ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Preliminary Exam 3
Thursday May 5, 2016
11:40AM–12:55PM
Phillips Hall Room 101
No calculator!
Only the provided formula sheet!
Work alone!

1. ($36 = 4 \times 3 + 12 + 2 \times 6$ pts.) Consider a system defined by the system function

$$H(z) = \frac{1 - az^{-1}}{(1 - b_1 z^{-1})(1 - b_2 z^{-1})}. \quad (133)$$

Please notice that no ROC is specified.

(a) What is the difference equation implied by this system function?

(b) What are the poles of this system function? Please remember that the poles are values of z where $H(z)$ is infinite, not values of z^{-1} where $H(z)$ is infinite.

(c) What are the zeros of this system function? Please remember that the zeros are values of z where $H(z)$ is zero, not values of z^{-1} where $H(z)$ is zero.

(d) Suppose that the system has input $x[n]$ and output $y[n]$. Please consider input signals of the form

$$x[n] = d^n. \quad (134)$$

Please list *all* values of $d \neq 0$ such that the output $y[n]$ is zero.

(e) Suppose that

$$|b_1| \quad < \quad |b_2|. \quad (135)$$

Please give all possible impulse responses $h[n]$ for this system. Please give the ROC for $H(z)$ along with the impulse response. Please write your answer in terms of the constants f_1 and f_2 listed below.

$$f_1 = \frac{1 - ab_1^{-1}}{1 - b_2 b_1^{-1}} \quad (136)$$

$$f_2 = \frac{1 - ab_2^{-1}}{1 - b_1 b_2^{-1}}. \quad (137)$$

Helpful information:

$$x[n] = \alpha^n u[n] \quad \leftrightarrow \quad X(z) = \frac{1}{1 - \alpha z^{-1}} \quad |\alpha| < |z| \quad (138)$$

$$x[n] = -\alpha^n u[-n-1] \quad \leftrightarrow \quad X(z) = \frac{1}{1 - \alpha z^{-1}} \quad |z| < |\alpha|. \quad (139)$$

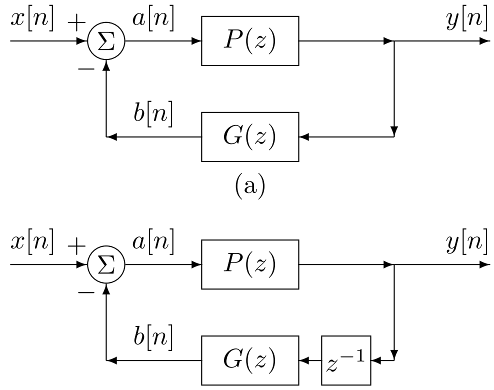
(f) For each impulse response in your answer to Part 1e, please state if the system is causal, anti-causal, or neither.

(g) Please continue to assume that

$$|b_1| \quad < \quad |b_2|. \quad (140)$$

For each impulse response in your answer to Part 1e, please give constraints on a, b_1, and b_2 such that the system is BIBO stable.

2. (36 = 6 × 6 pts.) Consider the following block diagrams where $P(z)$ is the "plant" and $G(z)$ is the "feedback controller":



(a) (b)

The Σ with + and − signs means that $a[n] = x[n] - b[n]$.

The problem is that the plant, $P(z)$, is BIBO unstable. The goal is to stablize the transformation from $x[n]$ to $y[n]$ by using the feedback controller $G(z)$. In System (b) there is an additional delay in the feedback loop in comparison with System (a) because there is communication time.

For System (a),

$$H_a(z) = \frac{Y(z)}{X(z)} = \frac{P(z)}{1 + P(z)G(z)}. \tag{141}$$

(a) Please give a formula similar to Eq. 141 for $H_b(z)$ for System (b).

(b) Consider $p[n] = \alpha^n u[n]$ with z transform $P(z) = 1/(1 - \alpha z^{-1})$ and ROC $|\alpha| < |z|$ with $|\alpha| > 1$. Consider $g[n] = \beta\delta[n]$ with z transform $G(z) = \beta$ for all z. Is $p[n]$ BIBO stable? The feedback controller $g[n]$ is just an amplifier with gain β.

(c) Continuing with the $p[n]$ and $g[n]$ from Problem 2b, the system function $H_a(z)$ for System (a) is

$$H_a(z) = \frac{1}{1+\beta} \frac{1}{1 - \frac{\alpha}{1+\beta} z^{-1}}. \tag{142}$$

What is the system function $H_b(z)$ for System (b)?

(d) Remembering that this is a causal system, what are the ROCs for System (a) and for System (b)?

(e) Please give conditions on β so that $H_a(z)$ is BIBO stable and such that $H_b(z)$ is BIBO stable.

(f) Suppose $\alpha = 2$. Give one value of β such that $H_a(z)$ is BIBO stable but $H_b(z)$ is *not* BIBO stable. This demonstrates that delay can destabilize a system!

3. $(28 = 4+6+4+6+4$ pts.) In this problem, $x[n]$ is the input to the system, $y[n]$ is the output of the system, and $h[n]$ is the impulse response of the system.

(a) Please consider

$$h_1[n] = \left(\frac{1}{2}\right)^n u[n+1]. \tag{143}$$

Is the system causal, anti-causal, or neither? Is the system BIBO stable?

(b) Please consider

$$y_2[n] = \sum_{k=-1}^{3} x_2[n-k]. \tag{144}$$

What is the impulse response $h_2[n]$? Is the system causal, anti-causal, or neither? Is the system BIBO stable?

(c) Please consider

$$h_3[n] = 2^{|n|}. \tag{145}$$

Is the system causal, anti-causal, or neither? Is the system BIBO stable?

(d) Please consider

$$h_4[n] = (-1)^n u[n]. \tag{146}$$

Is the system causal, anti-causal, or neither? Is the system BIBO stable?

(e) For a fixed integer $M > 0$, please consider

$$y_5[n] = \begin{cases} x_5[n/M], & n = Ml \text{ for some integer } l \\ 0, & \text{otherwise} \end{cases}. \tag{147}$$

Is this system linear? Is this system time invariant?