ECE 2200 / Spring 2016 / Preliminary Exam 3 / Solution

$$H(z) = \frac{1 - az^{-1}}{(1-b_1 z^{-1})(1-b_2 z^{-1})}$$

(a) $$\frac{Y(z)}{X(z)} = H(z) = \frac{1 - az^{-1}}{(1-b_1 z^{-1})(1-b_2 z^{-1})}$$

$$\Rightarrow (1-b_1 z^{-1})(1-b_2 z^{-1}) Y(z) = (1-az^{-1}) X(z)$$

$$\Rightarrow \left(1 - (b_1+b_2) z^{-1} + b_1 b_2 z^{-2}\right) Y(z) = (1-az^{-1}) X(z)$$

$$\Rightarrow y[n] - (b_1+b_2) y[n-1] + b_1 b_2 y[n-2] = x[n] - a x[n-1]$$

(b) $$H(z) = \frac{1 - az^{-1}}{(1-b_1 z^{-1})(1-b_2 z^{-1})} \frac{z^2}{z^2}$$

$$= \frac{z(z-a)}{(z-b_1)(z-b_2)}$$

poles: $z = b_1$, $z = b_2$

(c) zeros: $z = 0$, $z = a$

(d) $x[n] = d^n$

pick $d = a$ = location of zero.

$d = 0$ = location of second zero gives an input that is always zero

(e) $|b_1| < |b_2|$. Do partial fraction expansion

$$H(z) = \frac{1 - az^{-1}}{(1-b_1 z^{-1})(1-b_2 z^{-1})} = \frac{A}{1-b_1 z^{-1}} + \frac{B}{1-b_2 z^{-1}}$$

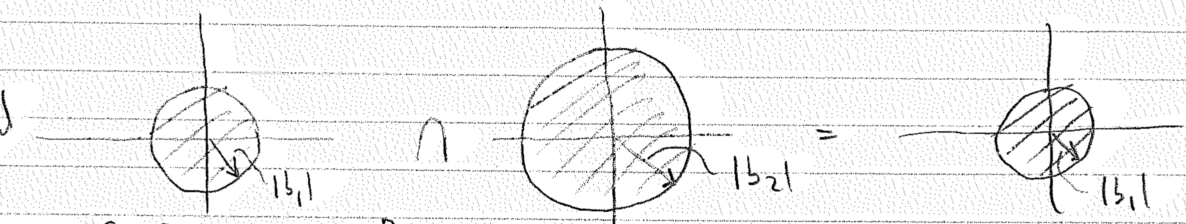
$$A = \left.\frac{1-az^{-1}}{1-b_2 z^{-1}}\right|_{z=b_1} = \frac{1-ab_1^{-1}}{1-b_2 b_1^{-1}} = f_1$$

$$B = \left.\frac{1-az^{-1}}{1-b_1 z^{-1}}\right|_{z=b_2} = \frac{1-ab_2^{-1}}{1-b_1 b_2^{-1}} = f_2$$

$$H(z) = \frac{f_1}{1-b_1 z^{-1}} + \frac{f_2}{1-b_2 z^{-1}} \qquad |b_1|<|b_2|$$

