Lecture 11

Single Stage FET Amplifiers:
Common Source (CS) Amplifier

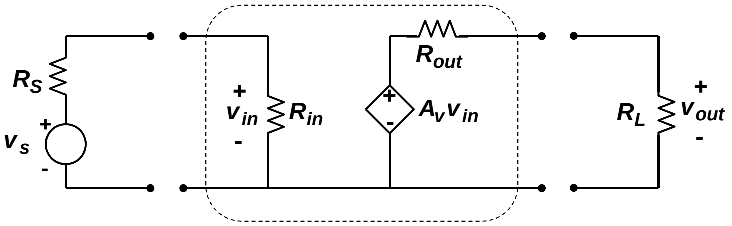
The Building Blocks of Analog Circuits - I

In this lecture you will learn:

- **General amplifier concepts (in terms of the two-port models)**
- **Common source amplifier (CS)**
- **Small signal models of amplifiers**

Two-Port Amplifier Models: A Voltage Amplifier

A Voltage Amplifier:



$$\frac{v_{out}}{v_s} = A_v \underbrace{\left(\frac{R_{in}}{R_{in}+R_S}\right)}_{\text{Input voltage divider}} \underbrace{\left(\frac{R_L}{R_{out}+R_L}\right)}_{\text{Ouput voltage divider}}$$

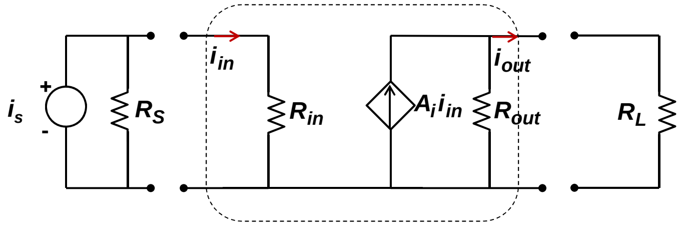
Requirements:
Large input resistance R_{in}
Small output resistance R_{out}

Open circuit output voltage gain (i.e. when $R_L = \infty$):

$$\frac{v_{out}}{v_{in}} = A_v = \text{Voltage gain}$$

Two-Port Amplifier Models: A Current Amplifier

A Current Amplifier:



$$\frac{i_{out}}{i_s} = A_i \underbrace{\left(\frac{R_S}{R_S + R_{in}}\right)}_{\text{Input current divider}} \underbrace{\left(\frac{R_{out}}{R_{out} + R_L}\right)}_{\text{Ouput current divider}}$$

Requirements:

Small input resistance R_{in}

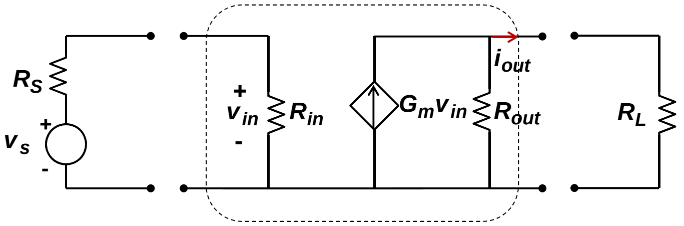
Large output resistance R_{out}

Short circuit output current gain (i.e. when $R_L = 0$):

$$\frac{i_{out}}{i_{in}} = A_i = \text{Current gain}$$

Two-Port Amplifier Models: A Transconductance Amplifier

A Transconductance Amplifier (or a Voltage-to-Current Amplifier):



$$\frac{i_{out}}{v_s} = G_m \underbrace{\left(\frac{R_{in}}{R_S + R_{in}}\right)}_{\text{Input voltage divider}} \underbrace{\left(\frac{R_{out}}{R_{out} + R_L}\right)}_{\text{Ouput current divider}}$$

Requirements:

