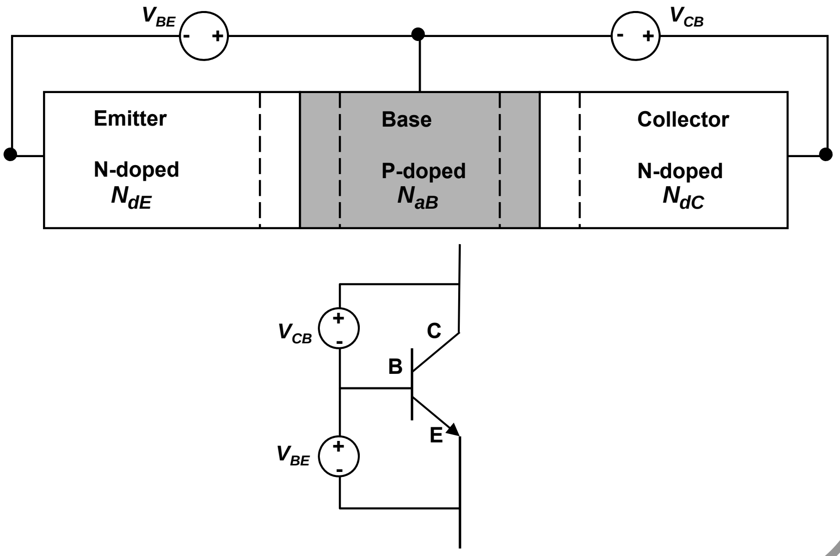
Lecture 18

PNP Bipolar Junction Transistors (BJTs)

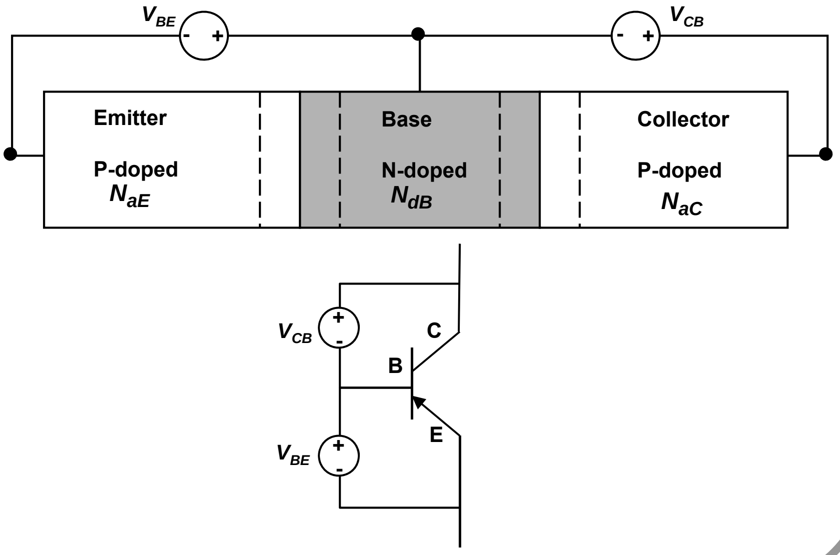
In this lecture you will learn:

- The operation of bipolar junction transistors
- Forward and reverse active operations, saturation, cutoff
- Ebers-Moll model

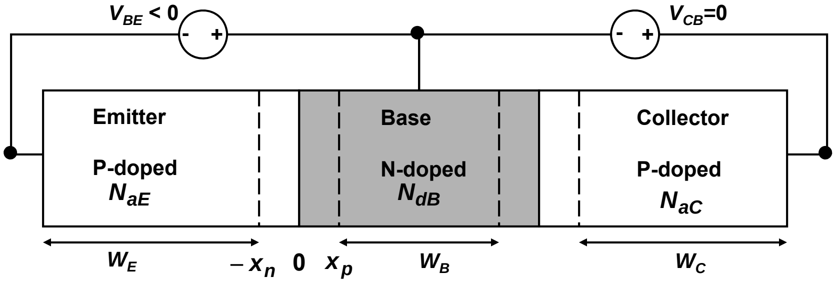
NPN Bipolar Junction Transistor



PNP Bipolar Junction Transistor



PNP BJT: Basic Operation



Suppose:

