Lecture 16

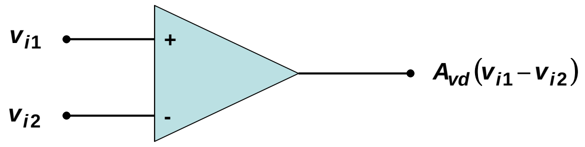
Differential Amplifiers – I
Basics

In this lecture you will learn:

- **Differential Amplifiers**
- **Differential FET Amplifiers**
- **Large Signal and Small Signal Analysis**
- **Half Circuit Techniques**

Ideal Differential Amplifiers

An ideal differential amplifier amplifies the difference signal between two inputs:



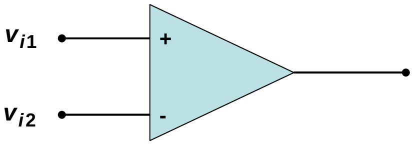
The need for differential amplifiers:

Differential amplifiers are used to remove unwanted signals that are common to both input signals.
For example, in many cases useful information is carried by the difference between two signal sources, 1 and 2, and unwanted noise signals that add to both the 1 and 2 signals will be rejected by a differential amplifier which will only amplify the difference of these signals.

These unwanted signals that add to both signals 1 and 2 could be a result of:

a) **Variation in the power supply voltage as a function of time**
b) **Variation in the substrate potential of the entire chip**
c) **Variation in the temperature of the chip**
d) **Electromagnetic interference signals from the environment**

Difference Mode and Common Mode Signals



The **difference-mode** and the **common-mode** components of two signals are:

$$v_{id} = v_{i1} - v_{i2} \longrightarrow \text{Difference-mode component}$$

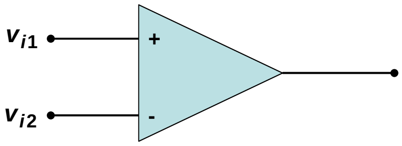
$$v_{ic} = \frac{v_{i1} + v_{i2}}{2} \longrightarrow \text{Common-mode component}$$

Any two signals can be written in terms of their **difference-mode** and **common-mode** components:

$$\Rightarrow v_{i1} = v_{ic} + \frac{v_{id}}{2}$$

$$\Rightarrow v_{i2} = v_{ic} - \frac{v_{id}}{2}$$

Differential Amplifier: Single-Ended Output



$$v_{out} = A_{vd}(v_{i1} - v_{i2}) + A_{vc}\left(\frac{v_{i1} + v_{i2}}{2}\right)$$

$$= A_{vd}v_{id} + A_{vc}v_{ic}$$

$$= \left(A_{vd} + \frac{A_{vc}}{2}\right)v_{i1} + \left(-A_{vd} + \frac{A_{vc}}{2}\right)v_{i2}$$

Difference-Mode Gain: A_{vd}

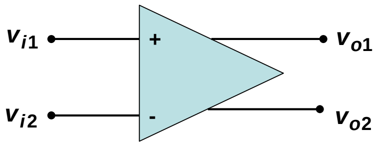
Common-Mode Gain: A_{vc}

One always wants the difference-mode gain to be much much larger than the common-mode gain (ideally one would want the common mode gain to be zero!)

Common-Mode Rejection Ratio (CMRR):

$$CMRR = \frac{A_{vd}}{A_{vc}}$$

Differential Amplifier: Double-Ended Output



$$v_{o1} = \frac{A_{vd}}{2}(v_{i1}-v_{i2}) + A_{vc}\left(\frac{v_{i1}+v_{i2}}{2}\right)$$

$$= A_{vd}\frac{v_{id}}{2} + A_{vc}v_{ic}$$

$$v_{o2} = -\frac{A_{vd}}{2}(v_{i1}-v_{i2}) + A_{vc}\left(\frac{v_{i1}+v_{i2}}{2}\right)$$

$$= -A_{vd}\frac{v_{id}}{2} + A_{vc}v_{ic}$$

Difference-Mode Output:

$$v_{od} = v_{o1} - v_{o2} = A_{vd}v_{id}$$

Common-Mode Output:

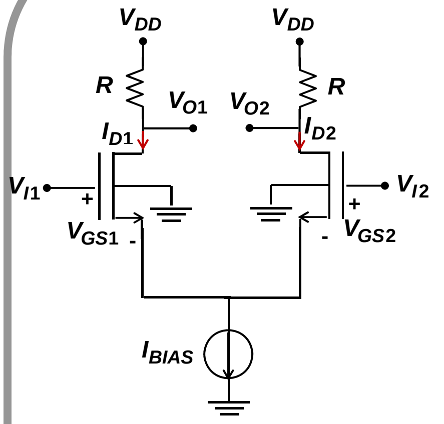
$$v_{oc} = \frac{v_{o1}+v_{o2}}{2} = A_{vc}v_{ic}$$

