ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Problem Set 3
Due Friday March 4, 2016 at 5:00PM.
Location to turn in: "ECE 2200" box on Phillips Hall 2nd floor

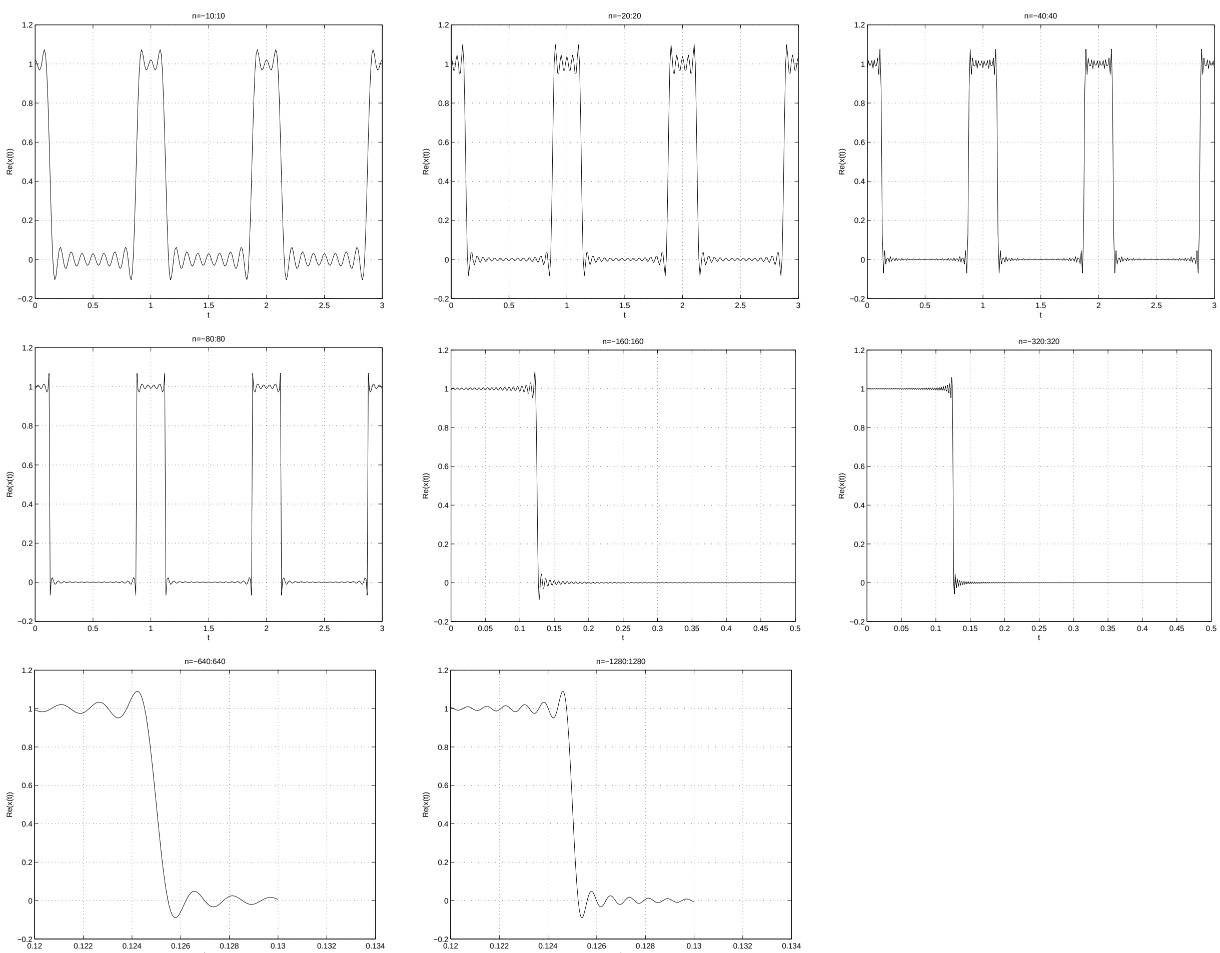
1. Consider a square wave signal denoted by $x(t)$ of amplitude $A = 1$ and period $T = 1$ which, within the time interval $[-T/2, +T/2]$ is nonzero only in the time interval $[-\tau/2, +\tau/2]$ where $\tau = 0.25$. In class we computed the Fourier series coefficients. The reconstruction equation is

$$\sum_{n=-\infty}^{+\infty} c_n \exp\left(j\frac{2\pi}{T}nt\right). \tag{42}$$

Suppose the sum in the reconstruction equation is symmetrically truncated to $\pm N$, i.e.,

$$\sum_{n=-N}^{+N} c_n \exp\left(j\frac{2\pi}{T}nt\right). \tag{43}$$

The resulting reconstructed versions of $x(t)$ are shown in the following plots where the time axis is successively magnified:

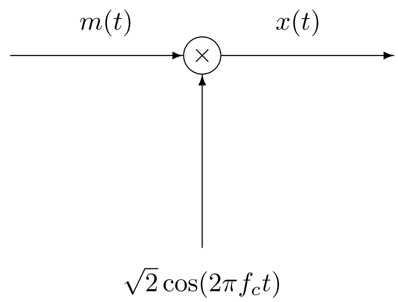


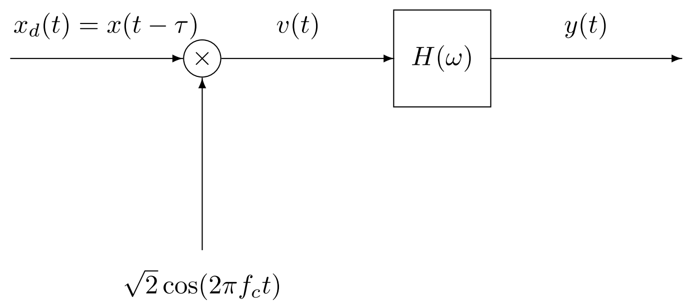
Questions:

(a) What do you see that is unusual about the pattern of oscilations in the reconstruction just before and just after the discontinuity at $t = 0.125$?

