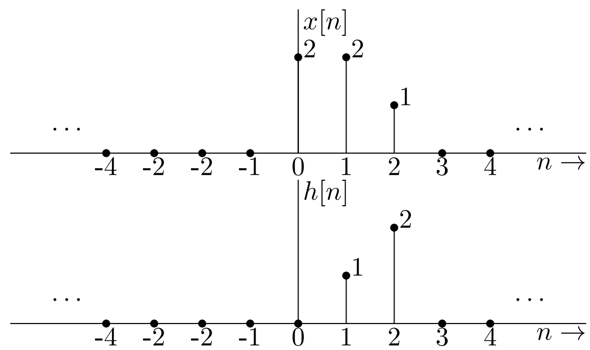
ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Problem Set 6
Due Friday March 25, 2016 at 5:00PM.
Location to turn in: "ECE 2200" box on Phillips Hall 2nd floor

1. Define $x[n]$ and $h[n]$ by



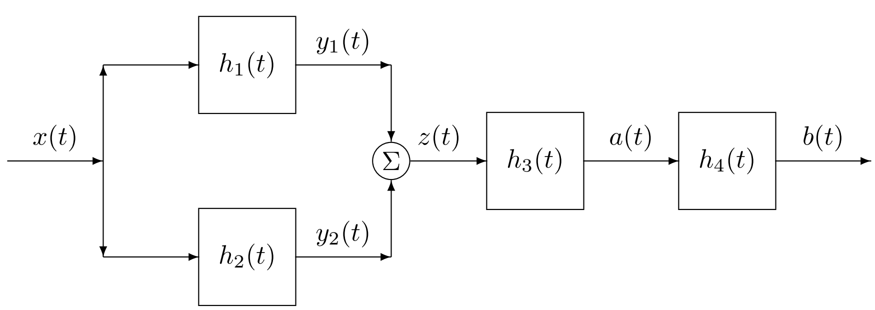
Plot $y[n] = x[n] * h[n]$.

2. Let $x(t)$ be defined by

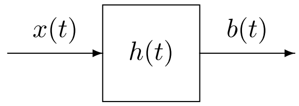
$$x(t) = \begin{cases} x_0, & |t| \le T \\ 0, & \text{otherwise} \end{cases} . \tag{70}$$

Give a formula and a plot for $y(t) = x(t)*x(t)$. Note that this result provides the fourth way to compute the Fourier transform of $y(t)$: (1) by direct integration of the Fourier transform integral, probably using integration by parts, (2) by taking one derivative of $y(t)$ and using the derivative property of Fourier transforms plus the Fourier transform of a rectangular pulse, (3) by taking two derivatives of $y(t)$ and using the derivative property of Fourier transforms plus the Fourier transform of a impulse, and (4) by using the result of this problem and the convolution property of Fourier transforms plus the the Fourier transform of a rectangular pulse.

3. Consider the following interconnection of four LTI systems where each system is described by its impulse response, denoted by $h_i(t)$ for $i \in \{1, 2, 3, 4\}$:



It is not hard, but is tedious, to show that an interconnection of LTI systems is LTI. Assuming this result, consider the system



where $x(t)$ and $b(t)$ are the same signals in the two block diagrams and $h(t)$ is the impulse response of this LTI system. Questions:

(a) Compute $h(t)$ in terms of $h_1(t)$, $h_2(t)$, $h_3(t)$, and $h_4(t)$.

(b) If $h_4(t) * h_1(t) = \delta(t)$, give a simplified formula for $h(t)$.

4. Part of McClellan, Schafer, Yoder, Signal Processing First (1st ed) Problem P-9.2. In order to show that a system has a property, you must show that it has the property for all inputs. However, in order to show that a system lacks a property, all you need to do is give one input for which the property fails.

   Please determine whether the following systems are linear and whether the following systems are time invariant.

   (a) Exponentiation: $x(t)$ is the input and the output is $y(t) = \exp(x(t+2))$.

   (b) Phase modulation: $x(t)$ is the input and the output is $y(t) = \cos(\omega_c t + x(t))$.

   (c) Amplitude modulation: $x(t)$ is the input and the output is $y(t) = (A + x(t))\cos(\omega_c t)$.

   (d) Take the even part: $x(t)$ is the input and the output is $y(t) = (1/2)(x(t) + x(-t))$.

5. Small modification of McClellan, Schafer, Yoder, Signal Processing First (1st ed) Problem P-9.8. The signal $u(t)$ is the unit step function: $u(t) = 0$ for $t < 0$ and $= 1$ for $t \geq 0$.

   (a) Use the convolution integral to determine the convolution of two exponential signals with the same time constant, specifically,

   $$y(t) = [\exp(-at)u(t))] * [\exp(-at)u(t))] \tag{71}$$

   with $a > 0$.

   (b) A standard result is that if

   $$x(t) = \exp(-at)u(t) \tag{72}$$
   $$h(t) = \exp(-bt)u(t) \tag{73}$$

   with $a \neq b$ and both $a > 0$ and $b > 0$ then

   $$y(t) = x(t) * h(t) = \frac{1}{b-a}\left(\exp(-at)u(t) - \exp(-bt)u(t)\right). \tag{74}$$

   Can you recover your answer to Problem 5a by using L'Hospital's rule on this result?