One always wants the difference-mode gain to be much much larger than the common-mode gain

Common-Mode Rejection Ratio (CMRR):

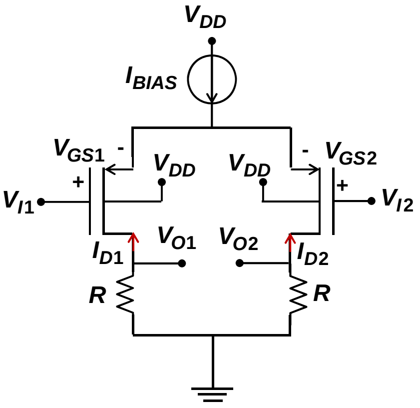
$$CMRR = \frac{A_{vd}}{A_{vc}}$$

FET Differential Amplifiers



Comments:

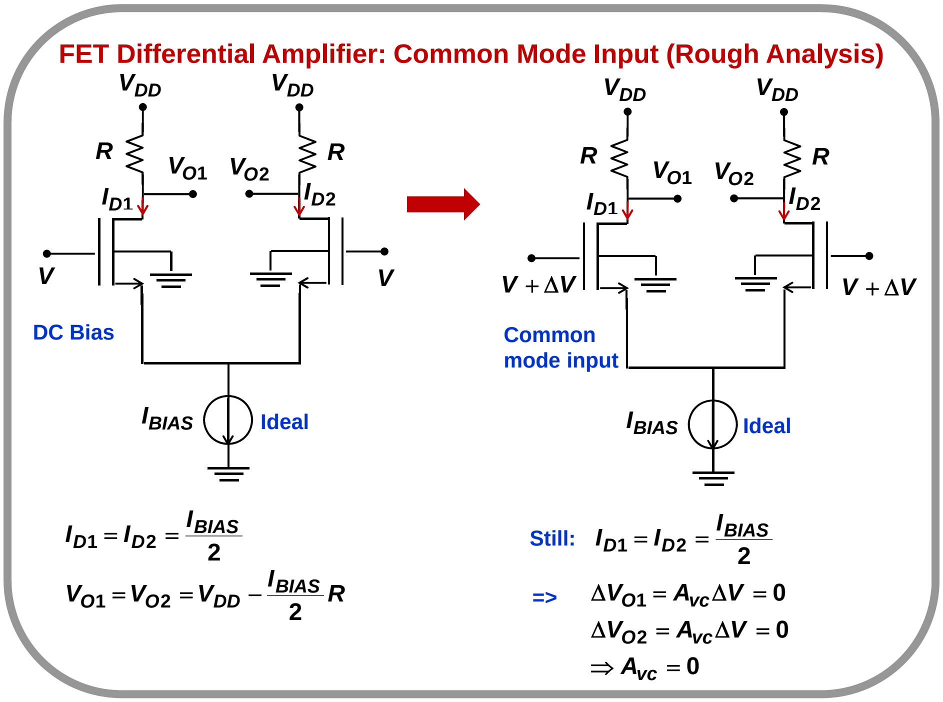
$$I_{D1} + I_{D2} = I_{BIAS}$$



Comments:

$$I_{D1} + I_{D2} = -I_{BIAS}$$

FET Differential Amplifier: Common Mode Input (Rough Analysis)