The base-emitter junction is forward biased

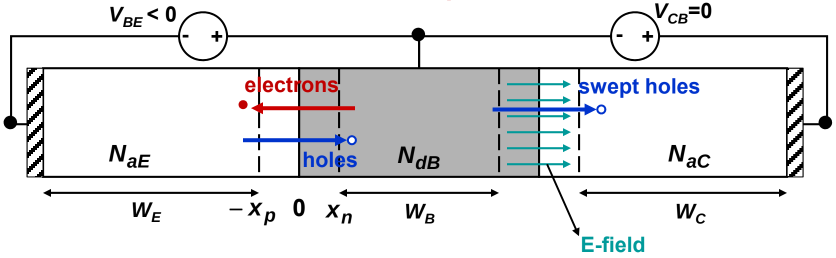
$$V_{BE} < 0$$

The base-collector junction is zero biased

$$V_{CB} = 0$$

This biasing scheme will put the device in the “forward active” operation (to be discussed fully later)

PNP BJT: Basic Operation



Consider the action in the base first ($V_{BE} < 0$ and $V_{CB} = 0$)

- The holes diffuse from the emitter, cross the depletion region, and enter the base

- In the base, the holes are the minority carriers

- In the base, the holes diffuse towards the collector

- As soon as the holes reach the base-collector depletion region they are immediately swept away into the collector by the strong electric fields in the depletion region

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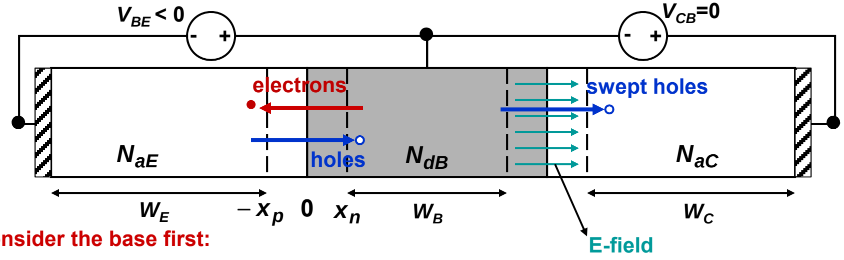
PNP BJT: Electron-Hole Populations



Consider the base first:

In the base, the hole population can be written as:

$$p(x) = p_{no} + p'(x) \qquad \Big\} \quad p_{no} = \frac{n_i^2}{N_{dB}}$$

Equilibrium hole density

Excess hole density

In the base, the excess electron population satisfies the differential equation:

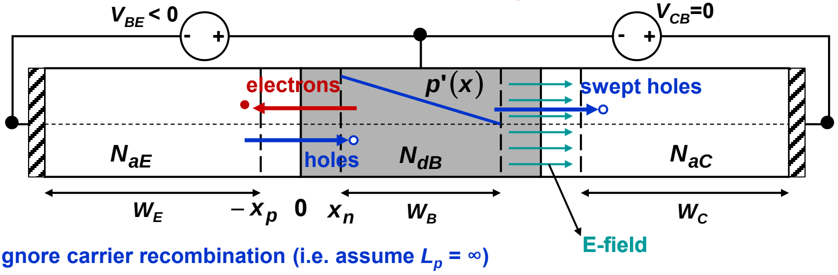
$$\frac{\partial^2 p'(x)}{\partial x^2} - \frac{p'(x)}{L_p^2} = 0 \quad \Big\}$$

Boundary conditions

$$p'(x_n) = \frac{n_i^2}{N_{dB}} \left(e^{\frac{-qV_{BE}}{KT}} - 1 \right)$$

$$p'(x_n + W_B) = \frac{n_i^2}{N_{dB}} \left(e^{\frac{qV_{BC}}{KT}} - 1 \right) = 0$$