p074 12

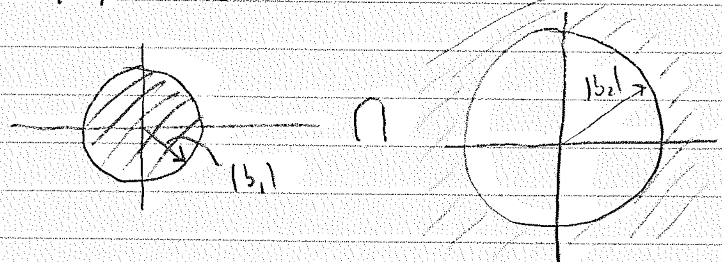
A) Anticausal



$h_A[n] = -f_1 b_1^n u[-n-1] - f_2 b_2^n u[-n-1]$

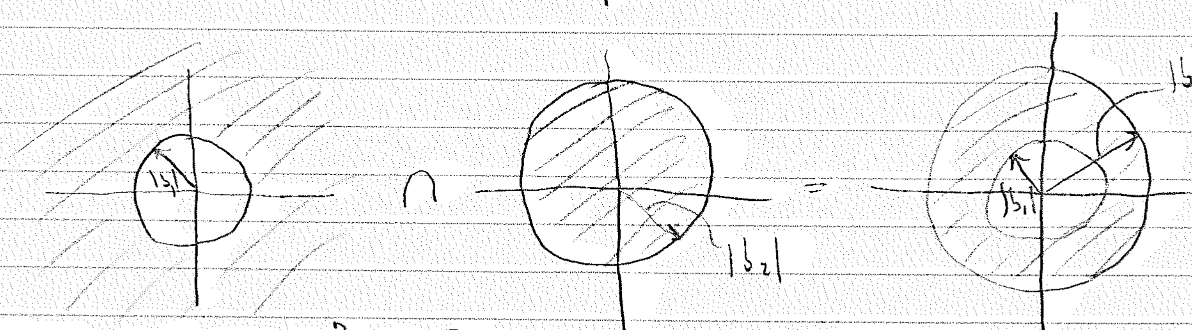
BIBO stable if $1<|b_1|$

B)



= no intersection

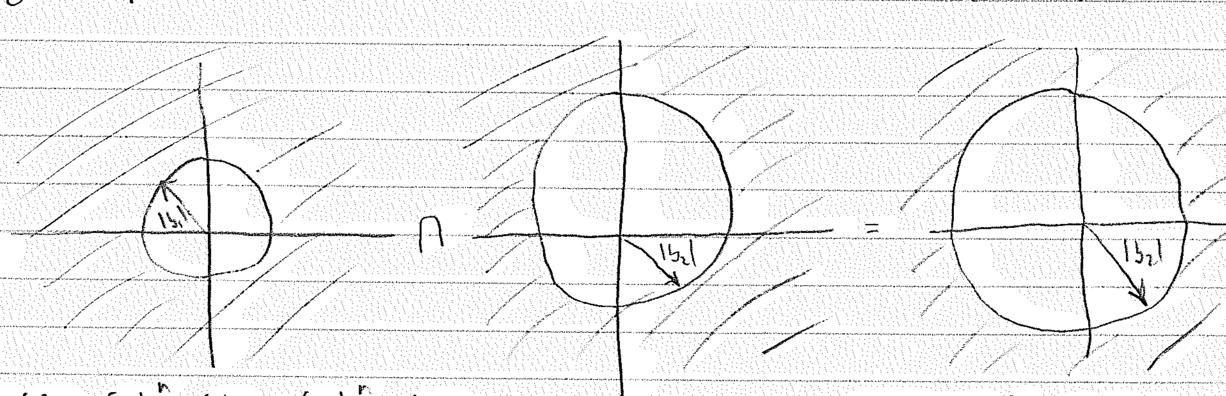
C) Neither



$h_C[n] = f_1 b_1^n u[n] - f_2 b_2^n u[-n-1]$

BIBO stable if $|b_1|<1<|b_2|$

D) Causal



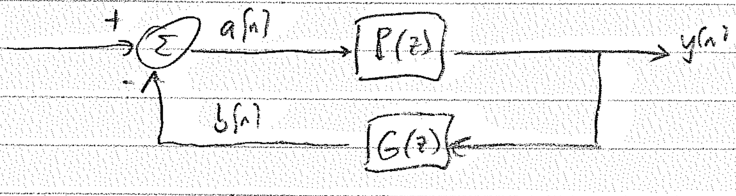
$h_c[n] = f_1 b_1^n u[n] + f_2 b_2^n u[n]$

BIBO stable if $|b_2|<1$

(f) See answers for causal, anticausal, and neither in (e)

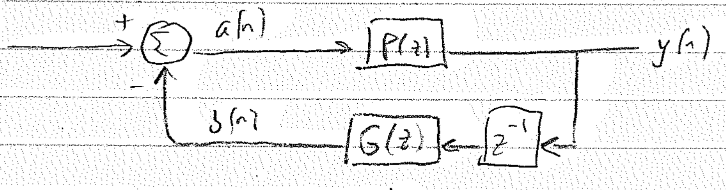
(g) See answers for BIBO stable in (e)

p075



(a)

$$H_a(z) = \frac{P(z)}{1 + P(z)G(z)}$$



(b)

$p(n) = \alpha^n u(n) \leftrightarrow P(z) = \frac{1}{1-\alpha z^{-1}}$

$|\alpha| < |z|$

$g(n) = \beta\delta(n) \leftrightarrow G(z) = \beta$ for all z.

