ECE 2200 / Spring 2016 / Problem Set 2 / Solutions

1

P-3.1 (SP-1) P-3.2 (DSP-2)

$$x(t) = 10\cos\left(800\pi t + \frac{\pi}{4}\right) + 7\cos\left(1200\pi t - \frac{\pi}{3}\right) - 3\cos(1600\pi t)$$

(b) Need to find the period T_0.

$$\left.\begin{aligned} 800\pi T_0 &= n_1 2\pi \\ 1200\pi T_0 &= n_2 2\pi \\ 1600\pi T_0 &= n_3 2\pi \end{aligned}\right\} \iff \left\{\begin{aligned} 400 T_0 &= n_1 \\ 600 T_0 &= n_2 \\ 800 T_0 &= n_3 \end{aligned}\right\} \iff T_0 = \frac{n_1}{400} = \frac{n_2}{600} = \frac{n_3}{800}$$

$400 = 2 * 200$

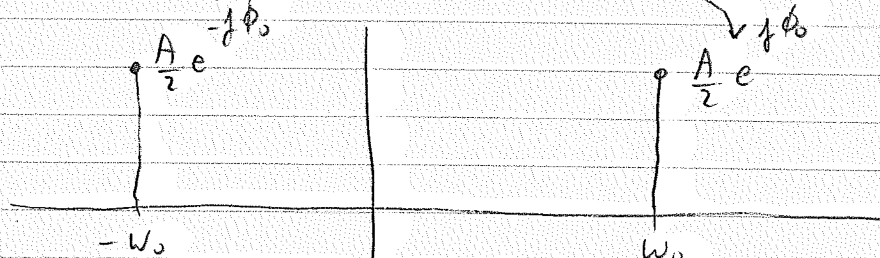
$600 = 3 * 200$

$800 = 4 * 200$

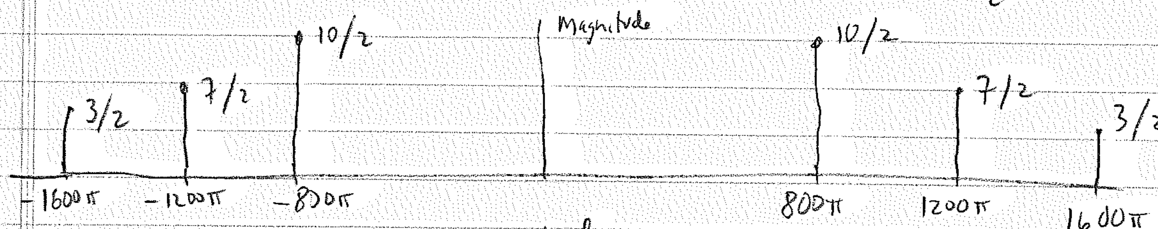
$T_0 = \frac{1}{200}$ works with $n_1 = 2$, $n_2 = 3$, $n_3 = 4$

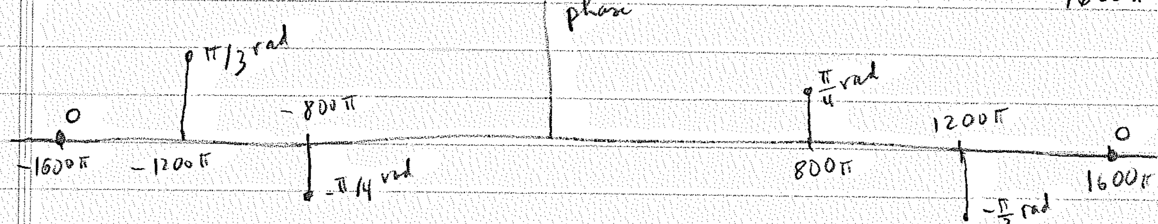
(a) For any A, ω_0, ϕ_0, the signal $A\cos(\omega_0 t + \phi_0)$ has a spectrum that you can determine by inspection from Euler's formula:

$$A\cos(\omega_0 t + \phi_0) = \underbrace{\frac{A}{2} e^{j(\omega_0 t + \phi_0)}} + \underbrace{\frac{A}{2} e^{-j(\omega_0 t + \phi_0)}}$$