PNP BJT: Electron-Hole Populations



- Ignore carrier recombination (i.e. assume $L_p = \infty$)

$$\frac{\partial^2 p'(x)}{\partial x^2} = 0 \quad \Bigg\}$$

Boundary conditions

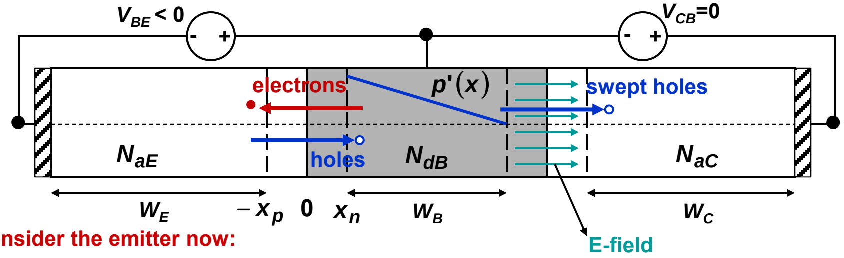
$$p'(x_n) = \frac{n_i^2}{N_{dB}}\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$

$$p'(x_n + W_B) = \frac{n_i^2}{N_{dB}}\left(e^{\frac{-qV_{BC}}{KT}} - 1\right) = 0$$

Solution is:

$$p'(x) = p'(x_p)\left(1 - \frac{x - x_n}{W_B}\right) = \frac{n_i^2}{N_{dB}}\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)\left(1 - \frac{x - x_n}{W_B}\right)$$

PNP BJT: Electron-Hole Populations



Consider the emitter now:

In the emitter, the electron population can be written as:

$$n(x) = n_{po} + n'(x)$$

Equilibrium electron density   Excess electron density

$$n_{po} = \frac{n_i^2}{N_{aE}}$$

In the emitter, the excess electron population satisfies the differential equation:

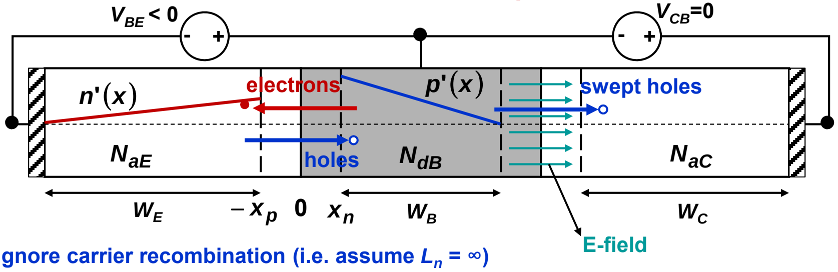
$$\frac{\partial^2 n'(x)}{\partial x^2} - \frac{n'(x)}{L_n^2} = 0 \quad \Bigg\}$$

Boundary conditions

$$n'(-x_p) = \frac{n_i^2}{N_{aE}}\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$

$$n'(-x_p - W_E) = 0$$

PNP BJT: Electron-Hole Populations



- Ignore carrier recombination (i.e. assume $L_n = \infty$)

$$\frac{\partial^2 n'(x)}{\partial x^2} = 0$$

Boundary conditions

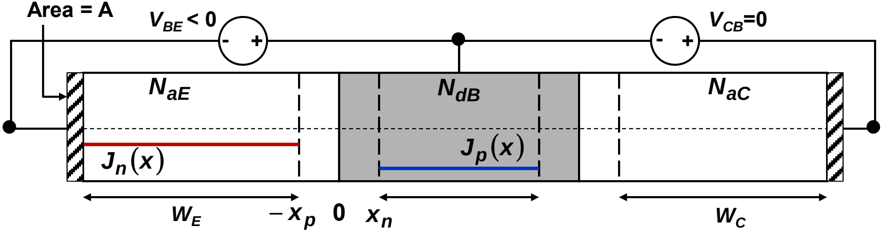
$$n'(-x_p) = \frac{n_i^2}{N_{aE}}\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$

$$n'(-x_p - W_E) = 0$$

Solution is:

$$n'(x) = n'(-x_p)\left(1+\frac{x+x_p}{W_E}\right) = \frac{n_i^2}{N_{aE}}\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)\left(1+\frac{x+x_p}{W_E}\right)$$

