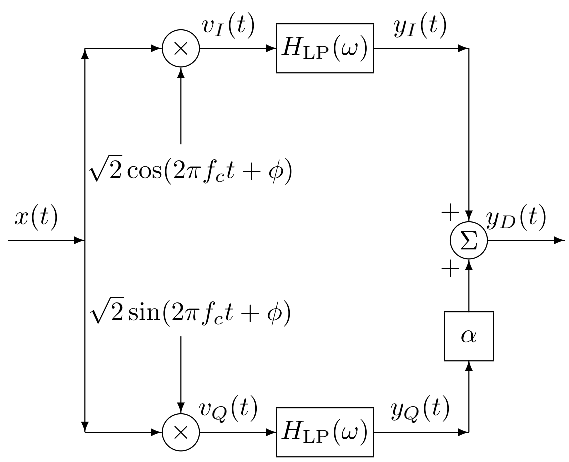
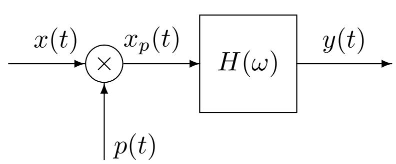
ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Preliminary Exam 2
Thursday April 7, 2016
11:40AM–12:55PM
Phillips Hall Room 201
No calculator!
Only the provided formula sheet!
Work alone!

1. (15 pts.) Please consider the following block diagram which is a part of a communication system. The block labeled "α" is an amplifier with gain α at all frequencies. This block diagram has two identical filters $H_{\mathrm{LP}}(\omega)$. Please draw a block diagram with the same input signal $x(t)$ and the same output signal $y_D(t)$ that contains only one filter $H_{\mathrm{LP}}(\omega)$.



2. (15 pts.) Consider the block diagram



where

$$x(t) = \cos(\omega_A t) \leftrightarrow X(\omega) = \pi\left[\delta(\omega - \omega_A) + \delta(\omega + \omega_A)\right], \tag{75}$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \leftrightarrow P(\omega) = \frac{2\pi}{T_s} \sum_{n=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi}{T_s} n\right), \tag{76}$$

and

$$H(\omega) = \begin{cases} T_s, & |\omega| < \omega_H \\ 0, & \text{otherwise} \end{cases}. \tag{77}$$

In terms of ω_A and ω_B, please give a value for T_s and a range for ω_H such that $y(t) = \cos(\omega_A t) + \cos(\omega_B t)$ where $\omega_B > \omega_A$. Please give a careful plot of $X_p(\omega)$, which is the continuous-time Fourier transform of $x_p(t)$, for your chosen value of T_s. Please give a careful plot of $H(\omega)$ for a particular value of ω_H in your chosen range of ω_H.

3. (30 = 10 + 10 + 10 pts.) The discrete-time signal $x[n]$ is defined by

$$x[n] = \begin{cases} A, & |n| \le n_0 \\ A\rho^{|n|-n_0}, & |n| > n_0 \end{cases} \tag{78}$$

where $A \in \mathbb{R}$, $\rho \in \mathbb{R}$, and $|\rho| < 1$.

(a) Please compute the Discrete-Time Fourier Transform (DTFT) of $x[n]$ which is denoted by $X(\Omega)$.

(b) Note that $x[n] \in \mathbb{R}$ and $x[n] = x[-n]$. What do these two properties imply for the DTFT $X(\Omega)$?

(c) Please verify that your answer in Problem 3a has the property that you describe in Problem 3b.

Helpful information:

(a) Define

$$y[n] = \begin{cases} 1, & |n| \le n_0 \\ 0, & |n| > n_0 \end{cases} . \tag{79}$$

Then the DTFT of $y[n]$, which is denoted by $Y(\Omega)$, is

$$Y(\Omega) = \frac{\sin\left(\Omega(2n_0+1)/2\right)}{\sin(\Omega/2)}. \tag{80}$$

(b) Define

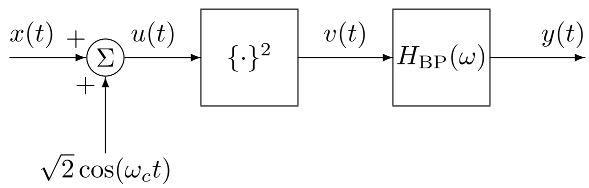
$$z[n] = \rho^n u[n] \tag{81}$$

with $|\rho| < 1$. Then the DTFT of $z[n]$, which is denoted by $Z(\Omega)$, is

$$Z(\Omega) = \frac{1}{1-\rho e^{-j\Omega}}. \tag{82}$$

4. (40 = 10 + 10 + 10 + 10 pts.) This problem concerns building modulators and demodulators for Double-Side-Band Suppressed-Carrier (DSB-SC) communication systems using squaring operations.

