(R \,||\, r_o) \qquad \frac{1}{\tau_3} = \frac{g_m}{C_{gd}}$$

1) Assuming $g_m R_S >> 1$, these poles are:

$$\frac{1}{\tau_1} \approx \frac{g_m}{C_{gs}} + \frac{1}{C_{gd}(R \,||\, r_o)} \qquad \frac{1}{\tau_2} \approx \frac{1}{\left[C_{gs} + C_{gd}\left(g_m(R \,||\, r_o)\right)\right] R_s}$$

This pole will likely determine the smallest frequency at which the total gain rolls over

2) Assuming $g_m R_S << 1$, these poles are:

$$\frac{1}{\tau_1} \approx \frac{1}{R_s C_{gs}} \qquad \frac{1}{\tau_2} \approx \frac{1}{C_{gd}(R \,||\, r_o)}$$

This pole will likely determine the smallest frequency at which the total gain rolls over

The Common Source Amplifier: The Miller Approximation

If the poles and zeros in $A_v(\omega)$ are at a higher frequency than the poles and zeros associated with the transfer function $v_{in}(\omega)/v_s(\omega)$ (which would be the case if $g_m R_S >> 1$) then one may approximate the open circuit gain $A_v(\omega)$ by its low frequency value:

$$A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = -\frac{g_m R - j\omega R C_{gd}}{1 + g_o R + j\omega R C_{gd}} \approx -g_m(R \,||\, r_o)$$

Miller approximation

And then:

$$H(\omega) = \frac{v_{out}(\omega)}{v_s(\omega)} = \frac{v_{out}(\omega)}{v_{in}(\omega)} \frac{v_{in}(\omega)}{v_s(\omega)} = A_v(\omega) \frac{Z_{in}(\omega)}{R_s + Z_{in}(\omega)}$$

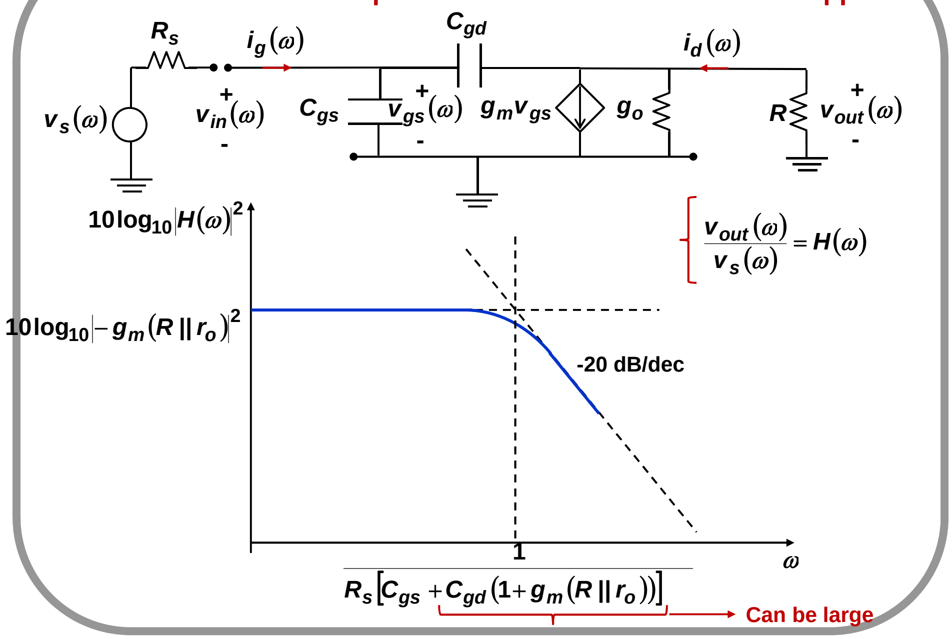
$$\approx \frac{-g_m(R \,||\, r_o)}{1 + j\omega\left[C_{gs} + \underbrace{C_{gd}\left(1 + g_m(R \,||\, r_o)\right)}\right] R_s}$$

Looking in from the input terminal the capacitance C_{gd} seems to be magnified by a factor proportional to the low frequency open circuit voltage gain of the amplifier!!

