ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Problem Set 5
Due Friday March 18, 2016 at 5:00PM.
Location to turn in: "ECE 2200" box on Phillips Hall 2nd floor

1. McClellan, Schafer, Yoder Problem P-5.5. Consider a system defined by

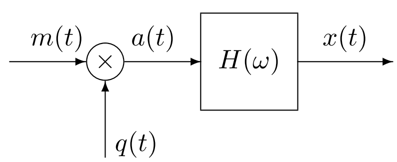
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]. \tag{57}$$

The support of a signal is the set of values of the independent variable (e.g., $t$ or $n$) such that the signal is nonzero.

   (a) Suppose that the input $x[n]$ is nonzero only for $0 \le n \le N-1$, i.e., it has a support of $N$ samples. Show that $y[n]$ is nonzero at most over a finite interval of the form $0 \le n \le P-1$. Determine $P$ and the support of $y[n]$ in terms of $M$ and $N$.

   (b) Suppose that the input $x[n]$ is nonzero only for $N_1 \le n \le N_2$. What is the support of $x[n]$? Show that $y[n]$ is nonzero at most over a finite interval of the form $N_3 \le n \le N_4$. Determine $N_3$ and $N_4$ and the support of $y[n]$ in terms of $N_1$, $N_2$, and $M$.

2. This problem concerns an alternative method of building a double side band suppressed carrier modulator. Consider the block diagram




where

$$q(t) = \sum_{n=-\infty}^{+\infty} q_0(t - nT_c) \tag{58}$$

$$q_0(t) = \begin{cases} 1, & |t| < \tau/2 \\ 0, & \text{otherwise} \end{cases} \tag{59}$$

and designing $H(\omega)$ is a part of the problem. It is important that $\tau < T_c$. Define $f_c = 1/T_c$. The continuous-time Fourier transform of $q_0(t)$ is denoted by $Q_0(\omega)$ and has the formula

$$Q_0(\omega) = \int_{t=-\infty}^{+\infty} q_0(t) \exp(-j\omega t) \mathrm{d}t \tag{60}$$

$$= \frac{1}{\omega} 2 \sin(\omega\tau/2) \tag{61}$$

$$= \tau \operatorname{sinc}(\omega\tau/(2\pi)) \tag{62}$$

which has zeros when

$$\omega = n2\pi/\tau \quad (n \in \mathcal{Z}, n \neq 0). \tag{63}$$

   (a) Sketch $q(t)$ for $0 \le t \le 3T_c$.

(b) Compute the Fourier series coefficients, denoted by $a_k$, of the periodic signal $q(t)$. Please write your answer in terms of $Q_0(\omega)$.

(c) Let $Q(\omega)$ be the continuous-time Fourier transform of $q(t)$. From the Fourier series coefficients $a_k$, please compute the Fourier transform of the periodic signal $q(t)$. Please write your answer in terms of $Q_0(\omega)$.

(d) Let $m(t)$ have continuous-time Fourier transform $M(\omega)$ which is defined by

$$M(\omega) = \begin{cases} M_0(1 - |\omega|/(2\pi f_m)), & |\omega| \leq 2\pi f_m \\ 0, & \text{otherwise} \end{cases} \tag{64}$$