(a) Let $x(t) = A_c m(t)$ where $m(t)$ is bandlimited to ω_m, i.e., $|M(\omega)| = 0$ for $|\omega| > \omega_m$. Consider the following block diagram:

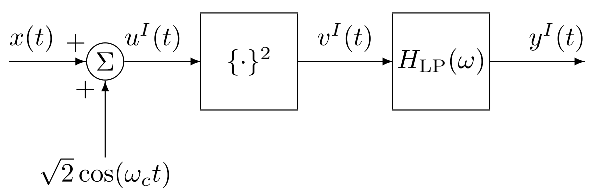


where $\omega_c \gg \omega_m$ and

$$H_{\mathrm{BP}}(\omega) = \begin{cases} 1, & |\omega - \omega_c| \le \omega_m \\ 1, & |\omega + \omega_c| \le \omega_m \\ 0, & \text{otherwise} \end{cases}.$$

The block labeled "$\{\cdot\}^2$" has output $v(t)$ that is the square of the input $u(t)$: $v(t) = u^2(t)$. This block diagram is attractive because it is easier to build a squarer than to build a multiplier. Give a formula for $y(t)$.

(b) Based on the successful use of a squaring operation in the modulator of Problem 4a, we decide to use the same squaring operation in the demodulator. Let $x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$ where $m(t)$ is bandlimited to ω_m, i.e., $|M(\omega)| = 0$ for $|\omega| > \omega_m$, and $\omega_c \gg \omega_m$. Consider the following block diagram:

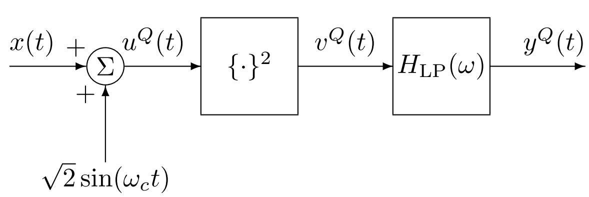


where $\omega_c \gg \omega_m$ and

$$H_{\mathrm{LP}}(\omega) = \begin{cases} 1, & |\omega| \le \omega_m \\ 0, & \text{otherwise} \end{cases}.$$

Give a formula for $y^I(t)$. Does this block diagram work as a demodulator, that is, is $y^I(t)$ proportional to $m(t)$?

(c) Due to the failure in Problem 4b, we have to think hard and it seems natural to consider also the block diagram with cos replaced by sin. Let $x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$ where $m(t)$ is bandlimited to ω_m, i.e., $|M(\omega)| = 0$ for $|\omega| > \omega_m$, and $\omega_c \gg \omega_m$. Consider the following block diagram:



where $\omega_c \gg \omega_m$ and

$$H_{\mathrm{LP}}(\omega) = \begin{cases} 1, & |\omega| \le \omega_m \\ 0, & \text{otherwise} \end{cases}.$$

Give a formula for $y^Q(t)$.

(d) Combining the results of Problems 4b and 4c, draw a block diagram of a successful DSB-SC demodulator using squaring operations instead of multipliers.

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Problem Set 7
Due Friday April 15, 2016 at 5:00PM.
Location to turn in: "ECE 2200" box on Phillips Hall 2nd floor

1. A modification of McClellan, Schafer, Yoder, Signal Processing First (1st ed) Problem P-9.17. A continuous-time system is defined by the input/output relation

$$y(t) = \int_{t-2}^{t+2} x(\tau)\mathrm{d}\tau. \tag{83}$$

   (a) Determine the impuluse resonse, $h(t)$, of this system. Hint: Substitute $x(t) = \delta(t)$ and compute $y(t)$. The key issue is to decide the values of t for which the integral is nonzero.

   (b) Use the convolution integral to determine the output of the system when the input is

$$x(t) = u(t+1). \tag{84}$$

   Plot your answer.

   (c) Please check your answer to Problem 1b by computing the same result directly from Eq. 83.

2. Let

$$x[n] = \rho^{|n|}. \tag{85}$$

   Assume that $|\rho| < 1$. However, ρ may be complex, i.e., $\rho = re^{j\phi}$. Compute the Discrete Time Fourier Transform of $x[n]$, i.e.,

$$X(\Omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\Omega n}. \tag{86}$$

   Hint: One possible approach is to use the geometric sum twice, once for non-negative values of n and once for negative values of n. The geometric sum is

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}. \tag{87}$$

3. McClellan, Schafer, Yoder, Signal Processing First (1st ed) Problem P-6.5 with hints. A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] + 2x[n-1] + x[n-2]. \tag{88}$$

   (a) Obtain an expression for the frequency response of this system.

   (b) Sketch the frequency response (magnitude and phase) as a function of frequency.

