ECE Final Exam Solutions

1

a) $V_{BH} = V_H - V_{TN} = 2.5\ V$

b) $C_B \dfrac{dV_B}{dt} = I_D = \dfrac{k_n}{2}\left(V_H - V_B - V_{TN}\right)^2$

c) $$\int_0^{0.9V_{BH}} \frac{dV_B}{(V_H - V_B - V_{TN})^2} = \frac{k_n}{2C_B}\int_0^{t_{0\to1}} dt = \frac{k_n}{2C_B}t_{0\to1} \Rightarrow \frac{9}{V_{BH}} = \frac{k_n}{2C_B}t_{0\to1} \Rightarrow t_{0\to1} = 1.2$$

d) $$C_B \frac{dV_B}{dt} = -I_D = k_n\left(\frac{kT}{q}\right)^2 (m-1)\, e^{\frac{q(-V_{TN})}{mkT}}\left(1 - e^{-\frac{q(V_B)}{kT}}\right)$$

Since $\left\{\begin{array}{l} V_B \gg \frac{kT}{q} \\ m = 2 \end{array}\right\} \Rightarrow \dfrac{dV_B}{dt} = -\dfrac{k_n}{C_B}\left(\dfrac{kT}{q}\right) e^{-q\frac{V_{TN}}{2kT}}$

Integrating

$$\Rightarrow V_B(t_L) - V_B(t=0) = -\frac{k_n}{C_B}\left(\frac{kT}{q}\right)^2 e^{-q\frac{V_{TN}}{2kT}}\, t_L = -1\ \text{Volt} \Rightarrow t_L = 4.2\ ms$$

2. a) $\phi_B - V = \int E_x\, dx = 1.55\ V \Rightarrow \phi_B = 1.55 - 0.8 = 0.75\ V$

b) $C_j = \dfrac{\varepsilon_s}{105\ nm} \cong 98\ nF/cm^2$

c) since $\dfrac{dE_x}{dx} = +\dfrac{\rho(x)}{\varepsilon_s} = 0$ for $0 < x < 50\ n$

$\Rightarrow$ no dopants in this range.

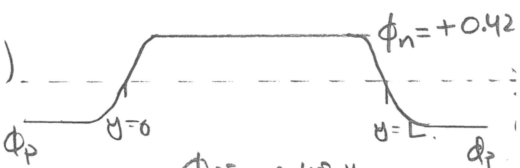
d) $\varepsilon_s \dfrac{dE_x}{dx} = \rho(x) = 4.14 \times 10^5\ c/m^3$

$\Rightarrow$ +ve depletion charge density $\Rightarrow$ N-type dopant $\Rightarrow N_d = \dfrac{4.14\times10^5}{q} = 2.58\times10^{24}$

$\Rightarrow N_d = 2.58 \times 10^{18}\ 1/cm^3$

e) Flatband $\Rightarrow p(x=0) = \dfrac{n_i^2}{n(x=0)} = \dfrac{n_i^2}{N_d} = 10^3\ 1/cm^3$

f)



g) At threshold $p(x=0) \approx N_d \Rightarrow n(x=0) = \dfrac{n_i^2}{p(x=0)} = \dfrac{n_i^2}{N_d} = 10^3\ cm^{-3}$

[Coulombs per unit area]

h) $Q_p = -C_{ox}\left[V_{GS} - V_{TP} - V_{CS}(y)\right]$ when $V_{DS} = 0$ $V_{CS}(y) = 0$ when $V_{DS} = -\Delta V_{DS}$

then $V_{CS}(y) \approx -\Delta V_{DS}\dfrac{y}{L} \Rightarrow V_{CS}(y=0) = 0$ & $V_{CS}(y=L) = V_{DS} = -\Delta V_{DS}$

Total inversion charge $Q_P\,[\text{Coulombs}] = -WC_{ox}\int_0^L dy\left[V_{GS} - V_{TP} - V_{CS}(y)\right]$

$$= -WC_{ox}(V_{GS} - V_{TP})L - WC_{ox}\Delta V_{DS}\frac{L}{2} \Rightarrow \Delta Q_P = -\frac{WL\,C_{ox}}{2}\Delta V_{DS}$$

$$\Rightarrow C_{gd} = -\frac{\Delta Q_P}{\Delta V_{DS}} = \frac{WL\,C_{ox}}{2}$$