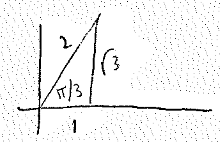


to do a magnitude & phase pair of plots or to do a real & imaginary parts pair of plots is just complex arithmetic on $\frac{A}{2} e^{\pm j\phi_0}$





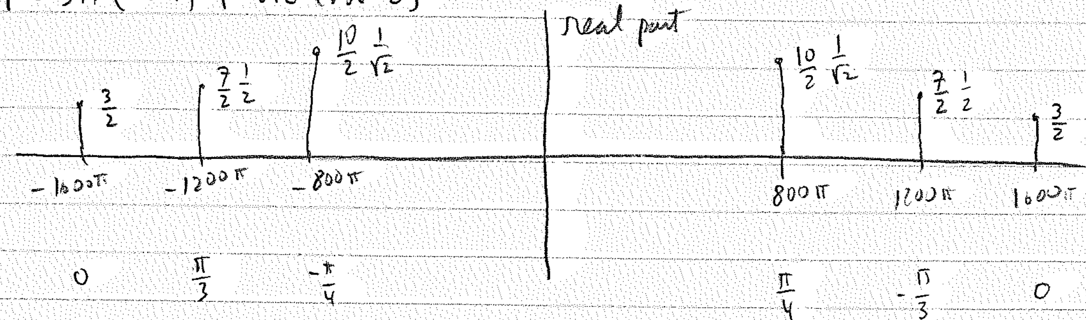
$\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

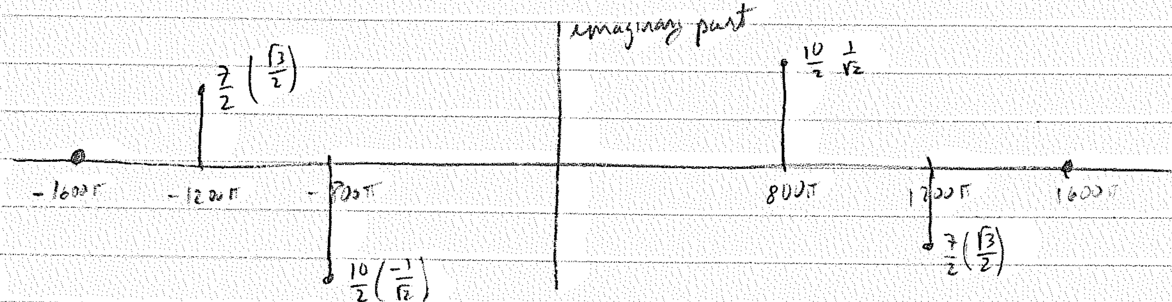


$\cos\frac{\pi}{3} = \frac{1}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

P - 3.1 (SP-1) P-3.2 (DSP-2)





(c) $y(t) = x(t) + 5\cos\left(1000\pi t + \frac{\pi}{2}\right)$

Now there is a fourth equation for determining the period:

$1000\pi T_0 = n_4 2\pi \iff 500 T_0 = n_4 \iff T_0 = \frac{n_4}{500}$

$400 = 4 \times 100$

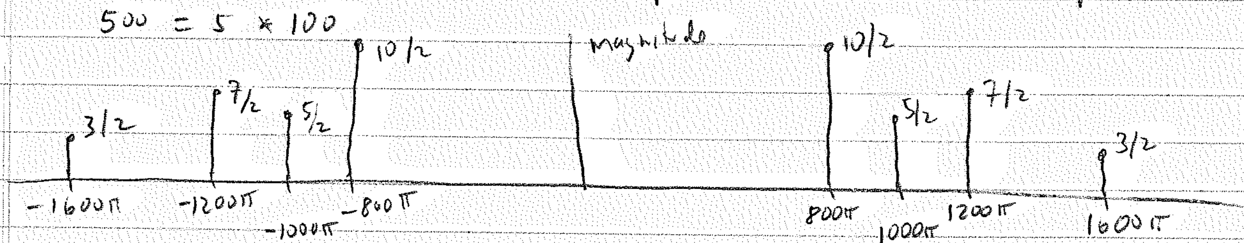
$600 = 6 \times 100$

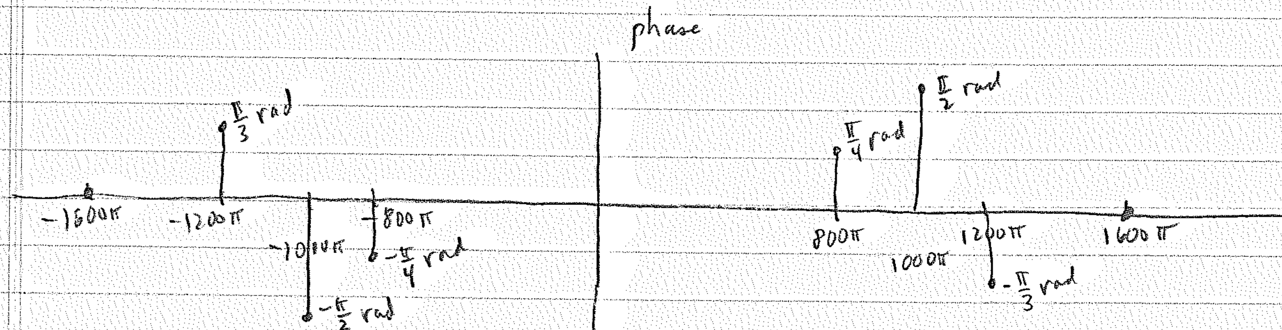
$800 = 8 \times 100$

$500 = 5 \times 100$

$T_0 = \frac{1}{100}$ works with $n_1 = 4$, $n_2 = 6$, $n_3 = 8$, $n_4 = 5$

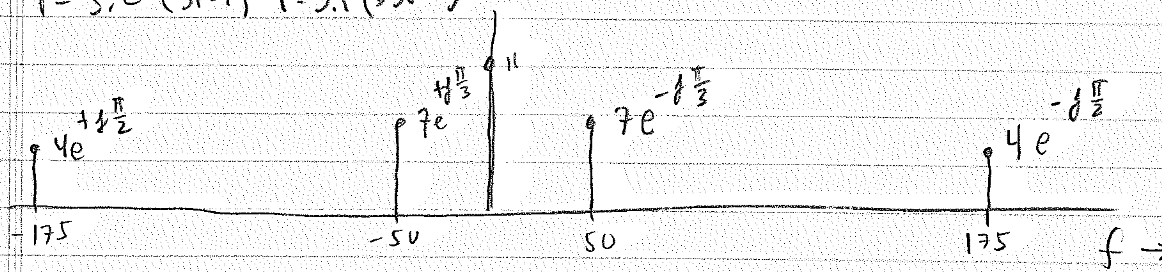
The new period is twice the old period.





Similar for real & imaginary parts

P-3.2 (SP-1) P-3.1 (DSP-2)



a) $x(t) = 11 + (7)(2)\cos\left(2\pi 50t - \frac{\pi}{3}\right) + (4)(2)\cos\left(2\pi 175t - \frac{\pi}{2}\right)$

I write "2π" because the axis is labeled "f" not "ω"

b) Is $x(t)$ periodic?

11 is periodic with any period you chose to use.

So the issue is the other two cosines.