Large input resistance R_{in}

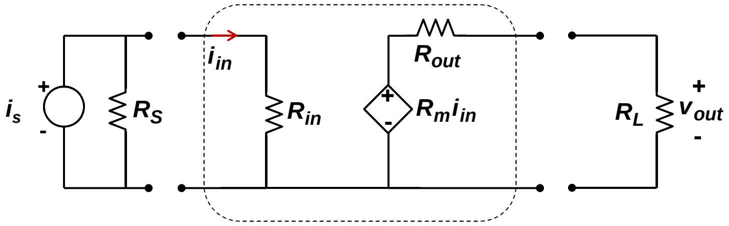
Large output resistance R_{out}

Short circuit output current and transconductance gain (i.e. when $R_L = 0$):

$$\frac{i_{out}}{v_{in}} = G_m = \text{Transconductance gain}$$

Two-Port Amplifier Models: A Transimpedance Amplifier

A Transimpedance (or a Transresistance) Amplifier (or a Current-to-Voltage Amplifier):



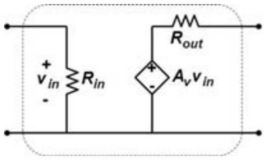
$$\frac{v_{out}}{i_s} = R_m \underbrace{\left(\frac{R_S}{R_S+R_{in}}\right)}_{\text{Input current divider}} \underbrace{\left(\frac{R_L}{R_{out}+R_L}\right)}_{\text{Ouput voltage divider}}$$

Requirements:
Small input resistance R_{in}
Small output resistance R_{out}

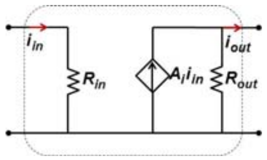
Open circuit output voltage and transimpedance gain (i.e. when $R_L = \infty$):

$$\frac{v_{out}}{i_{in}} = R_m = \text{Transimpedance gain}$$

Two-Port Amplifier Models: General Concepts



is equivalent to



$$R_{in} \longleftarrow R_{in}$$

$$R_{out} \longleftarrow R_{out}$$

$$A_v \longleftarrow A_i \frac{R_{out}}{R_{in}}$$

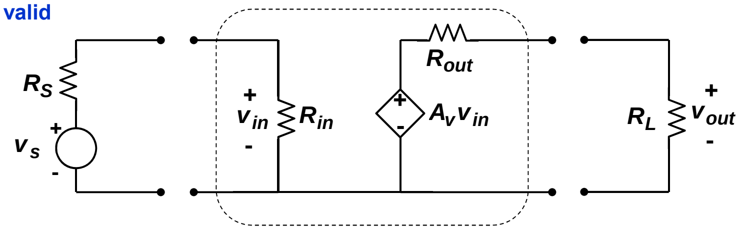
The two-port models are equivalent (inter-convertible)

The designation of an amplifier as a voltage, current, transconductance, or transimpedance amplifier depends on the values of the input and output resistances

Need to find the input resistance, output resistance, open circuit voltage gain, and short circuit current gain to characterize an amplifier

Unilateral Networks and Two-Port Amplifier Models

For many circuits and amplifiers, the kind of two-port models described here are not strictly valid



Reasons:

The input resistance R_{in} can depend on the load resistance R_L
The output resistance R_{out} can depend on the source resistance R_s

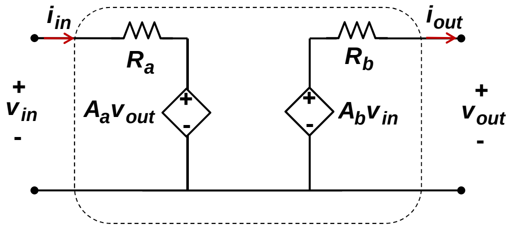
Circuits in which the above does not happen, and for which the two-port models described here are strictly valid, are unilateral

In many cases, even for non-unilateral networks, two-port models described here tend to be good approximations for hand-calculations

---

Example: A Two-Port Model for a Non-Unilateral Network

Consider the two-port model shown below:



One can write:

$$\begin{bmatrix} i_{in} \\ i_{out} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{R_a} & -\dfrac{A_a}{R_{in}} \\ \dfrac{A_b}{R_b} & -\dfrac{1}{R_b} \end{bmatrix} \begin{bmatrix} v_{in} \\ v_{out} \end{bmatrix}$$