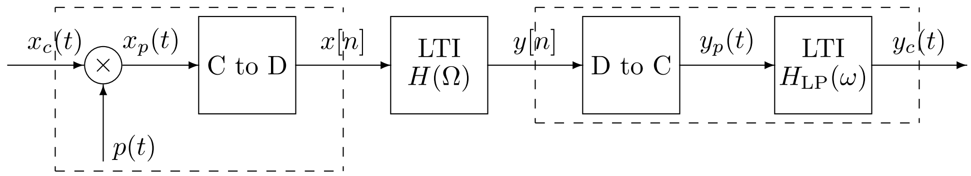
   (c) Determine the output when the input is

$$x[n] = 10 + 4\cos(0.5\pi n + \pi/4). \tag{89}$$

   (d) Determine the output when the input is the unit impulse sequence $\delta[n]$. This may be easier in the time domain.

   (e) Determine the output when the input is the unit-step sequence $u[n]$. This may be easier in the time domain.

4. Consider the block diagram



where

(a)

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \leftrightarrow P(\omega) = \frac{2\pi}{T_s} \sum_{n=-\infty}^{+\infty} \delta(\omega - \frac{2\pi}{T_s} n). \tag{90}$$

(b)

$$x[n] = x_c(nT_s). \tag{91}$$

(c)

$$y[n] = \frac{1}{2M+1} \sum_{m=-M}^{+M} x[n-m]. \tag{92}$$

(d)

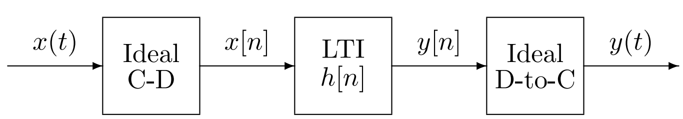
$$y_p(t) = \sum_{n=-\infty}^{+\infty} y[n]\delta(t - nT_s). \tag{93}$$

(e) $H_{\mathrm{LP}}(\omega)$ is the ideal reconstruction filter which is

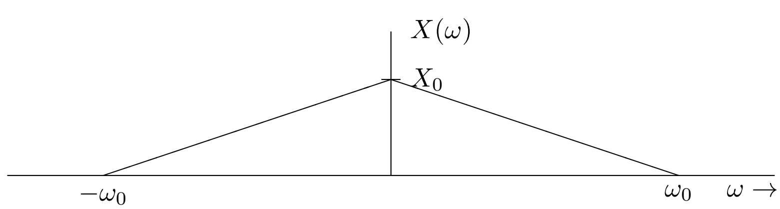
$$H_{\mathrm{LP}}(\omega) = \begin{cases} T_s, & |\omega| \le \pi/T_s \\ 0, & \text{otherwise} \end{cases} . \tag{94}$$

Assume that $x_c(t) \leftrightarrow X_c(\omega)$ satisfies the Nyquist frequency for this system. On the same ω axis scale, please draw accurate plots of $Y_c(\omega)/X_c(\omega)$ for $M \in \{1, 5, 10\}$. Hopefully you see that we are now in the business of low pass filter design, especially considering that the parts which you actually purchase approximate the two dashed boxes leaving only the central box, which might be a one-loop C program, to be built.

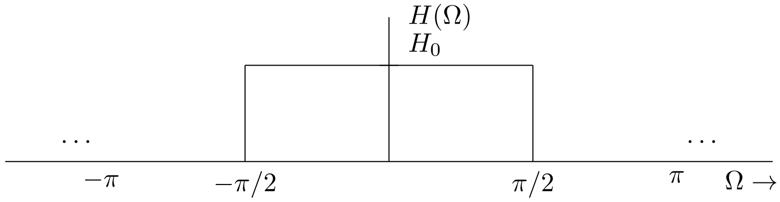
5. A modification of McClellan, Schafer, Yoder, Signal Processing First (1st ed) Problem P-12-15. Consider the system for discrete-time filtering of a continuous-time signal that is shown in the following block diagram:



For both the Ideal C-to-D (continuous to discrete) and Ideal D-to-C (discrete to continuous) blocks the sampling interval is T_s. The various signals have continuous-time (discrete-time) Fourier transforms $X(\omega)$, $X(\Omega)$, $H(\Omega)$, $Y(\Omega)$, and $Y(\omega)$ corresponding to $x(t)$, $x[n]$, $h[n]$, $y[n]$, and $y(t)$, respectively. The continuous-time Fourier transform of the input signal is shown in the following plot:



where $\omega_0 = 80\pi$. One period $(-\pi \le \Omega \le \pi)$ of the discrete-time Fourier transform of the impulse response of the LTI system is shown in the following plot:



where $H_0 = 1$.

(a) For this input signal, what is the smallest value of the sampling frequency $\omega_s = 2\pi/T_s$ such that the Fourier transforms of the input and output satisfy the relationship $Y(\omega) = H_{\text{eff}}(\omega)X(\omega)$ where $H_{\text{eff}}(\omega)$ is the effective frequency response of the entire system.

(b) If $f_s = 1/T_s = 100$samples/second, make a carefully labeled plot of $H_{\text{eff}}(\omega)$ and of $Y(\omega)$

(c) What is the smallest sampling rate such that the input signal passes through the lowpass filter unaltered, i.e., what is the minimum $f_s = 1/T_s$ such that $Y(\omega) = X(\omega)$?