ECE 3100 **PRELIM 2** **Fall 2014**

Wednesday, 05 November 2014

NAME: ______________________________ **netID:** ______

- Exam duration: 2:55 PM - 4:10 PM, that's 1 hour 15 minutes
- There are **6** questions to this exam
- Exams are collected at 4:10 PM, no extensions
- Write your answers on this set of sheets
- Write your calculations/reasonings *clearly*: no points awarded for what you *intended to write* or *meant*
- Write ALL calculations
- Circle your final answer
- There is a normal distribution table at the end of the exam that may be useful.
- Keep your cheat sheet after the exam

1. (10 Points). Consider a biased coin that has probability p of landing on Heads. You flip the coin a very large number of times (*i.e.*, an infinite number of times).

(a) Consider the first 20 tosses of the coin. Calculate the probability that the first 12 tosses will land on Tails. Give the answer in the form of an equation involving p.

(b) Let X be the number of tosses required to have the coin land on Tails. Calculate $E[X]$ in terms of p.

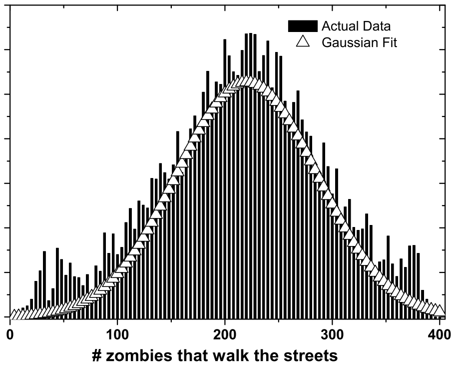
(c) Consider again the first 20 tosses of the coin. Calculate the probability that the 12^{th} Tails is on the 20^{th} toss. Give the answer in the form of an equation involving p.

2. (10 Points). One day, on Homer Simpson's watch at the nuclear power plant, one of the reactors begins to melt down. The emission of radioactive particles from the catastrophic event can be modeled as a Poisson process with rate $\lambda = 100$ particles/second.

(a) What is the expected *total* number of particles that are emitted in the time window $[10, 20] \cup [50, 60]$ seconds?

(b) Suppose that exactly 150 particles escaped in the first second. What is the PDF of the number of particles that escape in the next second?

3. (16 Points). In the year 2018, a zombie apocalypse arose across the world. As a survivor, having learned a great deal in ECE 3100 back in 2014, and to kill (haha!) some time, you put together statistics on the typical number of zombies that walk the streets during any given day: the result is the graph below. You want to do some calculations based on this data. Your first analysis reveals that you can estimate the graph with a Gaussian (normal) distribution, and that this normal distribution has a mean of 220, and a variance of 17161.



(a) What is the probability that there are 300 or less zombies walking the streets during any given day?

(b) What is the probability that there are 250 or more zombies walking the streets during any given day?

(c) What is the probability that there are 100 or less zombies walking the streets during any given day?

(d) During any given day, which is the most probable in the following (note that for these calculations, keep only the first 3 decimals in your calculations - show your calculations and explain your answer):
  (1) That 164 or less zombies are walking in the streets?
  (2) That 276 or more zombies are walking in the streets?
  (3) That between 164 and 276 zombies are walking in the streets?
  (4) none of the above.

Use the following page for your calculations and answers.

(Empty page for answers for Problem 3)

4. (20 Points).

(a) Let X_1 and X_2 be independent exponential random variables such that $X_1 = \lambda_1 e^{-\lambda_1 x}$ and $X_2 = \lambda_2 e^{-\lambda_2 x}$, for $x \geq 0$ for both cases (and $X_1 = X_2 = 0$ if $x < 0$). Calculate the PDF $Z = X - Y$.

(b) We have a random variable X that has the following properties:

$$E[X] = 0,\ E[X^2] = 1,\ E[X^3] = 0,\ E[X^4] = 3,$$

and we define $Y = a + bX + cX^2$ with a, b, c constants. Calculate the correlation coefficient $\rho(X, Y)$.

5. (30 Points). Consider the piecewise constant PDF given by

$$f_X(x) = \begin{cases} 0.2, & \text{if } 0 \leq x < 1, \\ 0.1, & \text{if } 1 \leq x < 5, \\ 0.08, & \text{if } 5 \leq x < 10, \\ 0, & \text{else.} \end{cases}$$

We have the following events:

$$A = \{X \text{ is in the interval } [0,1]\}$$
$$B = \{X \text{ is in the interval } (1,5]\}$$
$$C = \{X \text{ is in the interval } (5,10]\}$$
$$D = \{X \text{ is in the interval } [3,7]\}$$

Calculate the following:

(a) $P(A)$

(b) $P(D)$

(c) Calculate $E[X \mid A]$.

(d) Calculate $\mathrm{var}(X)$.

6. (14 Points). Answer the following with True or False. There are no partial points.

(a) The binomial distribution assumes that the probability of success is the same for each trial.

(b) The probability distribution of all continuous random variables are normally distributed.

(c) Given that the mean and standard deviation of a binomial distribution are 4 and $\sqrt{3}$, respectively, the parameter $p = 1/3$.

(d) $E[X^2] \leq (E[X])^2$

(e) For any two random variables X and Y, $E[E[X \mid Y]] = E[X]$.

(f) X is an exponential random variable with parameter λ=9 s^{-1}. Then:
$P\big((X > 3 \text{ seconds}) \mid (X > 1 \text{ second})\big) = e^{-27}$.

(g) For two random variables X and Y, if we have that $E[X - Y] = E[X + Y]$, then necessarily $E[Y] = 0$.

Standard Normal Probabilities

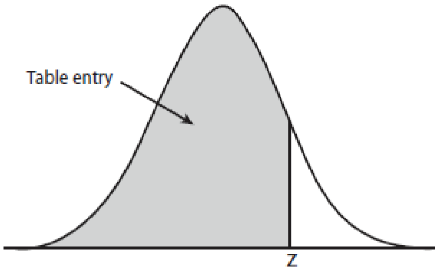


Table entry for Z is the area under the standard normal curve to the left of Z.

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	2/3	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998