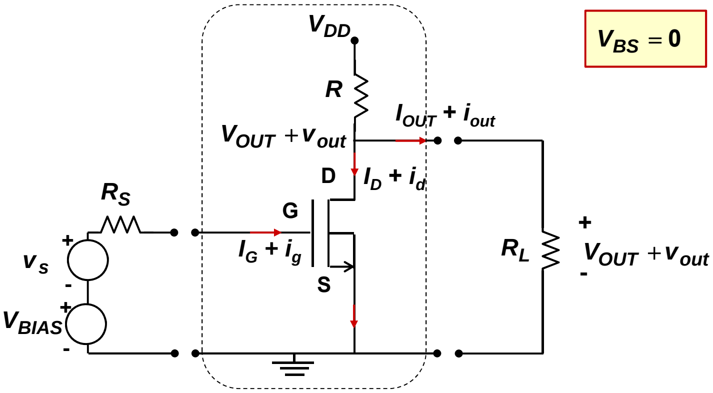
It is not difficult show that the input resistance, calculated as,

$$R_{in} = \frac{v_{in}}{i_{in}}$$

will depend on what load is connected at the output terminals

The Common Source Amplifier

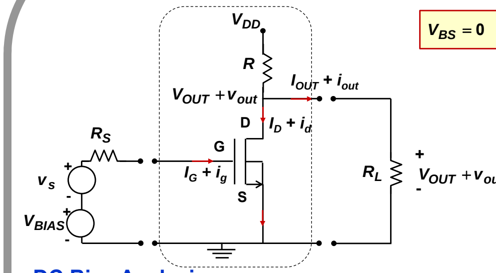
$V_{BS} = 0$

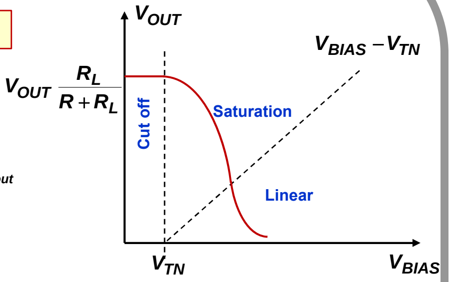


The source terminal is “common” between the input and the output

---

The Common Source Amplifier





DC Bias Analysis:

Make sure the output load resistance R_L is included in the DC bias analysis

Start by assuming the FET is in saturation (and then later verify):

$$V_{DD} - (I_{OUT} + I_D)R = V_{OUT}$$

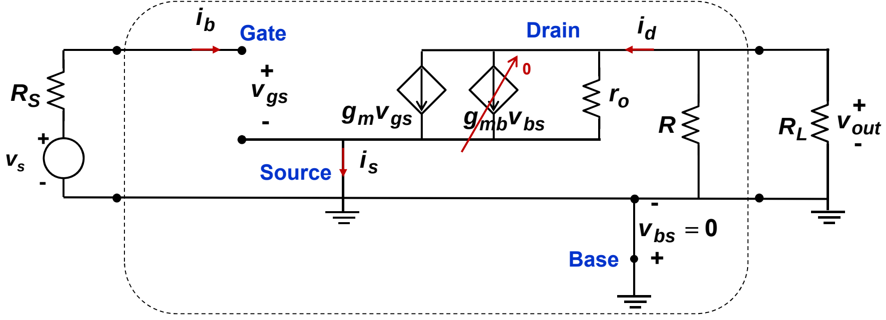
$$\Rightarrow V_{DD} - \left(\frac{V_{OUT}}{R_L} + I_D\right)R = V_{OUT}$$

$$\Rightarrow V_{OUT} = (V_{DD} - I_D R)\frac{R_L}{R+R_L}$$

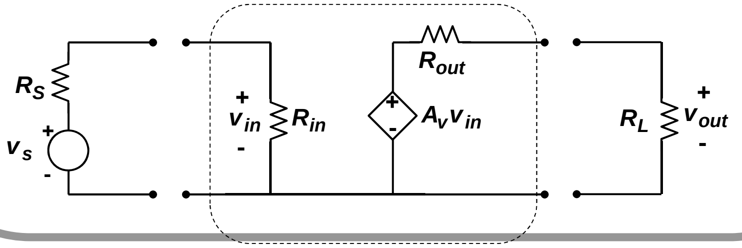
$$I_D = \frac{k_n}{2}(V_{BIAS} - V_{TN})^2(1+\lambda_n V_{OUT})$$