$$I_{D1} = I_{D2} = \frac{I_{BIAS}}{2}$$

$$V_{O1} = V_{O2} = V_{DD} - \frac{I_{BIAS}}{2}R$$

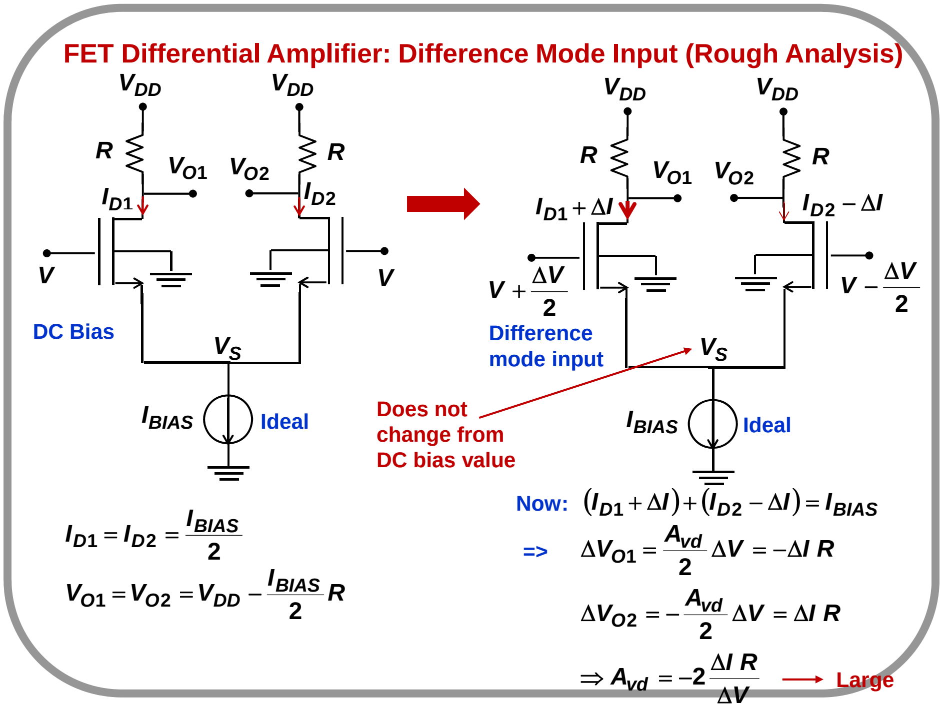
Still: $I_{D1} = I_{D2} = \frac{I_{BIAS}}{2}$

=> $\Delta V_{O1} = A_{vc}\Delta V = 0$

$\Delta V_{O2} = A_{vc}\Delta V = 0$

$\Rightarrow A_{vc} = 0$

FET Differential Amplifier: Difference Mode Input (Rough Analysis)



Does not change from DC bias value

$$I_{D1} = I_{D2} = \frac{I_{BIAS}}{2}$$

$$V_{O1} = V_{O2} = V_{DD} - \frac{I_{BIAS}}{2}R$$

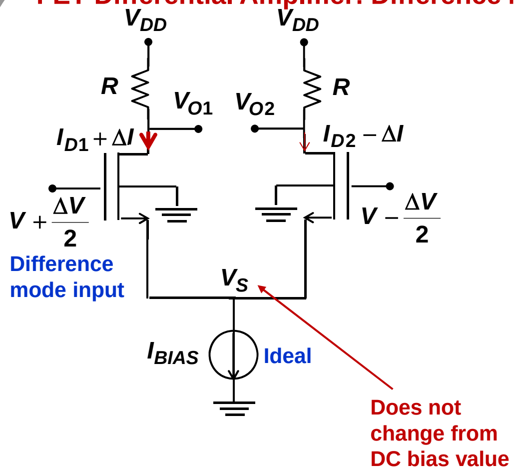
Now: $(I_{D1}+\Delta I)+(I_{D2}-\Delta I) = I_{BIAS}$

=> $\Delta V_{O1} = \frac{A_{vd}}{2}\Delta V = -\Delta I\,R$

$\Delta V_{O2} = -\frac{A_{vd}}{2}\Delta V = \Delta I\,R$

$\Rightarrow A_{vd} = -2\frac{\Delta I\,R}{\Delta V}$ ⟶ Large

FET Differential Amplifier: Difference Mode Input (Rough Analysis)



$$\Delta V_{GS1}=\frac{\Delta V}{2}$$

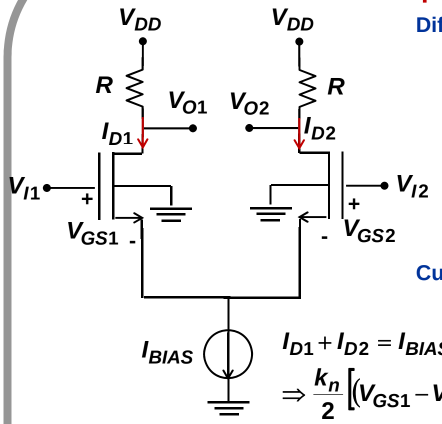
$$\Delta V_{GS2}=-\frac{\Delta V}{2}$$

$$\Rightarrow \Delta I = g_m \Delta V_{GS1} = g_m \frac{\Delta V}{2}$$

$$\Rightarrow A_{vd} = -2\frac{\Delta I\, R}{\Delta V} = -g_m R$$

Large

A FET Differential Amplifier: Large Signal Analysis



Difference-Mode and Common-Mode inputs:

$$V_{Id} = V_{I1}-V_{I2}$$

$$V_{Ic} = \frac{V_{I1}+V_{I2}}{2}$$

$$V_{I1}-V_{GS1}+V_{GS2} = V_{I2} \quad \Big\{ \text{KVL}$$

$$\Rightarrow V_{Id} = V_{I1}-V_{I2} = V_{GS1}-V_{GS2}$$

Currents:

$$I_{D1}+I_{D2} = I_{BIAS}$$

$$\Rightarrow \frac{k_n}{2}\left[(V_{GS1}-V_{TN})^2+(V_{GS2}-V_{TN})^2\right] = I_{BIAS}$$