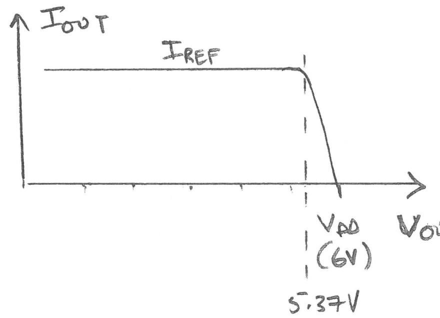
3a) $\frac{k_P}{2}\left(V_q - V_{DD} - V_{TP}\right)^2 = I_{REF} = 100\,\mu A \Rightarrow V_q = 5\,V$

$\Rightarrow V_q - V_{DD} = V_b - V_q \Rightarrow V_b = 4\,V$

b) $V_{OUT} > V_b - V_{TP} \Rightarrow V_{OUT} > 4.5\,V \Rightarrow$ when $V_{OUT} > 4.5\,V$ M3 will go into linear region.

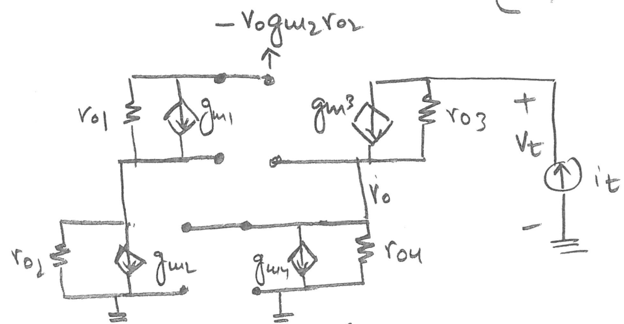
c) Let the voltage at the source of M3 be V_o. When $V_o > V_q - V_{TP} = 5.5\,V$ then M4 goes into linear region. At that point, M3 is already in the linear region so $I_{out} = k_P\left(V_b - V_o - V_{TP} - \left(\frac{V_{OUT} - V_o}{2}\right)\right)(V_{OUT} - V_o) = -I_{D3}$ but this must also equal $100\,\mu A$ since M4 is in saturation (or at the border b/w linear + saturation regime). Putting $V_o = 5.5\,V$ gives $V_{OUT} = 5.37\,V$. So if $V_{OUT} > 5.37\,V$ M4 will go into linear region.

d)



e) $R_{out} = r_{o3} + r_{o4} + g_{m3} r_{o3} r_{o4}$. { output resistance of a Cascode

f)



$V_o = \frac{i_t}{g_{m4}} \Rightarrow V_t = V_o + \left[i_t + g_{m3}(1 + g_{m2} r_{o2}) V_o\right] r_{o3} = i_t\left\{r_{o3} + \frac{g_{m3}}{g_{m4}} r_{o3} + \frac{g_{m3}}{g_{m4}} g_{m2} r_{o2} r_{o3}\right.$

4a) $\frac{k_{P4}}{2}\left(V_q - 5 - V_{TP}\right)^2 = 200\,\mu A \Rightarrow V_q = 4$ Volts.

b) In common mode $V_S \approx V_{IC} + 1 \Rightarrow$ M3 will go linear if $V_S > V_q - V_{TP}$

$\Rightarrow$ when $V_{IC} > 3.5\,V$

c) $V_{o1} = V_{o2} = 100\,\mu A \times R = 2V$, M1 + M2 will go linear if $V_{o1} > V_{IC} - V_{TP}$ or $V_{IC} < V_{o1} + V_{TP} = 1.5\,V$ d) $A_{vd} = -g_{mp}(r_{op} \| R)$ e) current magnitude is $g_{mp}\frac{V_{id}}{2}$ and current will flow from node V_{o2} to node V_{o1} if $V_{id} > 0$

f) Small signal circuit is symmetric $\Rightarrow$ half-circuit is a CS stage and $R_S = 0$. $\Rightarrow A_{vd} = -g_{mp}(R \| r_{op})\left\{\frac{1 - j\omega\frac{C_{gd}}{g_{mp}}}{1 + j\omega C_{gd}(r_{op} \| R)}\right\} \Rightarrow \omega_H = \frac{1}{C_{gd}(r_{op} \| R)}$