$$I_D \approx \frac{k_n}{2}(V_{BIAS} - V_{TN})^2$$

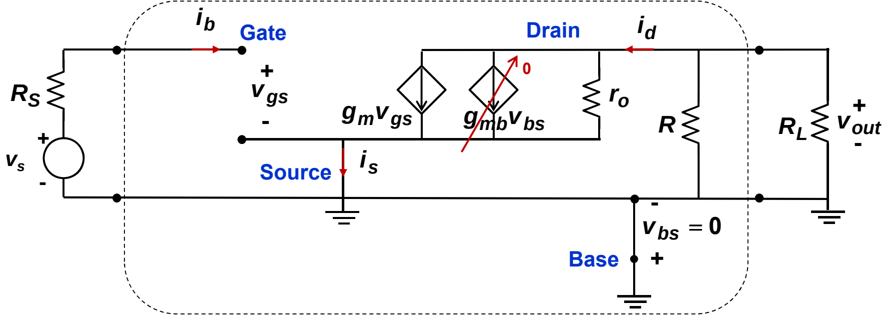
The Common Source Amplifier: Small Signal Model



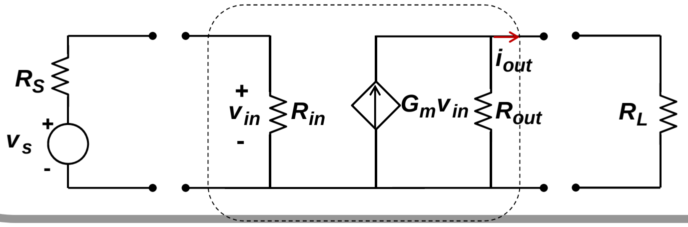
Compare with the standard voltage amplifier model:



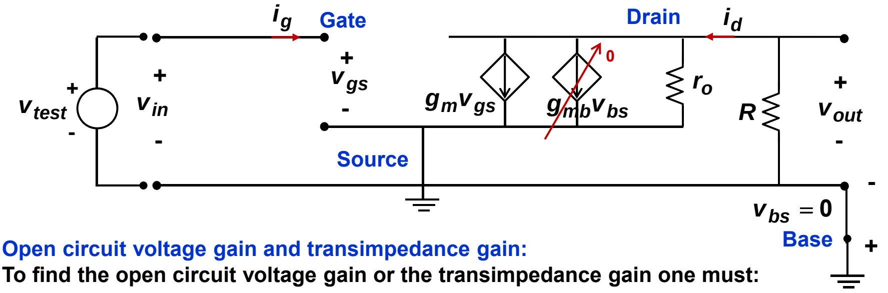
The Common Source Amplifier: Small Signal Model



Compare with the standard transconductance amplifier model:



The Common Source Amplifier: Open Circuit Voltage Gain



Open circuit voltage gain and transimpedance gain:
To find the open circuit voltage gain or the transimpedance gain one must:

i) Remove the load resistance R_L at the output that the circuit will drive
ii) Then apply a test voltage source at the input
iii) Then find the resulting open circuit output voltage
iv) Take the ratio of the output voltage and the input voltage to find the **open circuit voltage gain**:

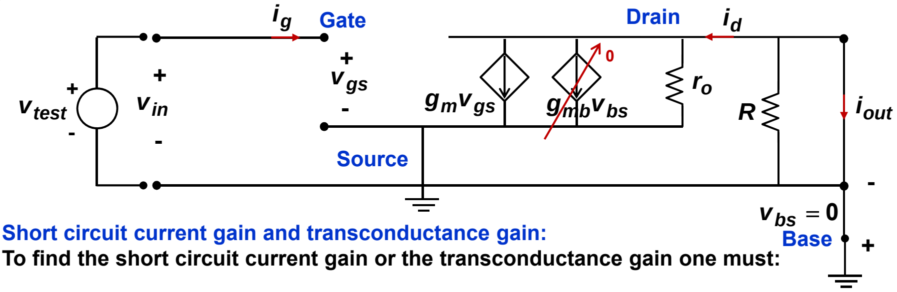
$$A_v = \frac{v_{out}}{v_{in}} = -\frac{i_d R}{v_{in}} = -g_m \left(r_o \, || \, R \right)$$

v) Or take the ratio of the output voltage and the input current to find the **transimpedance gain**:

$$R_m = \frac{v_{out}}{i_{in}} = -\infty$$

This result is somewhat artificial since at non-zero frequencies there will be a finite input current due to capacitances