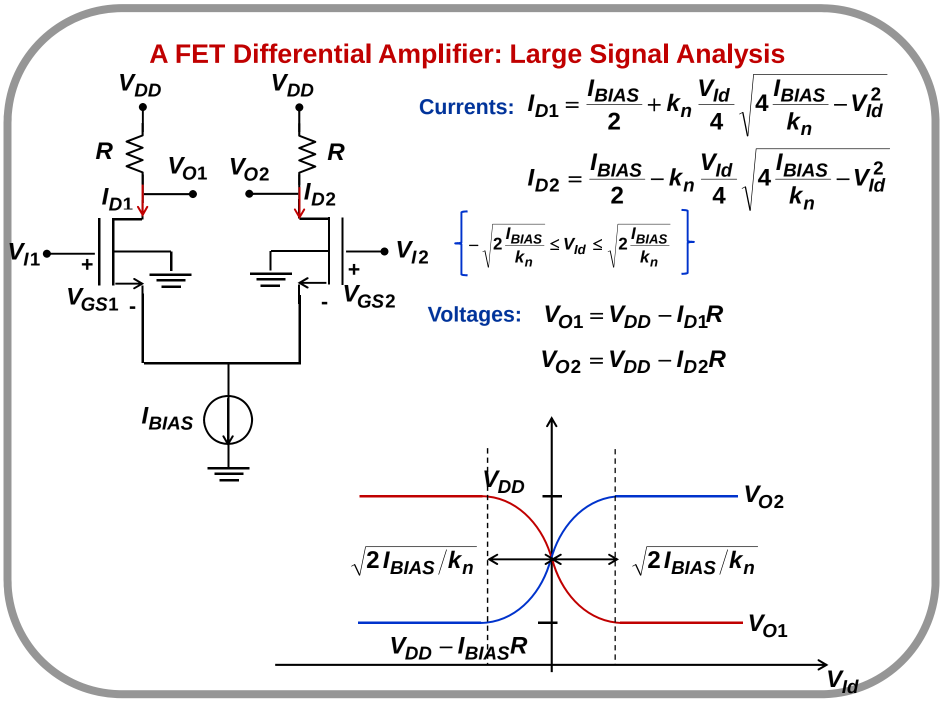
$$\Rightarrow I_{D1} = \frac{k_n}{2}\left[(V_{GS1}-V_{TN})^2\right] = \frac{I_{BIAS}}{2}+k_n\frac{V_{Id}}{4}\sqrt{4\frac{I_{BIAS}}{k_n}-V_{Id}^2}$$

$$\Rightarrow I_{D2} = \frac{k_n}{2}\left[(V_{GS2}-V_{TN})^2\right] = \frac{I_{BIAS}}{2}-k_n\frac{V_{Id}}{4}\sqrt{4\frac{I_{BIAS}}{k_n}-V_{Id}^2}$$

$$-\sqrt{2\frac{I_{BIAS}}{k_n}} \le V_{Id} \le \sqrt{2\frac{I_{BIAS}}{k_n}}\ \Big\}$$

Drain currents depend only on the difference-mode input signal

A FET Differential Amplifier: Large Signal Analysis



Currents:

$$I_{D1}=\frac{I_{BIAS}}{2}+k_n\frac{V_{Id}}{4}\sqrt{4\frac{I_{BIAS}}{k_n}-V_{Id}^2}$$

$$I_{D2}=\frac{I_{BIAS}}{2}-k_n\frac{V_{Id}}{4}\sqrt{4\frac{I_{BIAS}}{k_n}-V_{Id}^2}$$