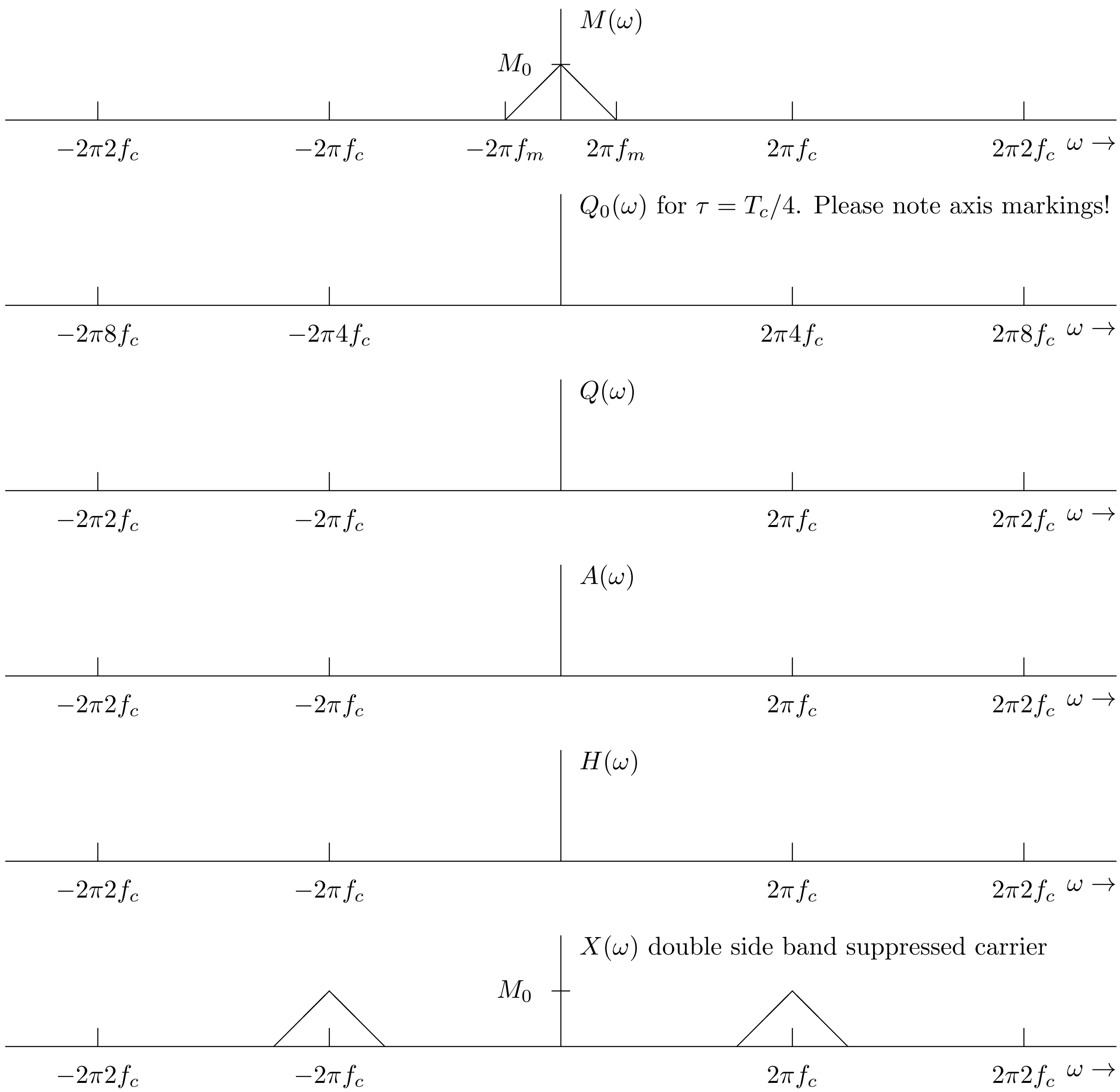
where $f_m \ll f_c$. Let $a(t)$ amd $x(t)$ have continuous-time Fourier transforms $A(\omega)$ and $X(\omega)$, respectively. On the following graphs, please plot the following Fourier transforms:

i. $Q_0(\omega)$ for the case where $\tau = T_c/4$.