The Common Source Amplifier: Short Circuit Current Gain



Short circuit current gain and transconductance gain:
To find the short circuit current gain or the transconductance gain one must:

i) Short the load resistance R_L at the output that the circuit will drive
ii) Then apply a test voltage source at the input
iii) Then find the resulting current at the shorted output
iv) Take the ratio of the output and the input currents to find the **short circuit current gain**:

$$A_i = \frac{i_{out}}{i_g} = -\frac{g_m v_{gs}}{0} = \infty \longrightarrow$$

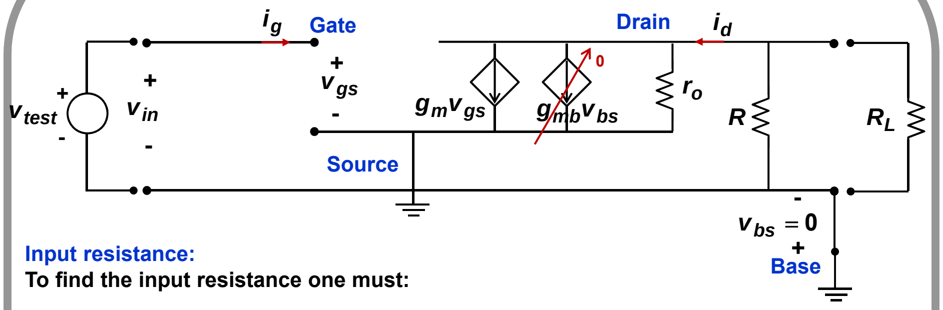
∞ for the CS amplifier (at DC)

This result is somewhat artificial since at non-zero frequencies there will be a finite input current due to capacitances

v) Or take the ratio of the output current and the input voltage to find the **transconductance gain**:

$$G_m = \frac{i_{out}}{v_{in}} = -\frac{g_m v_{gs}}{v_{in}} = -g_m$$

The Common Source Amplifier: Input Resistance



Input resistance:

To find the input resistance one must:

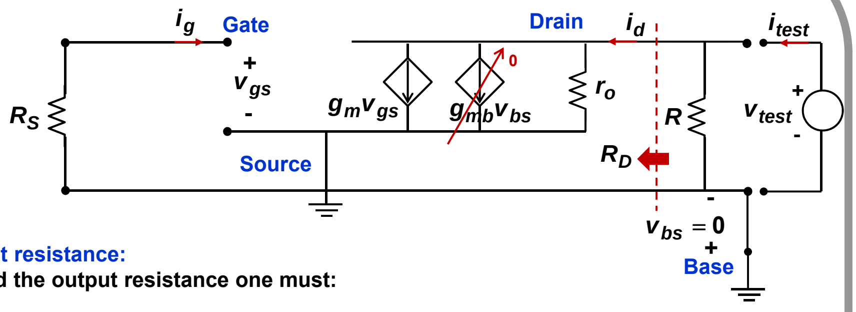
- i) Make sure the load resistance R_L that the circuit will drive is in place at the output
- ii) Then apply a test voltage source at the input
- iii) Then find the resulting current at the input
- iv) Then take the ratio of the input voltage and the input current

$$R_{in} = \frac{v_{in}}{i_g} = \infty$$

→ ∞ for the CS amplifier (at DC)

This result is somewhat artificial since at non-zero frequencies there will be a finite input current due to capacitances

The Common Source Amplifier: Output Resistance



Output resistance:

To find the output resistance one must:

- i) Remove the load resistance R_L and put a test voltage source in its place
- ii) Make sure the source resistance R_S is in place at the input
- iii) Then find the resulting test current at the output
- iv) Then take the ratio of the test voltage and the test current

$$R_{out} = \frac{v_{test}}{i_{test}} = r_o \,||\, R$$

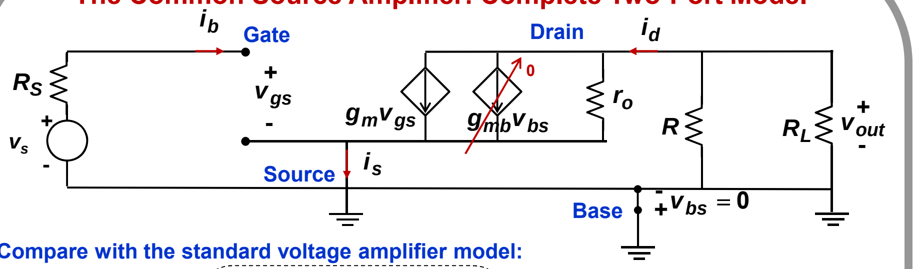
→ Fairly large for the CS amplifier

Analysis shows that CS (like CE) is a good transconductance amplifier!

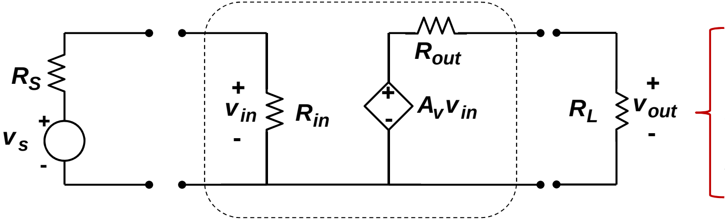
Resistance looking into the drain end of a FET:

$$R_D = r_o$$

The Common Source Amplifier: Complete Two-Port Model



Compare with the standard voltage amplifier model:

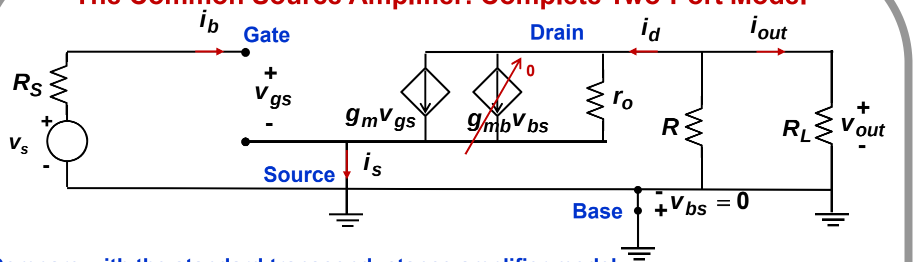


$$\left\{\begin{aligned} R_{in} &= \infty \\ A_v &= -g_m(r_o\,||\,R) \\ R_{out} &= (r_o\,||\,R) \end{aligned}\right.$$

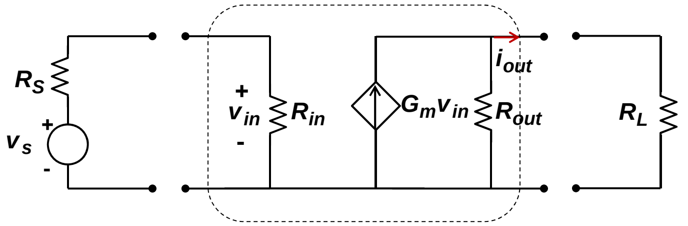
Now we can use the standard expression:

$$\frac{v_{out}}{v_s} = A_v\left(\frac{R_{in}}{R_{in}+R_S}\right)\left(\frac{R_L}{R_{out}+R_L}\right) = -g_m(r_o\,||\,R)\left(\frac{R_L}{(r_o\,||\,R)+R_L}\right)$$

The Common Source Amplifier: Complete Two-Port Model



Compare with the standard transconductance amplifier model:

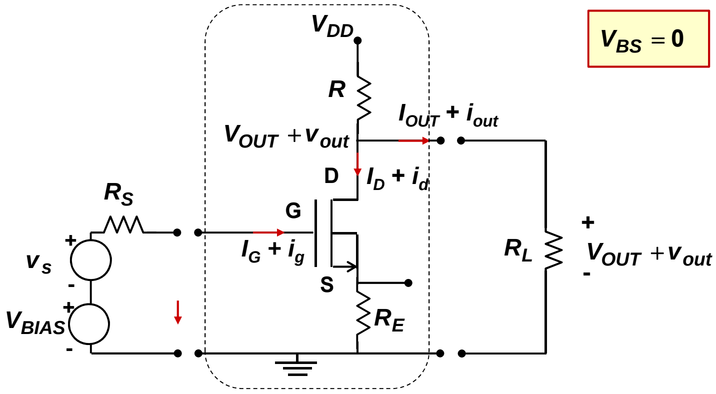


$$\left\{\begin{aligned} R_{in} &= \infty \\ G_m &= -g_m \\ R_{out} &= (r_o\,||\,R) \end{aligned}\right.$$

Now we can use the standard expression:

$$\frac{i_{out}}{v_s} = G_m\left(\frac{R_{in}}{R_{in}+R_S}\right)\left(\frac{R_{out}}{R_{out}+R_L}\right) = -g_m\left(\frac{(r_o\,||\,R)}{(r_o\,||\,R)+R_L}\right)$$

The Common Source Amplifier with Source Degeneration



$$V_{BS} = 0$$

$$V_{DD} - (I_{OUT} + I_D)R = V_{OUT}$$

$$\Rightarrow V_{DD} - \left(\frac{V_{OUT}}{R_L} + I_D\right)R = V_{OUT}$$

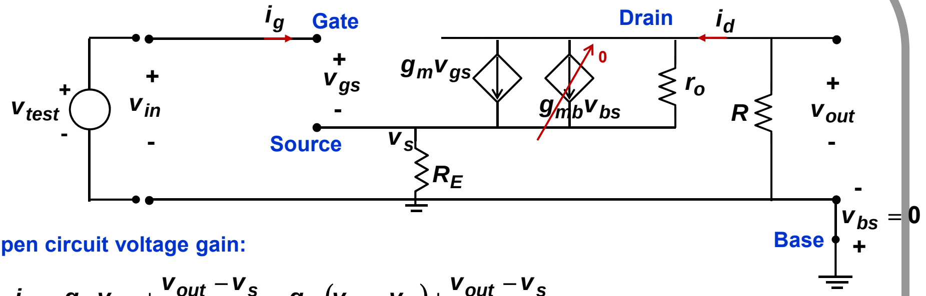
$$\Rightarrow V_{OUT} = (V_{DD} - I_D R)\frac{R_L}{R + R_L}$$

$$I_D = \frac{k_n}{2}(V_{GS} - V_{TN})^2(1 + \lambda_n V_{OUT})$$

$$I_D \approx \frac{k_n}{2}(V_{GS} - V_{TN})^2$$

$$V_{GS} = V_{BIAS} - I_D R_E$$

CS Amplifier with Source Degeneration: Open Circuit Voltage Gain



Open circuit voltage gain:

$$i_d = g_m v_{gs} + \frac{v_{out} - v_s}{r_o} = g_m(v_{in} - v_s) + \frac{v_{out} - v_s}{r_o}$$

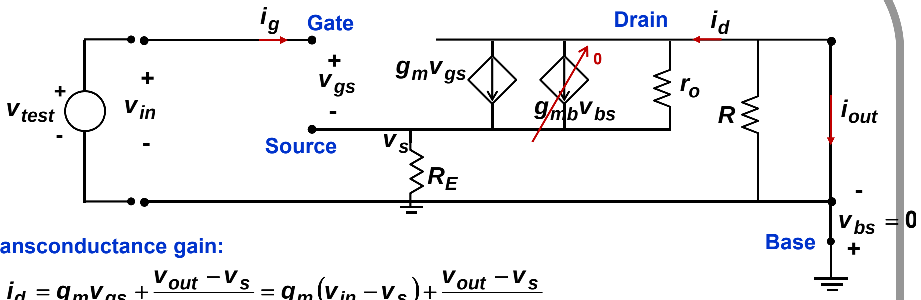
$$\Rightarrow i_d = g_m(v_{in} - i_d R_E) + \frac{-i_d R - i_d R_E}{r_o} = g_m v_{in} - i_d g_m R_E - i_d\frac{R + R_E}{r_o}$$

$$\Rightarrow i_d = \frac{g_m}{\left(1 + \frac{R + R_E}{r_o} + g_m R_E\right)} v_{in}$$