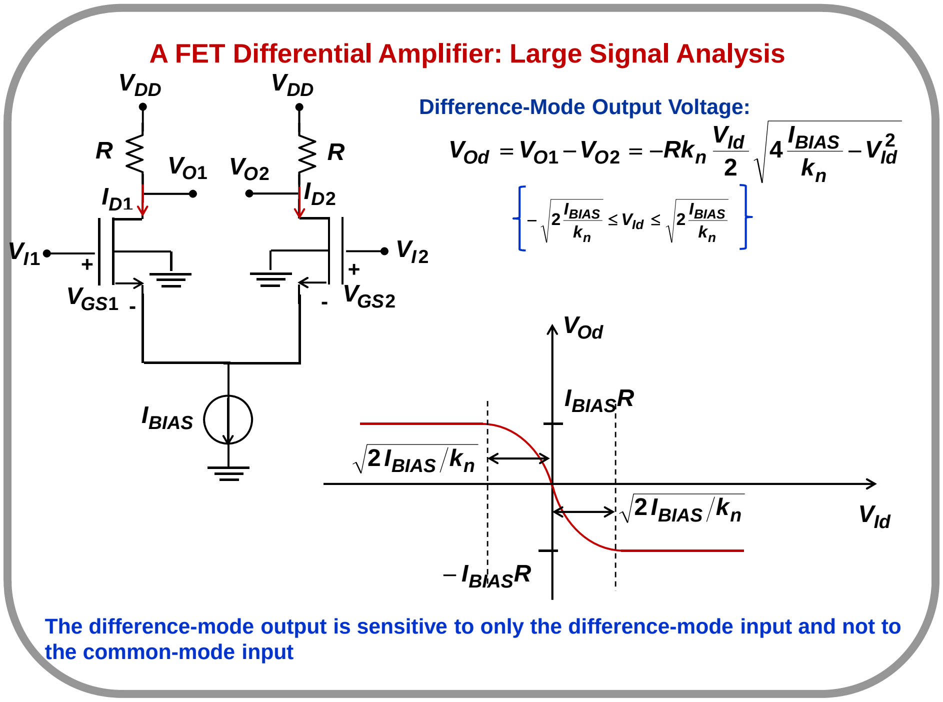
$$\left[-\sqrt{2\frac{I_{BIAS}}{k_n}}\le V_{Id}\le\sqrt{2\frac{I_{BIAS}}{k_n}}\right]$$

Voltages:

$$V_{O1}=V_{DD}-I_{D1}R$$

$$V_{O2}=V_{DD}-I_{D2}R$$

A FET Differential Amplifier: Large Signal Analysis



Difference-Mode Output Voltage:

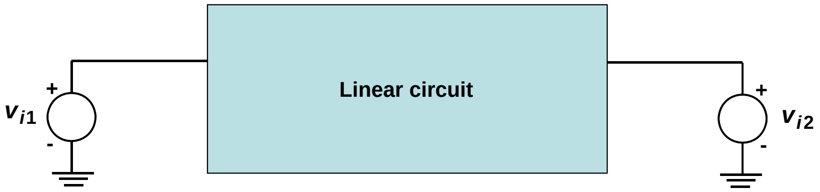
$$V_{Od}=V_{O1}-V_{O2}=-Rk_n\frac{V_{Id}}{2}\sqrt{4\frac{I_{BIAS}}{k_n}-V_{Id}^2}$$

$$\left[-\sqrt{2\frac{I_{BIAS}}{k_n}}\le V_{Id}\le\sqrt{2\frac{I_{BIAS}}{k_n}}\right]$$

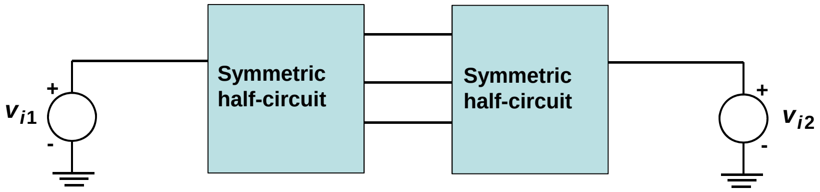
The difference-mode output is sensitive to only the difference-mode input and not to the common-mode input

Half-Circuit Techniques

Consider the following linear circuit



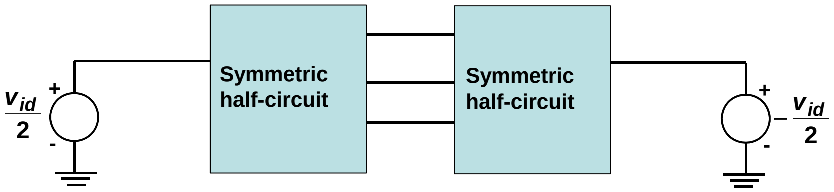
Suppose the circuit consists of identical parts that can be separated into two symmetric half-circuits as shown:

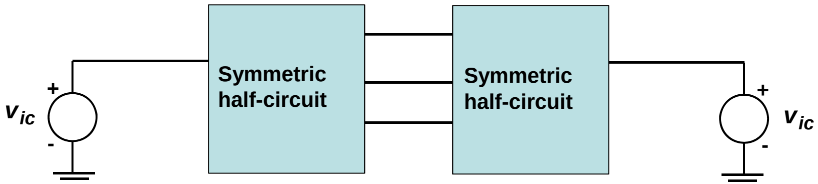


Half-Circuit Techniques

Since the circuit is linear, and superposition will hold, one can decompose the input signals into difference-mode and common-mode signals and then separately consider the circuit response to each signal

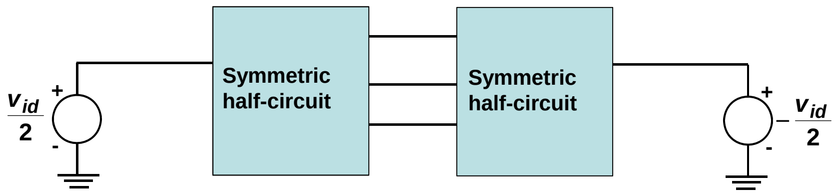
$$v_{id} = v_{i1} - v_{i2} \qquad v_{ic} = \frac{v_{i1} + v_{i2}}{2} \qquad \Longrightarrow \qquad v_{i1} = v_{ic} + \frac{v_{id}}{2} \qquad v_{i2} = v_{ic} - \frac{v_{id}}{2}$$





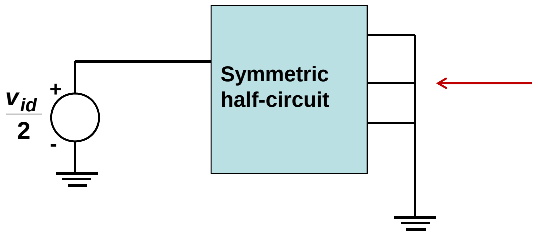
Half-Circuit Techniques: Difference-Mode Input

First consider the difference-mode input:



Because of symmetry, the nodes in the center will be at zero potential

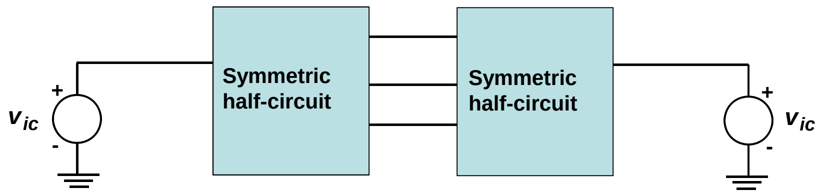
Therefore, one can use the following half-circuit to perform the analysis:



Shorted to ground

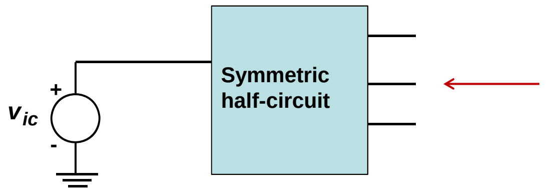
Half-Circuit Techniques: Difference-Mode Input

Now consider the common-mode input:



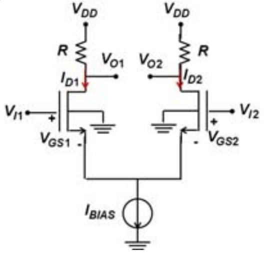
Because of symmetry, the wires in the center will carry no current

Therefore, one can use the following half-circuit to perform the analysis:



Open circuit

A FET Differential Amplifier: Small Signal Analysis



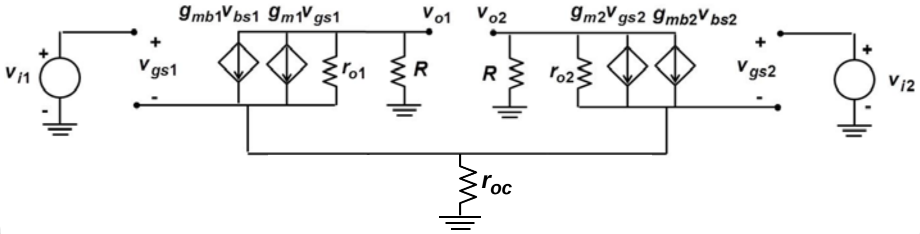
DC Bias:

$$V_{I1} = V_{I2}$$

$$I_{D1} = I_{D2} = \frac{I_{BIAS}}{2}$$

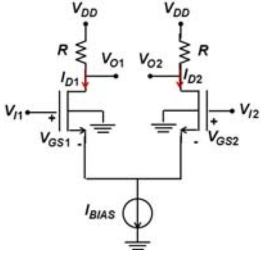
The small signal model can be built using the standard techniques

The small signal circuit models are always linear



Does this circuit consist of two identical and symmetric halves??

