ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Problem Set 4
Due Friday March 11, 2016 at 5:00PM.
Location to turn in: "ECE 2200" box on Phillips Hall 2nd floor

1. McClellan, Schafer, Yoder Problem P-4.8. The Fourier transform gives the frequency content of a signal.

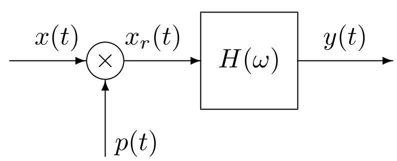
   (a) Draw a sketch of the Fourier transform of

   $$x(t) = \cos(50\pi t)\sin(700\pi t). \tag{50}$$

   Label the frequencies and complex amplitudes of each component.

   (b) Determine the minimum sampling rate that can be used to sample $x(t)$ without aliasing for any of the components.

2. Consider the block diagram



   where

   $$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \tag{51}$$

   $$H(\omega) = \begin{cases} T_s, & |\omega| < \pi/T_s \\ 0, & \text{otherwise} \end{cases}. \tag{52}$$

   Let $x(t) = \cos(2\pi f_0 t)$. The following questions can probably be more easily solved in the frequency domain rather than the time domain.

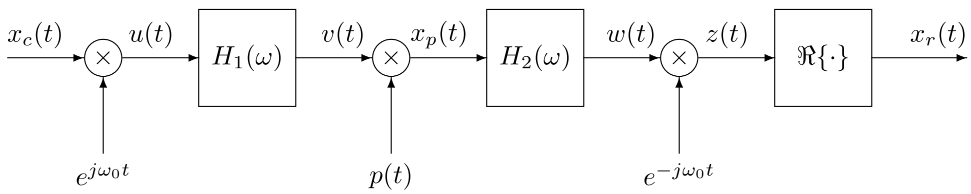
   (a) What is $y(t)$ when $f_0 = (1/4)(1/T_s)$? Does this sampling and reconstruction system make $y(t)$ equal to $x(t)$?

   (b) What is $y(t)$ when $f_0 = (3/4)(1/T_s)$? Does this sampling and reconstruction system make $y(t)$ equal to $x(t)$?

   (c) Give two different values of f_0 such that $y(t) = \cos(2\pi f_1 t)$ where $f_1 = (1/8)(1/T_s)$.

3. This problem considers the situation where you need to sample a bandpass signal, i.e., a signal for which the Fourier transform is non zero only in a limited range of frequencies that does not include 0. You can always use the Nyquist sampling rate, i.e., twice the highest frequency in the signal, but that seems wasteful if the Fourier transform is non zero over only a small range of frequencies but the highest frequency is very large. The approach taken here is a combination of a DSB-SC communication system and a sampling system. However, instead of multiplying by $\sqrt{2}\cos(2\pi f_c t)$, the block diagram includes multiplication by $\exp(j\omega_0 t)$ and $\exp(-j\omega_0 t)$. Because of Euler's formula, this means that both multiplication by sin and by cos is included. Note that using $\exp(\pm j\omega_0 t)$ implies that all of the intermediate signals are complex valued.

   Consider the following block diagram:



where

(a) $x_c(t)$ is real.

(b) $X_c(\omega) = 0$ except for $\omega_1 \leq |\omega| \leq \omega_2$.

(c) $\omega_0 = (\omega_1 + \omega_2)/2$.

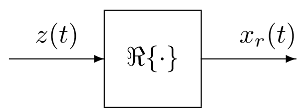
(d) $H_1(\omega) = 1$ if $|\omega| \leq (\omega_2 - \omega_1)/2$ and $H_1(\omega) = 0$ otherwise.

(e) $p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - n\frac{2\pi}{\omega_s})$.

(f) $\omega_s = 2(\omega_2 - \omega_1)$. $T_s = 2\pi/\omega_s$.

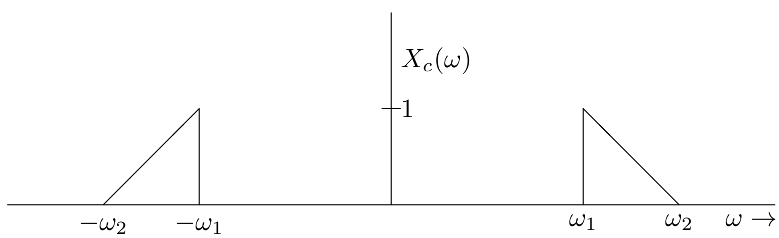
(g) $H_2(\omega) = 2T_s$ if $|\omega| \leq (\omega_2 - \omega_1)/2$ and $H_2(\omega) = 0$ otherwise.

(h)



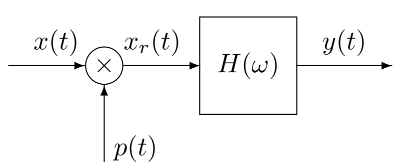
means that $x_r(t) = \Re\{z(t)\}$ which implies that $X_r(\omega) = \frac{1}{2}[Z(\omega) + Z^*(-\omega)]$. This is an important hint!

Let $X_c(\omega)$ be as shown in the following graph:



Plot the Fourier transforms of all the signals indicated on the block diagram. Does this sampling and reconstruction system make $x_r(t)$ equal to $x_c(t)$? Could the sampling rate ω_s be reduced and still have a successful system?

4. Consider the block diagram



where

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \tag{53}$$

Suppose that $x(t) = \exp(-\alpha t)u(t)$ where $\alpha \in \mathbb{R}$ and $\alpha > 0$ and $u(\cdot)$ is the unit step function. Consider the following different $H(\omega)$ (two of which are specified in the time domain):

$$H_1(\omega) = \begin{cases} T_s, & |\omega| < \pi/T_s \\ 0, & \text{otherwise} \end{cases} \tag{54}$$

$$h_2(t) = \begin{cases} 1, & 0 \leq t \leq T_s \\ 0, & \text{otherwise} \end{cases} \tag{55}$$

$$h_3(t) = \begin{cases} 1 - |t|/T_s, & |t| \leq T_s \\ 0, & \text{otherwise} \end{cases} . \tag{56}$$

Plot $Y(\omega)$, the Fourier transform of $y(t)$ for each of the three choices of filter when $\alpha = (1/10)(1/T_s)$ and when $\alpha = 1/T_s$.