PNP BJT: Electron and Hole Current Densities



In the base:

- The hole current is:

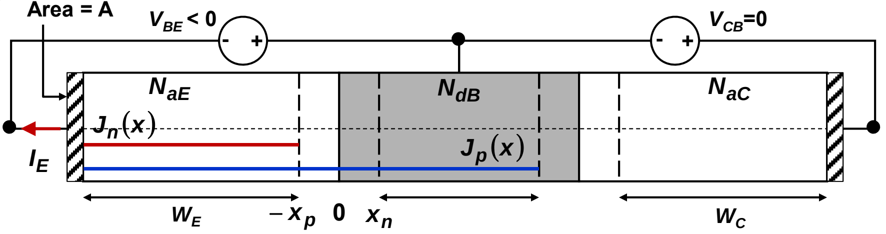
$$J_p(x) \approx -q\, D_p \frac{\partial p(x)}{\partial x} = -qn_i^2 \frac{D_p}{N_{dB}W_B}\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$

In the emitter:

- The electron current is:

$$J_n(x) \approx q\, D_n \frac{\partial n(x)}{\partial x} = -qn_i^2 \frac{D_n}{N_{aE}W_E}\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$

PNP BJT: Terminal Currents

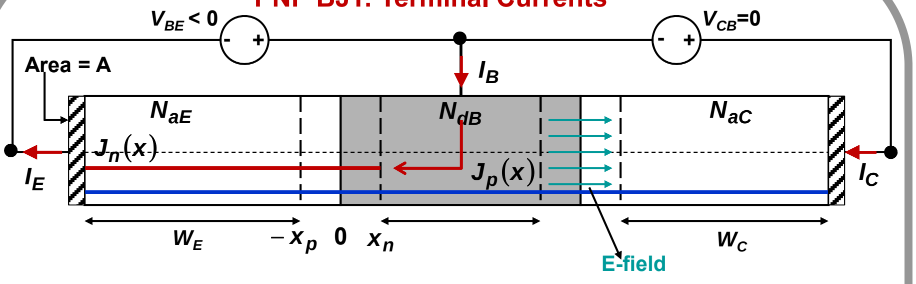


Emitter current:

- The current flowing out of the emitter is the sum of the total electron and total hole currents in the emitter:

$$I_E = -qn_i^2 A\left(\frac{D_n}{N_{aE}W_E} + \frac{D_p}{N_{dB}W_B}\right)\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$

PNP BJT: Terminal Currents



Collector Current:

- The current going into the collector is due to the holes that got swept from the Base through the Base-Collector depletion region by the electric-fields:

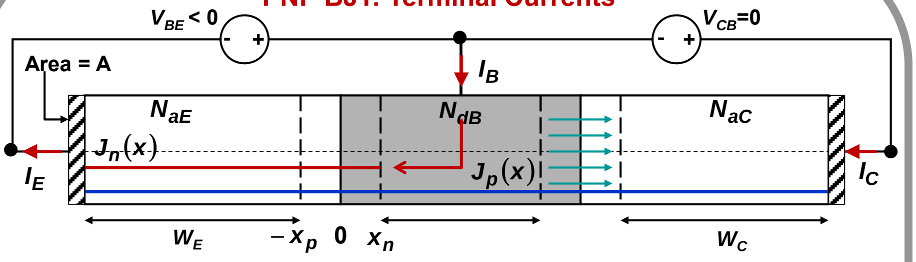
$$I_C = -qn_i^2 A\left(\frac{D_p}{N_{dB}W_B}\right)\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$

Base Current:

- The current going into the Base is due to the electrons that got injected from the base into the emitter:

$$I_B = -qn_i^2 A\left(\frac{D_n}{N_{aE}W_E}\right)\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$