Need

$$\left\{\begin{array}{l} 2\pi 50 T_0 = n_1 2\pi \\ 2\pi 175 T_0 = n_2 2\pi \end{array}\right\} \Longleftrightarrow \left\{\begin{array}{l} 50 T_0 = n_1 \\ 175 T_0 = n_2 \end{array}\right\} \Longleftrightarrow T_0 = \frac{n_1}{50} = \frac{n_2}{175}$$

$50 = 2 \times 25$ $\quad T_0 = \frac{1}{25}$ works with $n_1 = 2$, $n_2 = 7$

$175 = 7 \times 25$

Fundamental frequency is $f_0 = \frac{1}{T_0} = 25$ Hz

$$\omega_0 = \frac{2\pi}{T_0} = (2\pi)(25) \text{ rad/s}$$

c) negative frequencies are necessary in order to have real time functions.

For Fourier series, if the time function is real then $c_n = c_{-n}^*$

" " transforms, " " " " " " " $X(\omega) = X^*(-\omega)$

P- 3.17 (SP-1) P-3.21 (DSP-2)

Chirp signals

$\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time increases from $t=0$ to $t=T_2$

$x(t) = A\cos(\alpha t^2 + \beta t + \phi) = \cos(\Psi(t))$ where $\Psi(t) = \alpha t^2 + \beta t + \phi$

Instantaneous frequency is defined to be

$$\omega_i(t) = \frac{d}{dt}\Psi(t)$$

(a) $\omega_i(t) = \frac{d}{dt}\Psi(t) = \frac{d}{dt}(\alpha t^2 + \beta t + \phi) = 2\alpha t + \beta$

$\omega_i(0) = 2\alpha 0 + \beta = \beta$

$\omega_i(T_2) = 2\alpha T_2 + \beta$

(b) $x(t) = Re\{e^{j(40t^2 + 27t + 13)}\} = \cos(40t^2 + 27t + 13)$

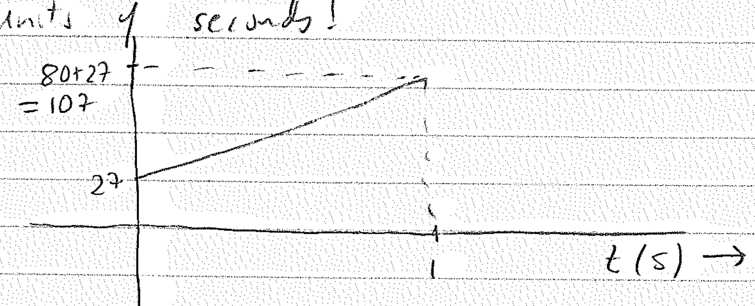
$\Rightarrow \Psi(t) = 40t^2 + 27t + 13$

$\Rightarrow \alpha = 40, \beta = 27, \phi = 13$

$\Rightarrow \omega_i(t) = 2\alpha t + \beta = 80t + 27$

the text does not specify any units for these constants!

(c) for the signal of part (b), plot ω_i. I will assume that the constants in part (b) have values appropriate for units of seconds!



answers:

a - 3
b - 5
c - 1
d - 2
e - 4

P-3.19 (SP-1) P-3.26 (DSP-2)
but with different numbers

(A) (a) (b) (c) time functions look like $x(t) = A + B\cos(2\pi f_0 t + \phi)$

So (a)(b)(c) match with (1)(3)(5) in some order.

(B) (b) has 3 periods in 2 sec $\Rightarrow 3T_0 = 2 \Rightarrow T_0 = \frac{2}{3} \Rightarrow f_0 = \frac{3}{2} = 1.5$ Hz

(b) has no DC level

both of these facts agree with $\boxed{(b) \longleftrightarrow (5)}$

(C) (1) and (3) are identical except for phase.