(b) If you were going to prove a theorem about the value of the reconstruction at $t = 0.125$, what value would you use? How is this related the the values just before and just after $t = 0.125$?

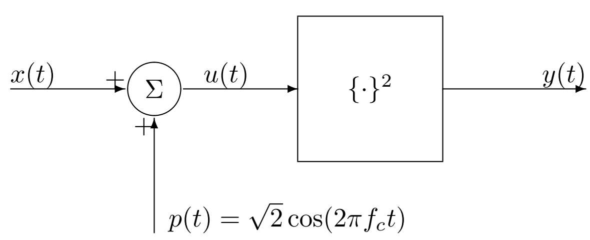
2. "DSB-SC" is an abbreviation for "double side band-suppressed carrier". This problem concerns the effect of a transmission delay between the transmitter and the receiver. Consider the following pair of block diagrams which are the modulator and demodulator for a DSB-SC system with a transmission time delay (denoted by τ):





The message $m(t)$ is bandlimited: $M(\omega) = 0$ for $|\omega| > W$. The filter $H(f)$ is an ideal low pass filter: $H(\omega) = 1$ for $|\omega| \le W$ and $H(\omega) = 0$ for $|\omega| > W$. Give a formula for $y(t)$ in terms of $m(t)$, f_c, and τ. Hint: first compute $Y(\omega)$.

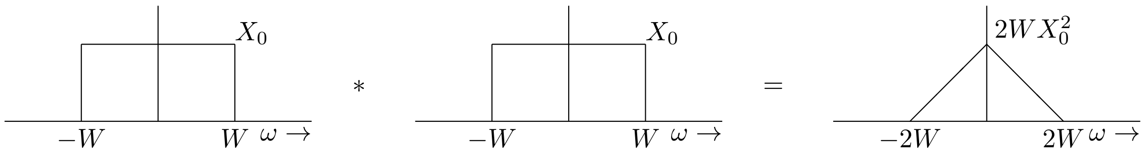
3. This problem concerns how to build a DSB-SC communication system for less money by avoiding building an expensive multiplier and, instead, using a less expensive squarer. Consider the following block diagram:



In the block diagram, the block $\{\cdot\}^2$ means that $y(t) = [u(t)]^2$. Let $X(\omega)$, the Fourier transform of $x(t)$, be

$$X(\omega) = \begin{cases} X_0, & |\omega| \le W \\ 0, & \text{otherwise} \end{cases}.$$

Please note that



(a) Plot $Y(\omega)$, the Fourier transform of $y(t)$.

(b) Plot the frequency response $H(\omega)$ of a filter such that when this filter is applied to $y(t)$, the resulting signal is DSB-SC.

(c) Does the filter $H(\omega)$ in Part 3b work only for this specific signal $x(t) \leftrightarrow X(f)$ or does it work for any signal $x(t) \leftrightarrow X(\omega)$ that is bandlimited to frequency W?

4. A fairly detailed list of Fourier transforms is given on the formula sheet for the exams. Signal Processing First P-11.3(a-c). For each of the following Fourier transforms, use a table of Fourier transform pairs to find $x(t)$:

(a)
$$X(\omega) = \frac{j\omega}{\alpha + j\omega} \exp(-j\omega\tau) \tag{44}$$

(b)
$$X(\omega) = 2 + 2\cos\omega \tag{45}$$

(c)
$$X(\omega) = \frac{1}{1 + j\omega} - \frac{1}{2 + j\omega} \tag{46}$$

5. Signal Processing First P-11.9.

(a) Prove that for a real $h(t)$, the magnitude $|H(\omega)|$ is an even function and the phase $\angle H(\omega)$ is an odd function, i.e.,

$$|H(\omega)| = |H(-\omega)| \tag{47}$$
$$\angle H(\omega) = -\angle H(-\omega). \tag{48}$$

(b) What is the result analogous to Problem 5a when $h(t)$ is purely imaginary?

6. Signal Processing First P-11.15 with small modifications by Doerschuk. Let $x(t)$ be a triangular pulse defined by

$$x(t) = \begin{cases} 1 - |t|/\tau, & |t| < \tau \\ 0, & \text{otherwise} \end{cases} . \tag{49}$$

(a) Please plot $x(t)$.

(b) Please compute the Fourier transform of $x(t)$, which is denoted by $X(\omega)$. Please perform this calculation by taking the derivative of $x(t)$. Hints:

i. The derivative should be the sum of two pulses.

ii. Use the linearity property and the delay property to write the Fourier transform of the derivative in terms of the Fourier transform of a standard rectangular pulse.

iii. Use the derivative property to compute $X(\omega)$.

(c) Repeat the calculation of Problem 6b by taking the second derivative of $x(t)$. Hints:

i. The second derivative should be the sum of three impulses.

ii. Use the linearity property and the delay property to write the Fourier transform of the second derivative in terms of the Fourier transform of an impulse.

iii. Use the derivative property twice to compute $X(\omega)$.