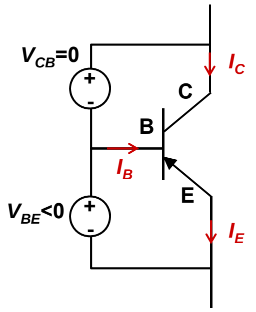
PNP BJT: Terminal Currents



$$I_E = -qn_i^2 A\left(\frac{D_n}{N_{aE}W_E} + \frac{D_p}{N_{dB}W_B}\right)\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$

$$I_C = -qn_i^2 A\left(\frac{D_p}{N_{dB}W_B}\right)\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$

$$I_B = -qn_i^2 A\left(\frac{D_n}{N_{aE}W_E}\right)\left(e^{\frac{-qV_{BE}}{KT}} - 1\right)$$



$$I_E = I_B + I_C$$

PNP BJT: Circuit Level Parameters

Current gain β_F :

Current gain of the BJT in the forward active operation is defined as the ratio of the collector and base currents:

$$\beta_F = \frac{I_C}{I_B} = \frac{D_p}{N_{dB}W_B}\frac{N_{aE}W_E}{D_n} \Rightarrow I_C = \beta_F I_B$$

Typical values of β_F are between 20-200 and:

$$N_{aE} >> N_{dB} > N_{aC}$$

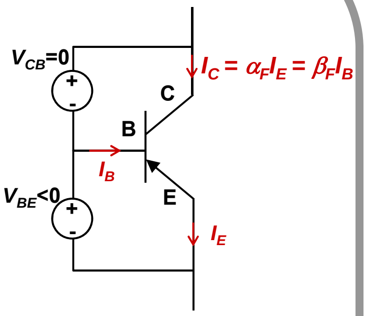
α_F :

In the forward active operation α_F is defined as the ratio of the collector and emitter currents:

$$\alpha_F = \frac{I_C}{I_E} = \frac{\frac{D_p}{N_{dB}W_B}}{\frac{D_n}{N_{aE}W_E} + \frac{D_p}{N_{dB}W_B}} \Rightarrow I_C = \alpha_F I_E$$

Transistor relation:

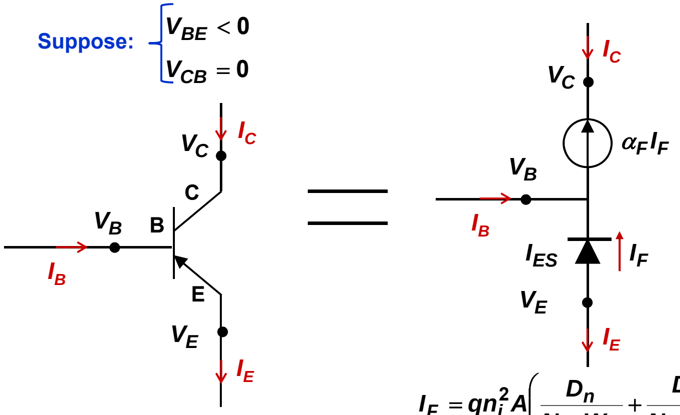
α_F and β_F are related: $\beta_F = \frac{\alpha_F}{1-\alpha_F}$



$$I_E = I_B + I_C$$

PNP BJT: Ebers-Moll Model for Forward Active Operation

Suppose: $\begin{cases} V_{BE} < 0 \\ V_{CB} = 0 \end{cases}$



$$I_F = qn_i^2 A\left(\frac{D_n}{N_{aE}W_E}+\frac{D_p}{N_{dB}W_B}\right)\left(e^{\frac{-qV_{BE}}{KT}}-1\right)$$

$$= I_{ES}\left(e^{\frac{-qV_{BE}}{KT}}-1\right)$$

The circuit level simplified model with an ideal diode and a current-controlled current source models the PNP transistor in the forward active operation