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$x_1(t) = 2 + (1.5)(2)\cos(2\pi 1.2t + \tfrac{\pi}{2})$$

$$= 2 + 3\left[\cos 2\pi 1.2t \underbrace{\cos\tfrac{\pi}{2}}_{=0} - \sin 2\pi 1.2t \underbrace{\sin\tfrac{\pi}{2}}_{=1}\right]$$

$$= 2 - 3\sin 2\pi 1.2t$$

Sine increases starting at $t=0$ so $x_1(t)$ decreases starting from $t=0$

$$x_3(t) = 2 + (1.5)(2)\cos(2\pi 1.2t - \tfrac{\pi}{4})$$

$$= 2 + 3\cos(2\pi 1.2t - \pi/4)$$

(a) increases at $t=0$ so $\boxed{(c) \longleftrightarrow (1)}$

this leaves $\boxed{(a) \longleftrightarrow (3)}$

(D) (e) has 2 periods in 5 sec ($t=-2$ to $t=+3$ is easy to see)

$$\Rightarrow 2T_0 = 5 \Rightarrow T_0 = \frac{5}{2} \Rightarrow f_0 = \frac{2}{5} = .4 \text{ Hz}$$

$$(4): \begin{Bmatrix} 2\pi 1.2 T_0 = n_1 2\pi \\ 2\pi\, 2\, T_0 = n_2 2\pi \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} 1.2 T_0 = n_1 \\ 2 T_0 = n_2 \end{Bmatrix} \Leftrightarrow T_0 = \frac{n_1}{1.2} = \frac{n_2}{2}$$

$T_0 = \frac{5}{2}$, $n_1 = 3$, $n_2 = 5$ works. Therefore $\boxed{(e) \longleftrightarrow (4)}$

(E) the remaining pair is $\boxed{(d) \longleftrightarrow (2)}$ (Note also that (d) is roughly lower frequency than (e) which matches (d) $\leftrightarrow$ (2) and (e) $\leftrightarrow$ (4)).

(5) $x(t)$ real

(a) $(\sim i)$ need $c_n = c_{-n}^*$

(b) $c_n = \rho^{|n|} \quad \rho \in \mathbb{R},\ |\rho| < 1$

$c_{-n}^* = \left(\rho^{|-n|}\right)^* = \left(\rho^{|n|}\right)^* = \rho^{|n|} = c_n \longrightarrow$ satisfies the constraint

(c) (a) yes

(b) no — $c_{-0}^* = c_0^* \Rightarrow c_0$ is real.

but $c_0 = 2+7j$ which is not real

(c) no — $c_1 = c_{-1}$ and $c_2 = c_{-2}$

(d) no — c_0 is complex, $c_1 = c_{-1}$, $c_2 = c_{-2}$

(e) yes

(6) $x(t)$ even

(a)(iv) need $c_n = c_{-n}$

(b) $c_n = \rho^{|n|}$ $\rho \in \mathbb{R}$, $|\rho| < 1$

$c_{-n} = \rho^{|-n|} = \rho^{|n|} = c_n$ satisfies the constraint

a) no $c_1 \neq c_{-1}$ $c_2 \neq c_{-2}$

b) no $c_1 \neq c_{-1}$ $c_2 \neq c_{-2}$

c) yes

d) yes

e) yes

Notice that there is no requirement on c_0 since

$$c_0 = c_{-0}$$

does not constrain c_0 in any way.

(7) $x(t)$ real and even

a) need both

$$\begin{cases} c_n = c_{-n}^* \\ c_n = c_{-n} \end{cases}$$

$$\Longrightarrow \quad c_{-n}^* = c_n = c_{-n}$$

$$\Longrightarrow \quad c_{-n}^* = c_{-n}$$

$$\Longleftrightarrow \quad c_n^* = c_n$$

$$\Longleftrightarrow \quad c_n \in \mathbb{R}$$

b) $c_n = \rho^{|n|} \quad \rho \in \mathbb{R},\ |\rho| < 1 \Rightarrow c_n \in \mathbb{R} \Rightarrow$ OK

(a) no

(b) no

(c) no

(d) no

(e) yes