A FET Differential Amplifier: Small Signal Analysis



DC Bias:

$$V_{I1} = V_{I2}$$

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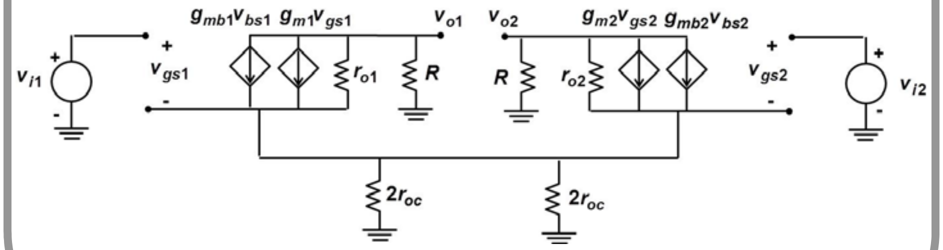
The small signal model can be built using the standard techniques

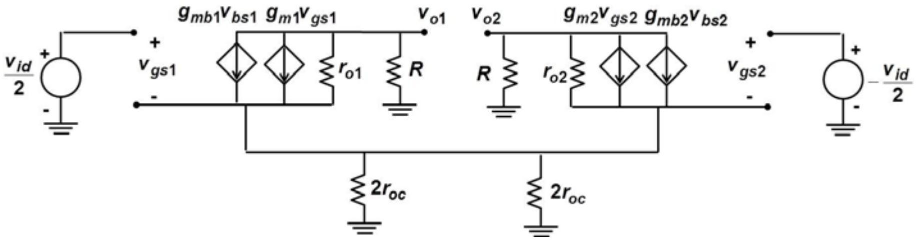
The small signal circuit models are always linear



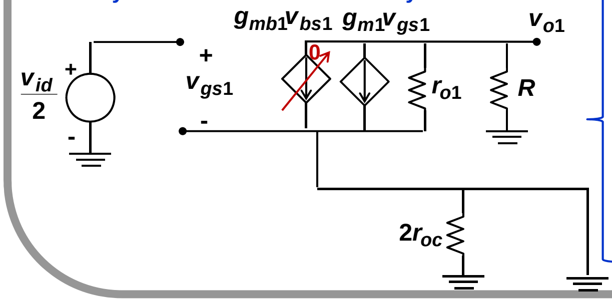
Now it does have two identical and symmetric halves!

A FET Differential Amplifier: Small Signal Analysis for Difference-Mode Input

Assume a difference-mode input



Use the symmetric half-circuit for analysis



$$v_{o1} = -g_{m1}(r_{o1} \,||\, R)\frac{v_{id}}{2}$$

$$\Rightarrow v_{o2} = g_{m1}(r_{o1} \,||\, R)\frac{v_{id}}{2}$$

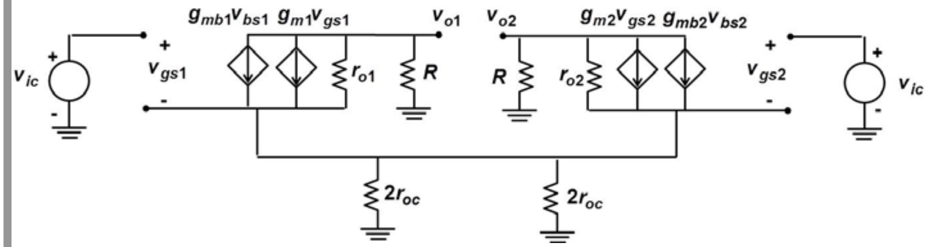
$$v_{od} = v_{o1} - v_{o2}$$

$$= -g_{m1}(r_{o1} \,||\, R)v_{id}$$

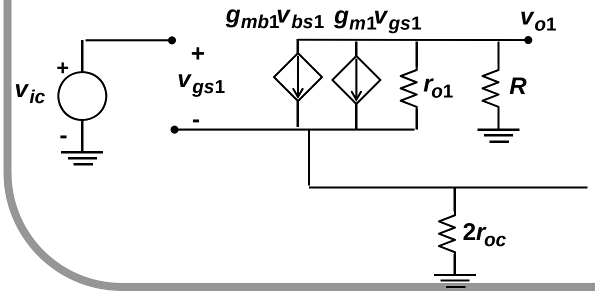
$$\Rightarrow A_{vd} = \frac{v_{od}}{v_{id}} = -g_{m1}(r_{o1} \,||\, R)$$

A FET Differential Amplifier: Small Signal Analysis for Common-Mode Input

Assume a common-mode input



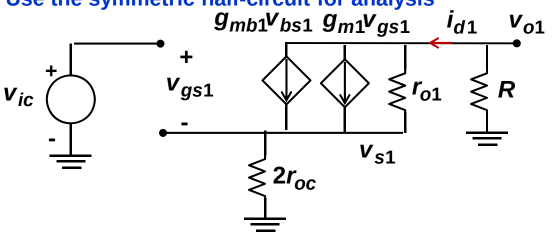
Use the symmetric half-circuit for analysis



This is like the small signal circuit of a CS amplifier with source degeneration…..!

