ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Problem Set 2
Due Friday February 19, 2016 at 5:00PM.
Location to turn in: "ECE 2200" box on Phillips Hall 2nd floor

1. Signal Processing First P-3.1. DSP First (2nd edition) P-3.2. A signal is composed of sinusoids is given by the equation

$$x(t) = 10\cos(800\pi t + \pi/4) + 7\cos(1200\pi t - \pi/3) - 3\cos(1600\pi t). \tag{1}$$

   (a) Sketch the spectrum of this signal, indicating the complex amplitude of each frequency component. Make separate plots for real/imaginary or magnitude/phase of the complex amplitudes at each frequency.

   (b) Is $x(t)$ periodic? If so, what is the period?

   (c) Now consider a new signal defined as $y(t) = x(t) + 5\cos(1000\pi t + \pi/2)$. How is the spectrum changed? Is $y(t)$ periodic? If so, what is the period?

2. Signal Processing First P-3.2. DSP First (2nd edition) P-3.1. A signal $x(t)$ has the two-sided spectrum representation shown in Fig. P-3.1.

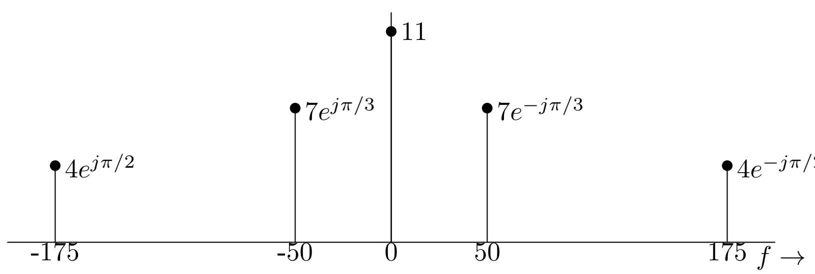

Figure P-3.1

   (a) Write an equation for $x(t)$ as a sum of cosines.

   (b) Is $x(t)$ a periodic signal? If so, determine its fundamental period and its fundamental frequency.

   (c) Explain why negative frequencies are needed in the spectrum.

3. Signal Processing First P-3.17. DSP First (2nd edition) P-3.21. Small changes by Doerschuk. A chirp signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. The general formula for a chirp is

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) = A\cos(\psi(t)) \tag{2}$$

where

$$\psi(t) = \alpha t^2 + \beta t + \phi \tag{3}$$

and the units of β are inverse seconds and the units of α are inverse seconds squared. The derivative of $\psi(t)$ with respect to t is the *instantaneous frequency* (in rad/s), which is also the frequency heard if the frequencies are in the audible range.

   (a) For the chirp in Eq. 2, determine formulas for the beginning ($t = 0$) instantaneous frequency (ω_1^{inst}) and the ending ($t = T_2$) instantaneous frequency (ω_2^{inst}) in terms of α, β, and T_2.

(b) For the chirp signal

$$x(t) = \Re\left\{e^{j(40t^2+27t+13}\right\} \tag{4}$$

derive a formula for the instantaneous frequency versus time.

(c) Make a plot of the instantaneous frequency (in Hz) versus time over $0 \le t \le 1$s for the signal defined in Eq. 4.

4. Signal Processing First P-3.19. DSP First (2nd edition) P-3.26 but with different numbers. The plots in Fig. P-3.19 show waveforms on the left and spectra on the right. Match the waveform letter with its corresponding spectrum number. In each case, write the formula for the signal as a sum of sinusoids.