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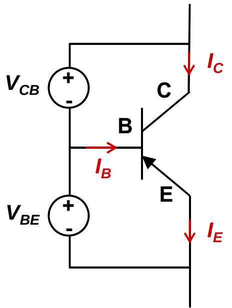
PNP BJT: Forward and Reverse Active Operations

$V_{BE} < 0$

$V_{CB} \leq 0$



Forward active operation

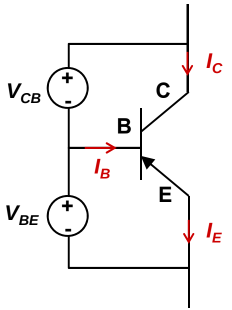
$$\beta_F = \frac{I_C}{I_B}$$

$$\alpha_F = \frac{I_C}{I_E}$$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F}$$

$V_{CB} > 0$

$V_{BE} \geq 0$



Reverse active operation

$$\beta_R = -\frac{I_E}{I_B} = \frac{D_p}{N_{dB}W_B}\frac{N_{aC}W_C}{D_n}$$

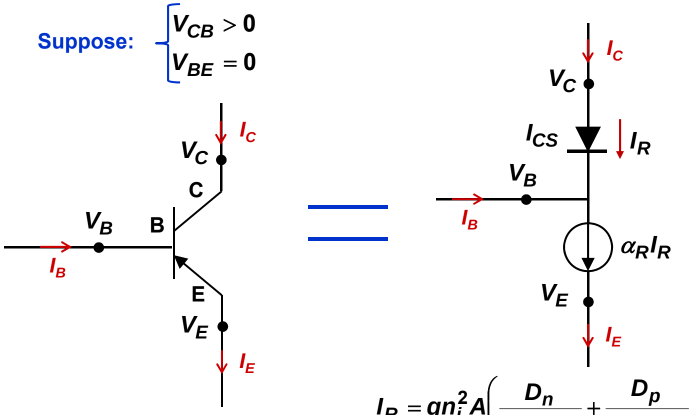
$$\alpha_R = \frac{I_E}{I_C}$$

$$\beta_R = \frac{\alpha_R}{1-\alpha_R}$$

In a well designed transistor: $\beta_F >> \beta_R$

PNP BJT: Ebers-Moll Model for Reverse Active Operation

Suppose: $\begin{cases} V_{CB} > 0 \\ V_{BE} = 0 \end{cases}$



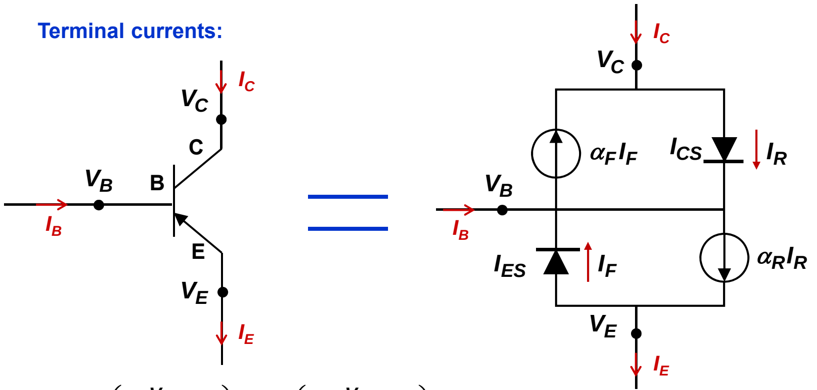
$$I_R = qn_i^2 A\left(\frac{D_n}{N_{aC}W_C}+\frac{D_p}{N_{dB}W_B}\right)\left(e^{\frac{qV_{CB}}{KT}}-1\right)$$

$$= I_{CS}\left(e^{\frac{qV_{CB}}{KT}}-1\right)$$

The circuit level simplified model with an ideal diode and a current-controlled current source models the PNP transistor in the reverse active operation

PNP BJT: Ebers-Moll Model and Terminal Currents

Terminal currents:



$$I_R = I_{CS}\left(e^{\frac{qV_{CB}}{KT}}-1\right) = I_{CS}\left(e^{\frac{-qV_{BC}}{KT}}-1\right)$$

$$I_F = I_{ES}\left(e^{\frac{-qV_{BE}}{KT}}-1\right) = I_{ES}\left(e^{\frac{qV_{EB}}{KT}}-1\right)$$