(a) Since $\rightarrow \boxed{z^{-1}} \rightarrow \boxed{G(z)} \rightarrow$ is equivalent to $\rightarrow \boxed{z^{-1}G(z)} \rightarrow$,

it follows that

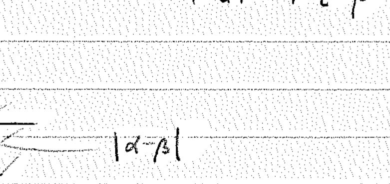
$$H_b(z) = \frac{P(z)}{1 + P(z)z^{-1}G(z)}$$

(b) $P(z)$ is BIBO unstable

(c) and (d)

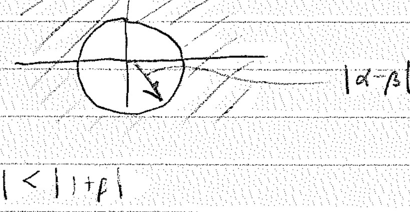
$$H_a(z) = \frac{P(z)}{1+P(z)G(z)} = \frac{\frac{1}{1-\alpha z^{-1}}}{1 + \frac{1}{1-\alpha z^{-1}}\beta} \; \frac{1-\alpha z^{-1}}{1-\alpha z^{-1}} = \frac{1}{1-\alpha z^{-1} + \beta}$$

$$= \frac{1}{(1+\beta) - \alpha z^{-1}} = \frac{1}{1+\beta} \; \frac{1}{1 - \frac{\alpha}{1+\beta} z^{-1}}$$



$$H_b(z) = \frac{P(z)}{1+P(z)z^{-1}G(z)} = \frac{\frac{1}{1-\alpha z^{-1}}}{1 + \frac{1}{1-\alpha z^{-1}} z^{-1}\beta} \; \frac{1-\alpha z^{-1}}{1-\alpha z^{-1}} = \frac{1}{1-\alpha z^{-1} + z^{-1}\beta}$$

$$= \frac{1}{1-(\alpha-\beta)z^{-1}}$$



(e) H_a: BIBO stable $\Leftrightarrow \left|\frac{\alpha}{1+\beta}\right| < 1 \Leftrightarrow |\alpha| < |1+\beta|$

H_b: BIBO stable $\Leftrightarrow |\alpha-\beta| < 1$

(f) $\alpha = 2,\ \beta = 5 \Rightarrow \begin{cases} |\alpha| < |1+\beta| \text{ is true since LHS} = 2,\ \text{RHS} = 6 \\ |\alpha-\beta| < 1 \text{ is false since } |\alpha-\beta| = |2-5| = |-3| = 3 \end{cases}$

p076

a) $h_1[n] = \left(\frac{1}{2}\right)^n u[n+1]$

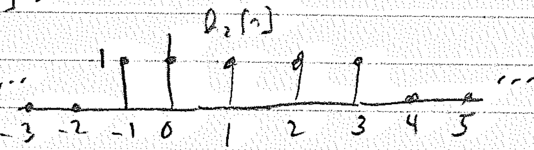
Starts at $n=-1 \Rightarrow$ Neither causal nor anticausal

is BIBO stable

b) $y_2[n] = \sum_{k=-1}^{3} x_2[n-k]$

to compute $h_2[n]$, set $x_2[n] = \delta[n]$:

$$h_2[n] = \sum_{k=-1}^{3} \delta[n-k] = \ldots$$



Starts at $n=-1 \Rightarrow$ Neither causal nor anticausal

is BIBO stable

c) $h_3[n] = 2^{|n|}$

$h_3[n]$ is always non zero — Neither causal nor anticausal

not BIBO stable: $\sum_{n=-\infty}^{+\infty} |h_3[n]| = \sum_{n=-\infty}^{+\infty} 2^{|n|} = +\infty$

d) $h_4[n] = (-1)^n u[n]$

Causal

not BIBO stable: $\sum_{n=-\infty}^{+\infty} |h_4[n]| = \sum_{n=0}^{\infty} |-1|^n = \sum_{n=0}^{\infty} 1 = \infty$

e) $M \in \mathbb{Z}$, $M > 0$

$$y_5[n] = \begin{cases} x_5[n/M], & n = Ml \text{ for some } l \in \mathbb{Z} \\ 0, & \text{otherwise.} \end{cases}$$

Is linear.

Is not time invariant: compare $x_5[n] = \delta[n] \Rightarrow y_5[n] = \delta[n]$ with $x_5[n] = \delta[n-1] \Rightarrow y_5[n] = \delta[n-M]$. The two x_5 are shifted by 1 in time but the two y_5 are shifted by M in time.