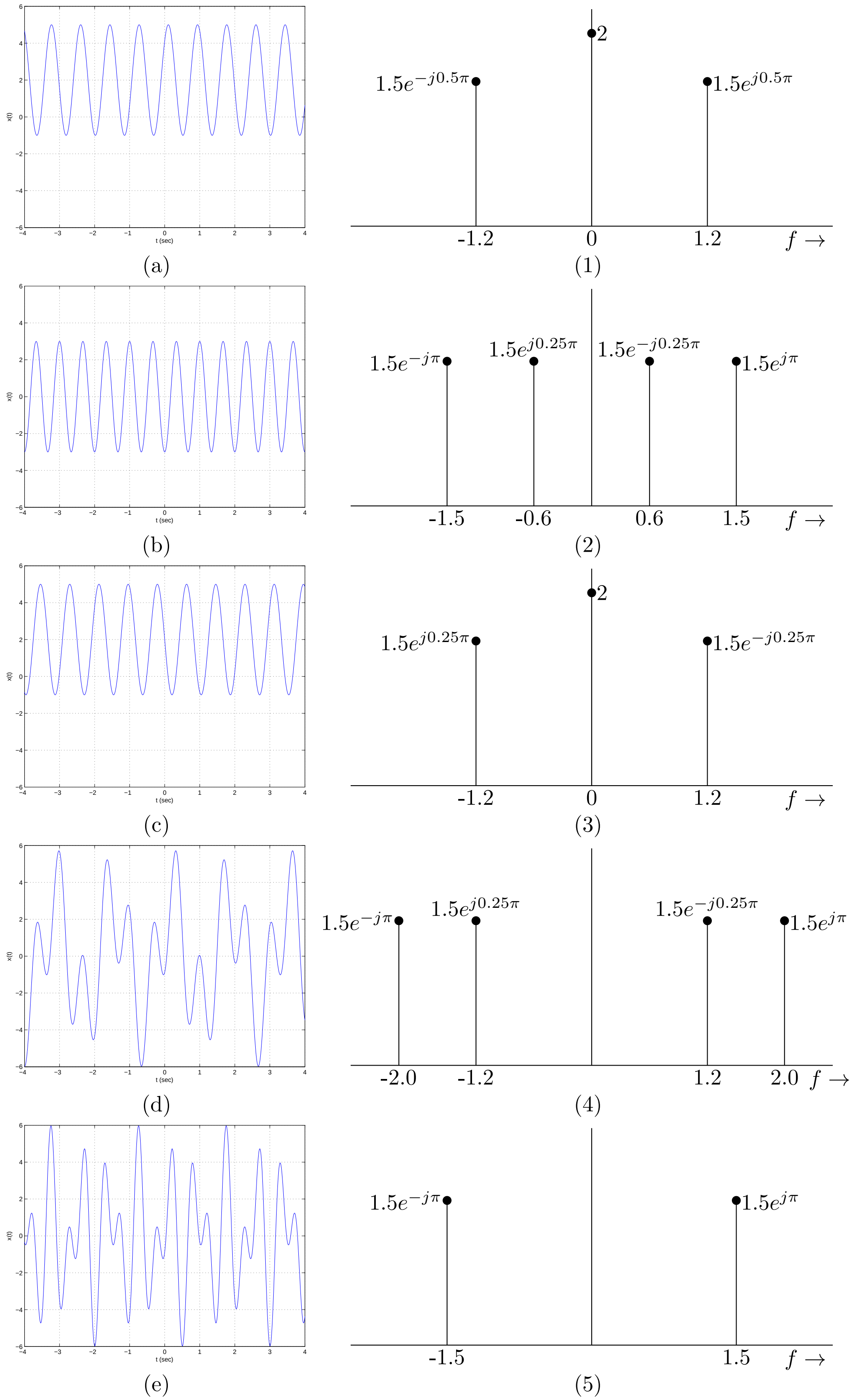
x(t)
t (sec)
(a)
(b)
(c)
(d)
(e)
2
1.5e^{-j0.5π}
1.5e^{j0.5π}
-1.2
0
1.2
f →
(1)
1.5e^{-jπ}
1.5e^{j0.25π}
1.5e^{-j0.25π}
1.5e^{jπ}
-1.5
-0.6
0.6
1.5
f →
(2)
2
1.5e^{j0.25π}
1.5e^{-j0.25π}
-1.2
0
1.2
f →
(3)
1.5e^{-jπ}
1.5e^{j0.25π}
1.5e^{-j0.25π}
1.5e^{jπ}
-2.0
-1.2
1.2
2.0
f →
(4)
1.5e^{-jπ}
1.5e^{jπ}
-1.5
1.5
f →
(5)

Figure P-3.19

5. Complex exponentials have many advantages over sines and cosines. The complex exponential form of the Fourier series is

$$x(t) = \sum_{n=-\infty}^{+\infty} c_n \exp\left(j\frac{2\pi}{T}nt\right) \tag{5}$$

$$c_n = \frac{1}{T}\int_T x(t)\exp\left(-j\frac{2\pi}{T}nt\right)\mathrm{d}t. \tag{6}$$

If $x(\cdot)$ takes only real values, then Eq. 5 is an unusual equation since it describes a real number [$x(t)$ for a particular value of t] by a sum of complex numbers [$c_n \exp\left(j\frac{2\pi}{T}nt\right)$ for the same particular value of t]. This requires that the c_n have a particular property, which is the subject of this problem.