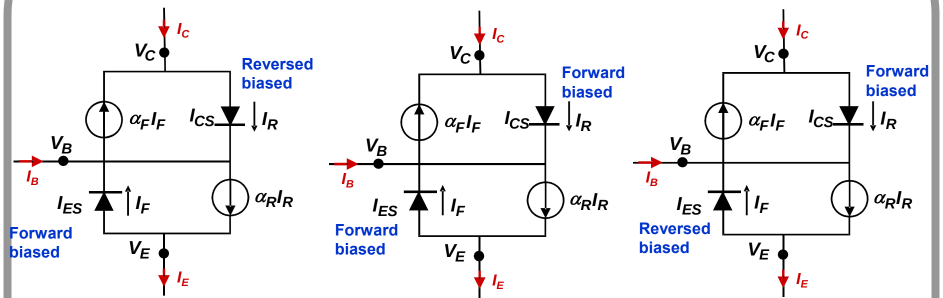
And

$$I_B = (1-\alpha_F)I_F + (1-\alpha_R)I_R$$

$$I_C = \alpha_F I_F - I_R$$

$$I_E = I_F - \alpha_R I_R$$

PNP BJT: Different Regimes of Operation



$V_{BE} > 0$
$V_{CB} \geq 0$

Forward Active

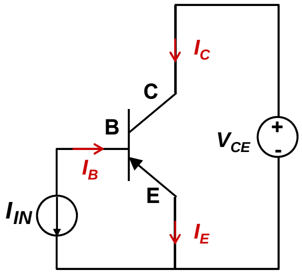
$V_{CB} < 0$
$V_{BE} > 0$

Saturation

$V_{CB} < 0$
$V_{BE} \leq 0$

Reverse Active

PNP BJT: Regimes of Operation - I



In forward active operation:

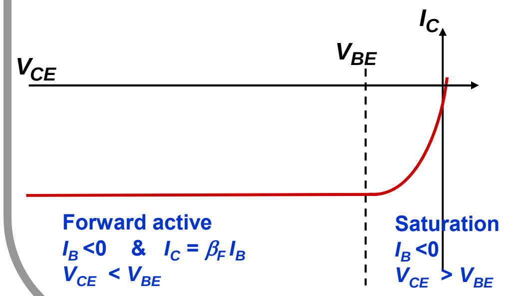
$I_B < 0 \qquad V_{BE} < 0 \qquad V_{CB} \leq 0$

Since: $V_{CE} = V_{CB} + V_{BE}$

$\Rightarrow$ In forward active operation: $V_{CE} \leq V_{BE}$

$$I_C = -qn_i^2 A\left(\frac{D_p}{N_{dB}W_B}\right)\left(e^{\frac{-qV_{BE}}{KT}} - 1\right) = \beta_F I_B$$

$\Rightarrow$ Independent of V_{CE}



Forward active:
Base-emitter junction forward biased
Base-collector junction reversed biased

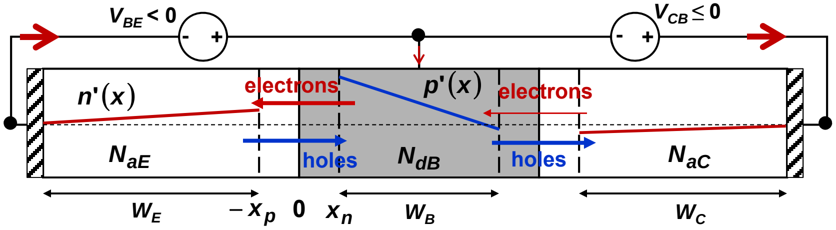
$I_B < 0 \quad V_{BE} < 0 \quad V_{CB} \leq 0$

Saturation:
Base-emitter junction forward biased
Base-collector junction forward biased