And now the single pole is at:

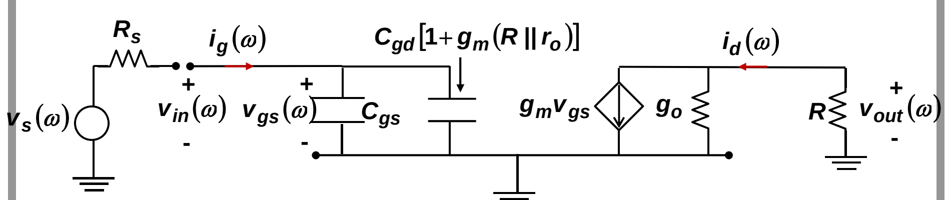
$$\frac{1}{\tau} \approx \frac{1}{\left[C_{gs} + C_{gd}\left(1 + g_m(R \,||\, r_o)\right)\right] R_s}$$

The Common Source Amplifier: Total Gain Under the Miller Approx



The Common Source Amplifier: Input Impedance

This approximate equivalent circuit will also work at not too high frequencies………

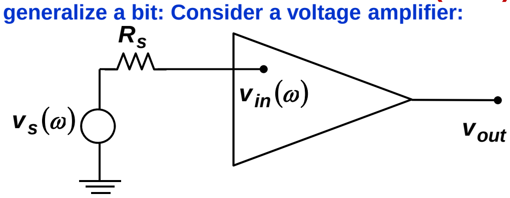


Looking in from the input terminal the capacitance C_{gd} seems to be magnified by a factor proportional to the low frequency open circuit voltage gain of the amplifier!!

$$Z_{in}(\omega) \approx \frac{1}{j\omega[C_{gs}+C_{gd}(1+g_m(R\,||\,r_o))]}$$

The Miller Effect and the Miller Capacitance
John A. Miller (1920)

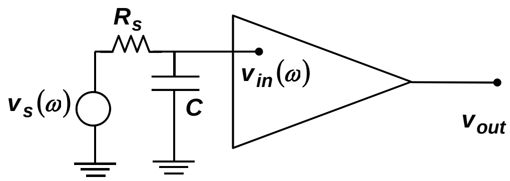
Lets generalize a bit: Consider a voltage amplifier:



$$v_{out}(\omega) = A v_{in}(\omega) = A v_s(\omega)$$

{ $|A| >> 1$

Consider now a capacitor sitting at the input of an amplifier:

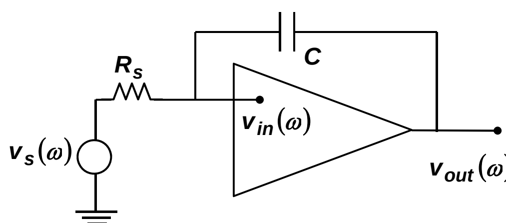


$$v_{out}(\omega) = A v_{in}(\omega) = \frac{A}{1 + j\omega R_S C} v_s(\omega)$$

Input voltage to the amplifier decreases and so does the output voltage of the amplifier at high frequencies (but not too bad…..)

The Miller Effect and the Miller Capacitance

Consider now a capacitor straddling the input and the output of an amplifier:



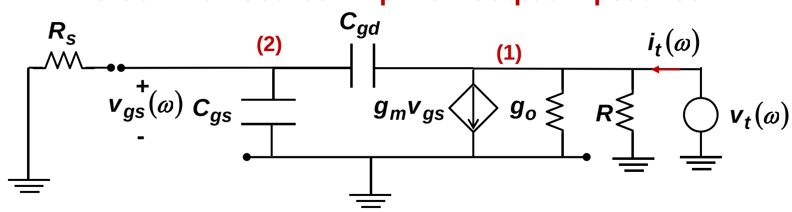
{ $|A| >> 1$

$$v_{out}(\omega) = A v_{in}(\omega) = \frac{A}{1 + j\omega R_S C(1 - A)} v_s(\omega)$$