(a) One way to derive this property is to do the following calculation:

i. Take the complex conjugate of Eq. 5 to get

$$x^*(t) = \left[\sum_{n=-\infty}^{+\infty} c_n \exp\left(j\frac{2\pi}{T}nt\right)\right]^*. \tag{7}$$

ii. Use the fact that $x(\cdot)$ is real to get

$$x(t) = \left[\sum_{n=-\infty}^{+\infty} c_n \exp\left(j\frac{2\pi}{T}nt\right)\right]^*. \tag{8}$$

iii. Use $(z_1+z_2)^* = z_1^* + z_2^*$ to get

$$x(t) = \sum_{n=-\infty}^{+\infty} \left[c_n \exp\left(j\frac{2\pi}{T}nt\right)\right]^*. \tag{9}$$

iv. Use $(z_1 z_2)^* = z_1^* z_2^*$ to get

$$x(t) = \sum_{n=-\infty}^{+\infty} c_n^* \left[\exp\left(j\frac{2\pi}{T}nt\right)\right]^*. \tag{10}$$

v. Use $[\exp(j\theta)]^* = \exp(-j\theta)$ to get

$$x(t) = \sum_{n=-\infty}^{+\infty} c_n^* \exp\left(-j\frac{2\pi}{T}nt\right). \tag{11}$$

vi. Change dummy summation index from n to m where $m = -n$ to get

$$x(t) = \sum_{m=-\infty}^{+\infty} c_{-m}^* \exp\left(j\frac{2\pi}{T}mt\right). \tag{12}$$

By comparing Eqs. 5 and 12, describe the constraint that the Fourier series coefficients c_n must satisfy if $x(\cdot)$ is to be real valued.

(b) Does $c_n = \rho^{|n|}$ where $\rho \in \mathbb{R}$ and $|\rho| < 1$ satisfy the constraint?

(c) For each of the sets of c_n described in Table 1, does the c_n represent a real valued signal?

6. A signal $x(\cdot)$ is defined to be "even" if $x(t) - x(-t)$ for all values of t. This is the only so-called "space group symmetry" when the independent variable, here denoted by t, is a real number. When the independent variable is a vector of 2 or of 3 real numbers then there are many space groups (230 for the case of vectors with three components) and these symmetries are very important in the atomic

n	c_n
0	2
1	3+6j
2	1.5+2.5j
-1	3-6j
-2	1.5-2.5j

(a)

n	c_n
0	2+7j
1	3+6j
2	1.5+2.5j
-1	3-6j
-2	1.5-2.5j

(b)

n	c_n
0	2
1	3+6j
2	1.5+2.5j
-1	3+6j
-2	1.5+2.5j

(c)

n	c_n
0	2+7j
1	3+6j
2	1.5+2.5j
-1	3+6j
-2	1.5+2.5j

(d)

n	c_n
0	2
1	3
2	1.5
-1	3
-2	1.5

(e)

Table 1: Sets of Fourier series coefficients. Coefficients not listed have value zero.

structure of materials. The usual signal/image processing terminology is to refer to these as 1-D, 2-D, or 3-D signals.

Suppose $x(\cdot)$ is even. The complex exponential form of the Fourier series is

$$x(t) = \sum_{n=-\infty}^{+\infty} c_n \exp\left(j\frac{2\pi}{T}nt\right) \tag{13}$$

$$c_n = \frac{1}{T}\int_T x(t) \exp\left(-j\frac{2\pi}{T}nt\right) \mathrm{d}t. \tag{14}$$

The fact that $x(\cdot)$ is even requires that the c_n have a particular property, which is the subject of this problem.

(a) One way to derive this property is to do the following calculation:

i. Replace t by $-t$ in Eq. 13 gives

$$x(-t) = \sum_{n=-\infty}^{+\infty} c_n \exp\left(j\frac{2\pi}{T}n(-t)\right). \tag{15}$$

ii. Use the fact that $x(\cdot)$ is even to get

$$x(t) = \sum_{n=-\infty}^{+\infty} c_n \exp\left(j\frac{2\pi}{T}n(-t)\right). \tag{16}$$

iii. Change dummy summation index from n to m where $m = -n$ to get

$$x(t) = \sum_{m=-\infty}^{+\infty} c_{-m} \exp\left(j\frac{2\pi}{T}(-m)(-t)\right). \tag{17}$$

iv. By comparing Eqs. 13 and 17, describe the constraint that the Fourier series coefficients c_n must satisfy if $x(\cdot)$ is to be even.

(b) For the examples of c_n given in Problem 5, do the c_n belong to a signal that is even?

7. (a) By combining the results of Problems 5 and 6, what are the constraints on the c_n if a signal is both real valued and even?

(b) For the examples of c_n given in Problem 5, do the c_n belong to a signal that is both real valued and even?