A_v is reduced when R_E is present

$$\Rightarrow A_v = \frac{v_{out}}{v_{in}} = \frac{-i_d R}{v_{in}} = -\frac{g_m R r_o}{(r_o + R + R_E(1 + g_m r_o))} = -\frac{g_m(r_o \,||\, R)}{\left(1 + \frac{R_E}{(r_o + R)}(1 + g_m r_o)\right)} v_{in}$$

CS Amplifier with Source Degeneration: Transconductance Gain



Transconductance gain:

$$i_d = g_m v_{gs} + \frac{v_{out} - v_s}{r_o} = g_m (v_{in} - v_s) + \frac{v_{out} - v_s}{r_o}$$

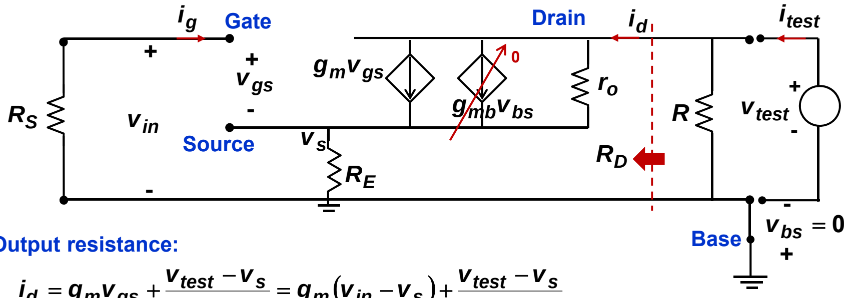
$$\Rightarrow i_d = g_m (v_{in} - i_d R_E) + \frac{0 - i_d R_E}{r_o} = g_m v_{in} - i_d g_m R_E - i_d \frac{R_E}{r_o}$$

$$\Rightarrow i_d = \frac{g_m}{\left(1 + \frac{R_E}{r_o} + g_m R_E\right)} v_{in}$$

G_m is reduced when R_E is present

$$\Rightarrow G_m = \frac{i_{out}}{v_{in}} = \frac{-i_d}{v_{in}} = -\frac{g_m r_o}{(r_o + R_E(1 + g_m r_o))} = -\frac{g_m}{\left(1 + \frac{R_E}{r_o}(1 + g_m r_o)\right)}$$

CS Amplifier with Source Degeneration: Output Resistance



Output resistance:

$$i_d = g_m v_{gs} + \frac{v_{test} - v_s}{r_o} = g_m (v_{in} - v_s) + \frac{v_{test} - v_s}{r_o}$$

$$\Rightarrow i_d = g_m (i_g R_s - i_d R_E) + \frac{v_{test} - i_d R_E}{r_o} = -i_d \left(g_m + \frac{1}{r_o}\right) R_E + \frac{v_{test}}{r_o}$$

$$\Rightarrow i_d = \frac{v_{test}}{r_o + R_E (1 + g_m r_o)}$$

$$\Rightarrow R_D = \frac{v_{test}}{i_d} = r_o + R_E (1 + g_m r_o) \longleftarrow$$

R_D is increased when R_E is present

$$\Rightarrow R_{out} = \frac{v_{test}}{i_{test}} = (R \,||\, R_D) = \frac{R[r_o + R_E (1 + g_m r_o)]}{r_o + R + R_E (1 + g_m r_o)}$$

Relations to Remember

For any small signal amplifier model, the following always hold:

(Transconductance) X (Output resistance) = (Open circuit voltage gain)

(Transimpedance) / (Output resistance) = (Short circuit current gain)

The above follows from the equivalent Thevenin and Norton models of the amplifier

***All quantities must be calculated assuming the same value of R_s (typically zero)