$I_B < 0 \quad V_{BE} < 0 \quad V_{CB} > 0$

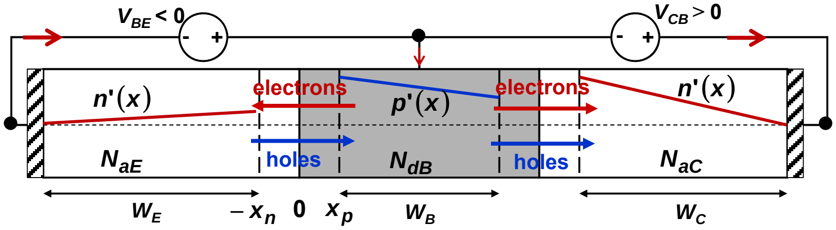
Carrier Densities in Different Regimes of Operation

Forward active:



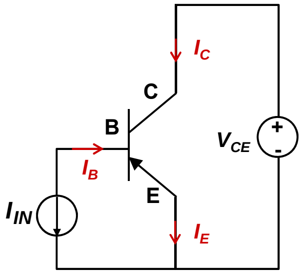
$N_{aE} >> N_{dB} > N_{aC}$

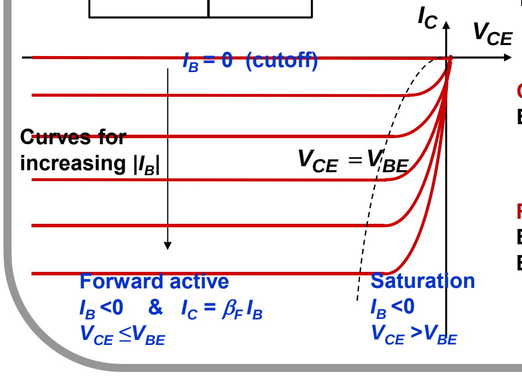
Saturation:



The forward biased base-collector junction reduces the magnitude of the collector current!

PNP BJT: Regimes of Operation - II





Forward active:
Base-emitter junction forward biased
Base-collector junction reversed biased

$I_B < 0 \quad V_{BE} < 0 \quad V_{CB} \le 0$

Saturation:
Base-emitter junction forward biased
Base-collector junction forward biased

$I_B < 0 \quad V_{BE} < 0 \quad V_{CB} > 0$

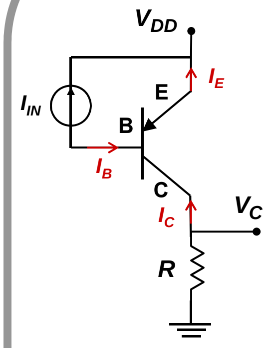
Cutoff:
Base current zero

$I_B = 0$

Reverse active:
Base-emitter junction reverse biased
Base-collector junction forward biased

$I_B < 0 \quad V_{BE} \ge 0 \quad V_{CB} > 0$

PNP BJT: A Simple Amplifier Circuit



Lesson: Don’t let the base-collector junction become forward biased

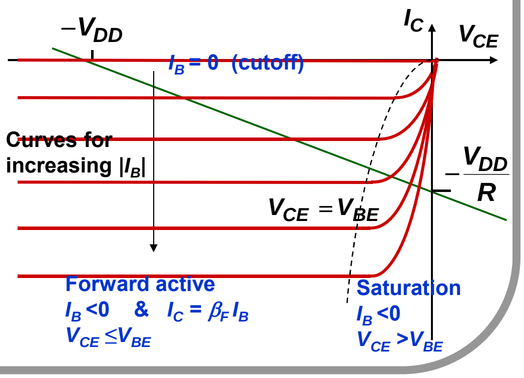
Current gain (in forward active regime):

$$\frac{I_C}{I_B} = \beta_F$$

Load line equation:

$$V_{CE} = -I_C R - V_{DD}$$

$$\Rightarrow \quad I_C = -\frac{V_{DD} + V_{CE}}{R}$$



A Silicon PNP BJT