ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Problem Set 8
Due Friday April 22, 2016 at 5:00PM.
Location to turn in: "ECE 2200" box on Phillips Hall 2nd floor

1. Slightly modified version of McClellan, Schafer, Yoder Exercise 12.12 on p. 364. If $x(t)$ is the input and $y(t)$ is the output, a so-called half-wave rectifier is defined by

$$y(t) = \begin{cases} x(t), & x(t) > 0 \\ 0, & \text{otherwise} \end{cases} . \tag{95}$$

Please show that the half-wave rectifier is a nonlinear system.

2. Slightly modified version of McClellan, Schafer, Yoder Exercise 12.13 on p. 365. Let $x(t) = \cos(\omega_c t)$. Let $y(t)$ be the output of the half-wave rectifier defined in Problem 1. Please show that the Fourier coefficients for $y(t)$ are

$$a_k = \begin{cases} \frac{\cos(\pi k/2)}{\pi(1-k^2)}, & k \neq \pm 1 \\ \frac{1}{4}, & k = \pm 1 \end{cases} . \tag{96}$$

3. Please determine the inverse continuous-time Fourier transform of

$$H(\omega) = \frac{86 + 28j\omega - 2\omega^2}{24 + 10j\omega - \omega^2}. \tag{97}$$

I suggest doing this via partial fraction expansion.

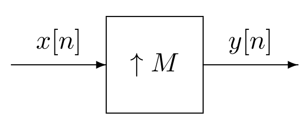
4. Please consider the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 10\frac{\mathrm{d}y}{\mathrm{d}t} + 24y(t) = 2x(t) + 8\frac{\mathrm{d}x}{\mathrm{d}t} \tag{98}$$

with input $x(t)$ and output $y(t)$. What is the frequency response $H(\omega)$ (recall that if $x(t) \leftrightarrow X(\omega)$ and $y(t) \leftrightarrow Y(\omega)$ then the frequency response is $H(\omega) = Y(\omega)/X(\omega)$)?

5. This problem concerns oversampling Digital to Analog converters. Such converters are commonly used in CD players.

   (a) Define an up-converter block by the following block diagram and equation:



$$y[n] = \begin{cases} x[n/M], & n = Ml \text{ for some } l \in \mathcal{Z} \\ 0, & \text{otherwise} \end{cases} . \tag{99}$$

   If $x[n]$ has Discrete-Time Fourier Transform $X(\Omega)$, then what is the Discrete-Time Fourier Transform of $y[n]$?
   Answer:

$$Y(\Omega) = \sum_{n=-\infty}^{+\infty} y[n] \exp(-in\Omega) \tag{100}$$

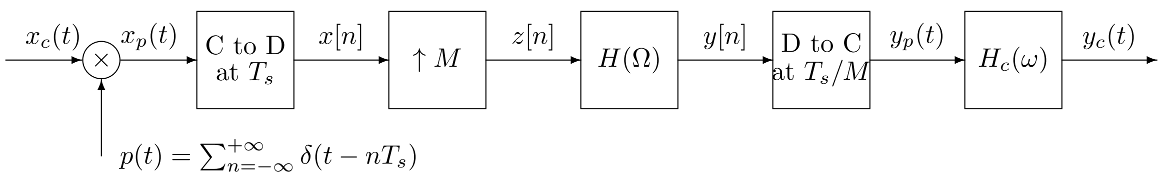
$$
\begin{aligned}
&= \sum_{n=-\infty, n=Ml}^{+\infty} y[n]\exp(-in\Omega) + \sum_{n=-\infty, n\neq Ml}^{+\infty} y[n]\exp(-in\Omega) && (101)\\
&= \sum_{n=-\infty, n=Ml}^{+\infty} x[n/M]\exp(-in\Omega) + \sum_{n=-\infty, n\neq Ml}^{+\infty} 0\exp(-in\Omega) && (102)\\
&= \sum_{l=-\infty}^{+\infty} x[l]\exp(-iMl\Omega) && (103)\\
&= \sum_{l=-\infty}^{+\infty} x[l]\exp(-il(M\Omega)) && (104)\\
&= X(M\Omega). && (105)
\end{aligned}
$$

(b) Suppose that

$$
\begin{aligned}
X_0(\Omega) &= \begin{cases} \bar{X}_0\left(1-|\Omega|/\pi\right), & -\pi \leq \Omega \leq \pi \\ 0, & \text{otherwise} \end{cases} && (106)\\
X(\Omega) &= \sum_{k=-\infty}^{+\infty} X_0(\Omega + k2\pi). && (107)
\end{aligned}
$$

Please plot $X(\Omega)$. Please plot the corresponding $Y(\Omega)$ for the case of $M = 8$.

(c) Please consider the following block diagram:



where

i. $x[n] = x_c(nT_s)$.

ii.

$$
z[n] = \begin{cases} x[n/M], & n - Ml \text{ for some } l \in \mathcal{Z} \\ 0, & \text{otherwise} \end{cases} . \qquad (108)
$$

iii.

$$
\begin{aligned}
H_0(\Omega) &= \begin{cases} 1, & -\pi/M \leq \Omega \leq \pi/M \\ 0, & \text{otherwise} \end{cases} && (109)\\
H(\Omega) &= \sum_{k=-\infty}^{+\infty} H_0(\Omega + k2\pi). && (110)
\end{aligned}
$$

iv. $z[n] \leftrightarrow Z(\Omega)$, $y[n] \leftrightarrow Y(\Omega)$, and $Y(\Omega) = H(\Omega)Z(\Omega)$.

v.

$$
y_p(t) = \sum_{n=-\infty}^{+\infty} y[n]\delta(t - nT_s/M). \qquad (111)
$$

vi. $y_p(t) \leftrightarrow Y_P(\omega)$, $y_c(t) \leftrightarrow Y_c(\omega)$, and $Y(\omega) = H_c(\omega)Y_P(\omega)$.

For the case of

i.

$$
X_c(\omega) = \begin{cases} \bar{X}_c\left(1 - |\omega|/\omega_X\right), & -\omega_X \leq \omega \leq \omega_X \\ 0, & \text{otherwise} \end{cases} . \qquad (112)
$$

ii. $T_s < (1/2)(2\pi/\omega_X)$ and $\bar{X}_c = 1$.

iii. $M = 8$.

please plot $X_c(\Omega)$, $X(\Omega)$, $Z(\Omega)$, and $Y(\Omega)$. What are the constraints on $H_c(\omega)$ such that $y_c(t) = x_c(t)$?

(d) Why do the manufacturers of CD players often use this approach?

(e) What is the impulse response $h[n]$ corresponding to the frequency response $H(\Omega)$? Probably you will agree with the statement that some approximation to this impulse response will be needed in order to build a practical system. Note, however, that in many play-back applications, a delay can be tolerated.