A FET Differential Amplifier: Small Signal Analysis for Common-Mode Input

Use the symmetric half-circuit for analysis



$$v_{oc} = \frac{v_{o1}+v_{o2}}{2}$$

$$v_{o2} = v_{o1}$$

$$\Rightarrow v_{oc} = v_{o1}$$

$$\Rightarrow A_{vc} = \frac{v_{oc}}{v_{ic}} = \frac{v_{o1}}{v_{ic}}$$

$$v_{ic} = v_{gs1} + v_{s1} = v_{gs1} + i_{d1}(2r_{oc})$$

$$i_{d1} = g_{m1}v_{gs1} - g_{mb1}v_{s1} + \frac{v_{o1}-v_{s1}}{r_{o1}} =$$

$$= g_{m1}(v_{ic} - i_{d1}(2r_{oc})) - g_{mb1}i_{d1}(2r_{oc}) - \frac{i_{d1}R + i_{d1}(2r_{oc})}{r_{o1}}$$

$$\Rightarrow i_{d1} = \frac{g_{m1}r_{o1}}{r_{o1}+R+2r_{oc}+(g_{m1}+g_{mb1})r_{o1}(2r_{oc})}v_{ic}$$

$$\Rightarrow A_{vc} = \frac{v_{o1}}{v_{ic}} = \frac{-i_{d1}R}{v_{ic}} = -\frac{g_{m1}r_{o1}R}{r_{o1}+R+2r_{oc}+(g_{m1}+g_{mb1})r_{o1}(2r_{oc})}$$

A FET Differential Amplifier: CMRR

Difference-Mode Gain:

$$A_{vd} = \frac{v_{od}}{v_{id}} = -g_{m1}(r_{o1}\,||\,R)$$

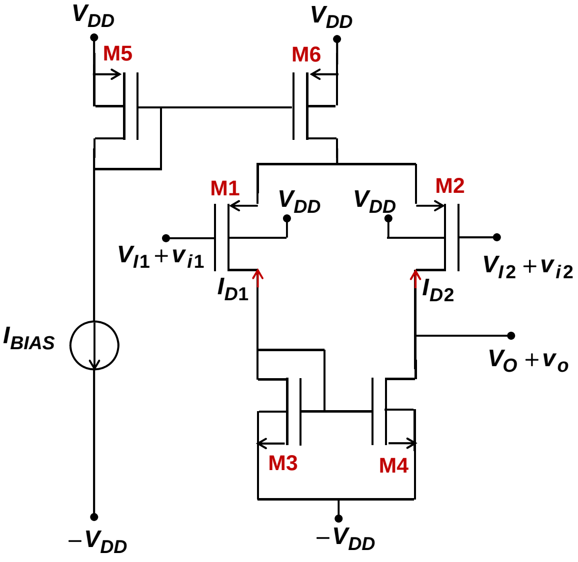
Common-Mode Gain:

$$A_{vc} = \frac{v_{oc}}{v_{ic}} = -\frac{g_{m1}r_{o1}R}{r_{o1}+R+2r_{oc}+(g_{m1}+g_{mb1})r_{o1}(2r_{oc})} = -\frac{g_{m1}(r_{o1}\,||\,R)}{1+\frac{2r_{oc}}{r_{o1}+R}\left[1+(g_{m1}+g_{mb1})r_{o1}\right]}$$

Common-Mode Rejection Ratio (CMRR):

$$CMRR = \frac{A_{vd}}{A_{vc}} = 1+\frac{2r_{oc}}{r_{o1}+R}\left[1+(g_{m1}+g_{mb1})r_{o1}\right]$$

$$\sim g_{m1}(2r_{oc}) \longrightarrow \text{Large if } r_{oc} \text{ is large}$$


V_{DD}
M5
V_{DD}
M6
M1
V_{DD}
V_{DD}
M2
$V_{I1} + v_{i1}$
$V_{I2} + v_{i2}$
I_{D1}
I_{D2}
I_{BIAS}
$V_O + v_o$
M3
M4
$-V_{DD}$
$-V_{DD}$