Input voltage to the amplifier decreases and so does the output voltage of the amplifier at high frequencies
But the effective capacitance seen from the input side now is bigger (compared to the case on the previous slide) by a factor proportional to the gain of the amplifier

The amplifier input voltage, and the output voltage, will now begin to drop-off at a much lower frequency!!
This is the Miller effect and the capacitance positioned this way is called the Miller capacitance

The Common Source Amplifier: Output Impedance



Need to find:

$$Z_{out}(\omega)$$

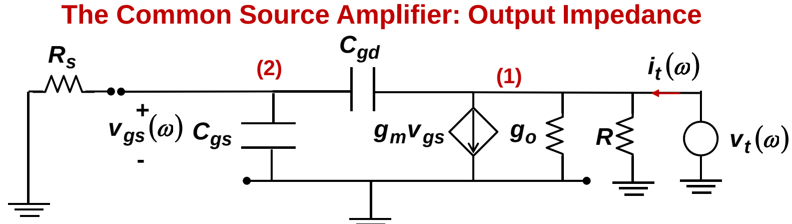
KCL at (2) gives:

$$[v_t(\omega) - v_{gs}(\omega)]j\omega C_{gd} = j\omega C_{gs} v_{gs} + \frac{v_{gs}}{R_s}$$

This gives:

$$v_{gs}(\omega) = \frac{j\omega C_{gd} R_s}{j\omega(C_{gs} + C_{gd})R_s + 1} v_t(\omega)$$

The Common Source Amplifier: Output Impedance



$$v_{gs}(\omega) = \frac{j\omega C_{gd} R_s}{j\omega(C_{gs} + C_{gd})R_s + 1} v_t(\omega)$$

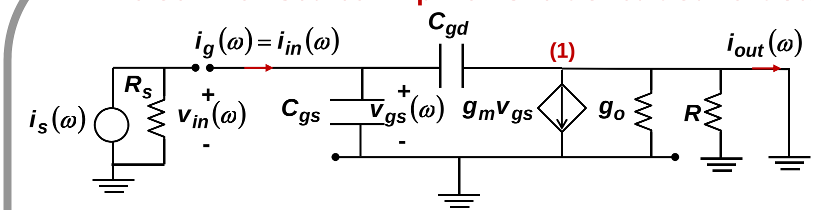
KCL at (1) gives:

$$i_t(\omega) = \frac{v_t(\omega)}{(R \,||\, r_o)} + g_m v_{gs}(\omega) + [v_t(\omega) - v_{gs}(\omega)]j\omega C_{gd}$$

$$\Rightarrow \frac{1}{Z_{out}(\omega)} = \frac{1}{(R \,||\, r_o)} + \frac{(g_m + j\omega C_{gs}) j\omega C_{gd} R_s}{j\omega(C_{gs} + C_{gd})R_s + 1}$$

↑ Not a bad approximation even at moderately high frequencies

The Common Source Amplifier: Short Circuit Current Gain



Need to find:

$$\frac{i_{out}(\omega)}{i_{in}(\omega)}$$

$$Z_{in}(\omega) \approx \frac{1}{j\omega[C_{gs}+C_{gd}]}$$

Start from KCL at (1):

$$(0 - v_{in}(\omega))j\omega C_{gd} + g_m v_{in}(\omega) + i_{out}(\omega) = 0$$

$$\Rightarrow \frac{i_{out}(\omega)}{v_{in}(\omega)} = -(g_m - j\omega C_{gd})$$

$$\Rightarrow \frac{i_{out}(\omega)}{i_{in}(\omega)} = \frac{i_{out}(\omega)}{v_{in}(\omega)}\frac{v_{in}(\omega)}{i_{in}(\omega)} = -(g_m - j\omega C_{gd})Z_{in}(\omega) \approx -g_m Z_{in}(\omega)$$