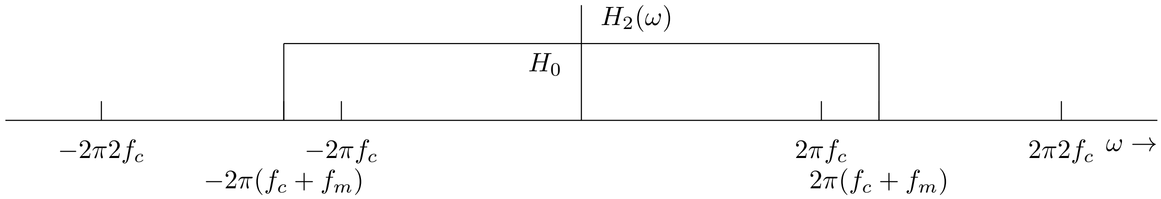
ii. $Q(\omega)$.

iii. $A(\omega)$.

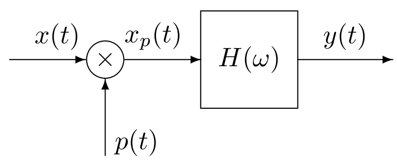
iv. $H(\omega)$ such that $X(\omega)$ is as shown in the plot provided below which shows that $x(t)$ is the double side band suppressed carrier signal corresponding to $m(t)$.



(e) Now consider replacing $q(t)$ by $q^{\text{new}}(t) = q(t) - q_*$ where $q_*$ is a constant. What is the continuous-time Fourier transform of $q^{\text{new}}(t)$? Can you choose a value of $q_*$ such that you can replace the filter $H(\omega)$ of Part 2d by the filter $H_2(\omega)$ shown below without altering $X(\omega)$? What is the value of $H_0$?




3. Consider the block diagram




where

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \tag{65}$$

and $H(\omega)$ is one of the following three possibilities:

(a) The ideal reconstruction filter with frequency response

$$H_1(\omega) = \begin{cases} T_s, & |\omega| \leq \pi/T_s \\ 0, & \text{otherwise} \end{cases} . \tag{66}$$

(b) The sample-and-hold reconstruction filter with impulse response

$$h_2(t) = \begin{cases} 1, & 0 \leq t < T_s \\ 0, & \text{otherwise} \end{cases} \tag{67}$$

and frequency response $H_2(\omega)$.

(c) The linear-interpolation reconstruction filter with impulse response

$$h_3(t) = \begin{cases} 1 - |t|/T_s & |t| \leq T_s \\ 0, & \text{otherwise} \end{cases} \tag{68}$$

and frequency response $H_3(\omega)$.

Let $x(t) = \sin\left(\frac{2\pi}{T_0} t\right)$. Questions:

(a) With $T_s = (1/4)T_0$ make a careful plot of $y(t)$ when $H(\omega) = H_1(\omega)$ for at least the range $0 \leq t \leq T_0$.

(b) With $T_s = (1/4)T_0$ make a careful plot of $y(t)$ when $H(\omega) = H_2(\omega)$ for at least the range $0 \leq t \leq T_0$.

(c) With $T_s = (1/4)T_0$ make a careful plot of $y(t)$ when $H(\omega) = H_3(\omega)$ for at least the range $0 \leq t \leq T_0$.

(d) With $T_s = (3/4)T_0$ make a careful plot of $y(t)$ when $H(\omega) = H_1(\omega)$ for at least the range $0 \leq t \leq 3T_0$.

(e) With $T_s = (3/4)T_0$ make a careful plot of $y(t)$ when $H(\omega) = H_2(\omega)$ for at least the range $0 \leq t \leq 3T_0$.

(f) With $T_s = (3/4)T_0$ make a careful plot of $y(t)$ when $H(\omega) = H_3(\omega)$ for at least the range $0 \leq t \leq 3T_0$.

4. Let a system with input $x[n]$ and output $y[n]$ be defined by

$$y[n] = \frac{1}{3}\left(x[n-1] + x[n] + x[n+1]\right). \tag{69}$$

What is the impulse response of this system? In other words, if $x[n] = \delta[n]$, what is $y[n]$? Traditionally, the impulse response would be named $h[n]$.