ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Problem Set 7
Due Friday April 15, 2016 at 5:00PM.
Location to turn in: "ECE 2200" box on Phillips Hall 2nd floor

1. A modification of McClellan, Schafer, Yoder, Signal Processing First (1st ed) Problem P-9.17. A continuous-time system is defined by the input/output relation

$$y(t) = \int_{t-2}^{t+2} x(\tau) \mathrm{d}\tau. \tag{83}$$

   (a) Determine the impuluse resonse, $h(t)$, of this system. Hint: Substitute $x(t) = \delta(t)$ and compute $y(t)$. The key issue is to decide the values of t for which the integral is nonzero.

   (b) Use the convolution integral to determine the output of the system when the input is

$$x(t) = u(t+1). \tag{84}$$

   Plot your answer.

   (c) Please check your answer to Problem 1b by computing the same result directly from Eq. 83.

2. Let

$$x[n] = \rho^{|n|}. \tag{85}$$

Assume that $|\rho| < 1$. However, ρ may be complex, i.e., $\rho = re^{j\phi}$. Compute the Discrete Time Fourier Transform of $x[n]$, i.e.,

$$X(\Omega) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\Omega n}. \tag{86}$$

Hint: One possible approach is to use the geometric sum twice, once for non-negative values of n and once for negative values of n. The geometric sum is

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}. \tag{87}$$

3. McClellan, Schafer, Yoder, Signal Processing First (1st ed) Problem P-6.5 with hints. A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] + 2x[n-1] + x[n-2]. \tag{88}$$

   (a) Obtain an expression for the frequency response of this system.

   (b) Sketch the frequency response (magnitude and phase) as a function of frequency.

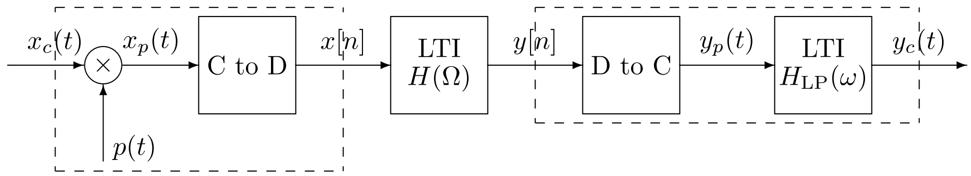
   (c) Determine the output when the input is

$$x[n] = 10 + 4\cos(0.5\pi n + \pi/4). \tag{89}$$

   (d) Determine the output when the input is the unit impulse sequence $\delta[n]$. This may be easier in the time domain.

   (e) Determine the output when the input is the unit-step sequence $u[n]$. This may be easier in the time domain.

4. Consider the block diagram



where

(a)
$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \leftrightarrow P(\omega) = \frac{2\pi}{T_s} \sum_{n=-\infty}^{+\infty} \delta(\omega - \frac{2\pi}{T_s} n). \tag{90}$$

(b)
$$x[n] = x_c(nT_s). \tag{91}$$

(c)
$$y[n] = \frac{1}{2M+1} \sum_{m=-M}^{+M} x[n-m]. \tag{92}$$

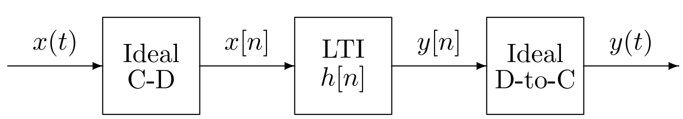
(d)
$$y_p(t) = \sum_{n=-\infty}^{+\infty} y[n]\delta(t - nT_s). \tag{93}$$

(e) $H_{\mathrm{LP}}(\omega)$ is the ideal reconstruction filter which is

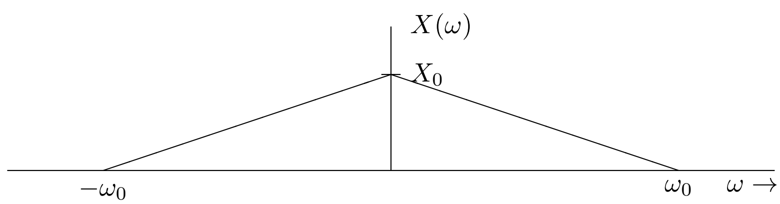
$$H_{\mathrm{LP}}(\omega) = \begin{cases} T_s, & |\omega| \leq \pi/T_s \\ 0, & \text{otherwise} \end{cases} . \tag{94}$$

Assume that $x_c(t) \leftrightarrow X_c(\omega)$ satisfies the Nyquist frequency for this system. On the same ω axis scale, please draw accurate plots of $Y_c(\omega)/X_c(\omega)$ for $M \in \{1, 5, 10\}$. Hopefully you see that we are now in the business of low pass filter design, especially considering that the parts which you actually purchase approximate the two dashed boxes leaving only the central box, which might be a one-loop C program, to be built.

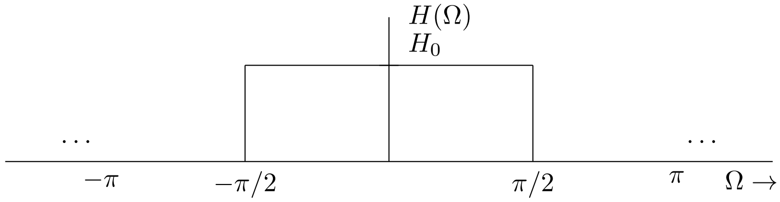
5. A modification of McClellan, Schafer, Yoder, Signal Processing First (1st ed) Problem P-12-15. Consider the system for discrete-time filtering of a continuous-time signal that is shown in the following block diagram:



For both the Ideal C-to-D (continuous to discrete) and Ideal D-to-C (discrete to continuous) blocks the sampling interval is T_s. The various signals have continuous-time (discrete-time) Fourier transforms $X(\omega)$, $X(\Omega)$, $H(\Omega)$, $Y(\Omega)$, and $Y(\omega)$ corresponding to $x(t)$, $x[n]$, $h[n]$, $y[n]$, and $y(t)$, respectively. The continuous-time Fourier transform of the input signal is shown in the following plot:



where $\omega_0 = 80\pi$. One period $(-\pi \leq \Omega \leq \pi)$ of the discrete-time Fourier transform of the impulse response of the LTI system is shown in the following plot:



where $H_0 = 1$.

(a) For this input signal, what is the smallest value of the sampling frequency $\omega_s = 2\pi/T_s$ such that the Fourier transforms of the input and output satisfy the relationship $Y(\omega) = H_{\text{eff}}(\omega)X(\omega)$ where $H_{\text{eff}}(\omega)$ is the effective frequency response of the entire system.

(b) If $f_s = 1/T_s = 100$samples/second, make a carefully labeled plot of $H_{\text{eff}}(\omega)$ and of $Y(\omega)$

(c) What is the smallest sampling rate such that the input signal passes through the lowpass filter unaltered, i.e., what is the minimum $f_s = 1/T_s$ such that $Y(\omega) = X(\omega)$?