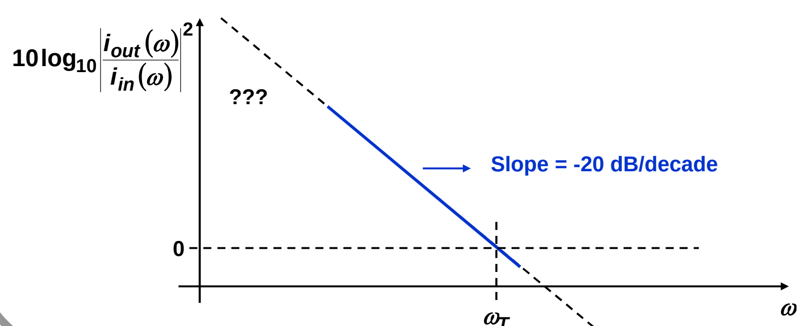
$$= -\frac{g_m}{j\omega(C_{gs}+C_{gd})}$$

Short Circuit Current Gain and the Transition Frequency (f_T or ω_T)

For most transistors, the short circuit current gain falls of with frequency with a -20 dB/dec slope (at high enough frequencies)

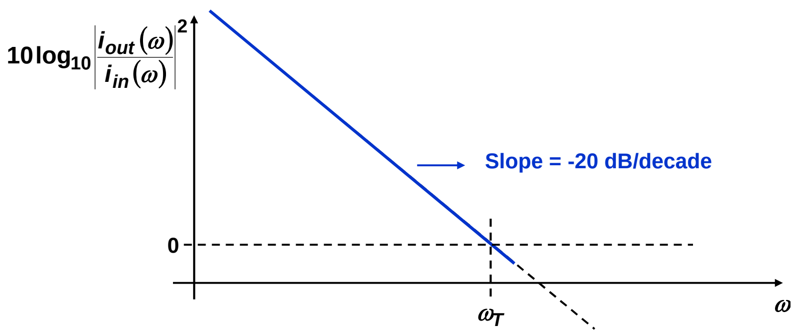
The frequency at which the short circuit current gain equals unity is called the transition frequency (f_T or ω_T)

The transition frequency expresses the maximum useful frequency of operation of the transistor



The Common Source Amplifier: The Transition Frequency

$$\frac{i_{out}(\omega)}{i_{in}(\omega)} = -\frac{g_m}{j\omega\left(C_{gs} + C_{gd}\right)}$$



The transition frequency is:

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \approx \frac{g_m}{C_{gs}}$$

The Common Source Amplifier: The Transition Frequency

The FET transition frequency is:

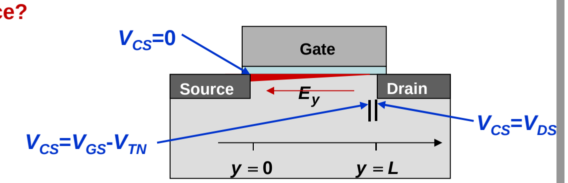
$$\omega_T \approx \frac{g_m}{C_{gs}}$$

This is the highest frequency at which the transistor is still useful

Q: What is its physical significance?

$$g_m = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_{TN}\right)$$

$$C_{gs} = \frac{2}{3}WLC_{ox}$$



Therefore the transition frequency is:

$$\omega_T = \frac{g_m}{C_{gs}} = \frac{3}{2}\mu_n \frac{\left(V_{GS} - V_{TN}\right)}{L^2}$$

$$\omega_T \propto \mu_n \frac{\left(V_{GS} - V_{TN}\right)}{L^2} = \frac{\mu_n \frac{\left(V_{GS} - V_{TN}\right)}{L}}{L} = \frac{\mu_n \left|E_y\right|}{L} = \frac{v_n}{L} = \frac{1}{\tau_t}$$

Electron transit time through the FET

Therefore, the electron transit time sets the maximum frequency of operation of the FET!!