ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Preliminary Exam 1 Solution

1. (a) $g(t)$ will "blow up" twice, once at $t = -a$ and once at $t = a$.
   (b)
$$\delta(t^2 - a^2) = \frac{1}{2|a|} \left[\delta(t + a) + \delta(t - a)\right] \tag{25}$$
   since then if you multiply both sides of Eq. 25 by a smooth function $f(t)$ and integrate from $-\infty$ to $+\infty$ you will get the same answer, which is Eq. 24.

2. (a) The period is 4.
   (b) It is not possible to draw a second signal for Figures 2–5. For Figures 6–9, second signals that are lower frequency cosines are shown in Figures 10–13.
   (c) From these plots, it appears that you need $T_s < \frac{1}{2}T$ in order to not have aliasing, where T is the period of the original signal and T_s is the sampling interval.

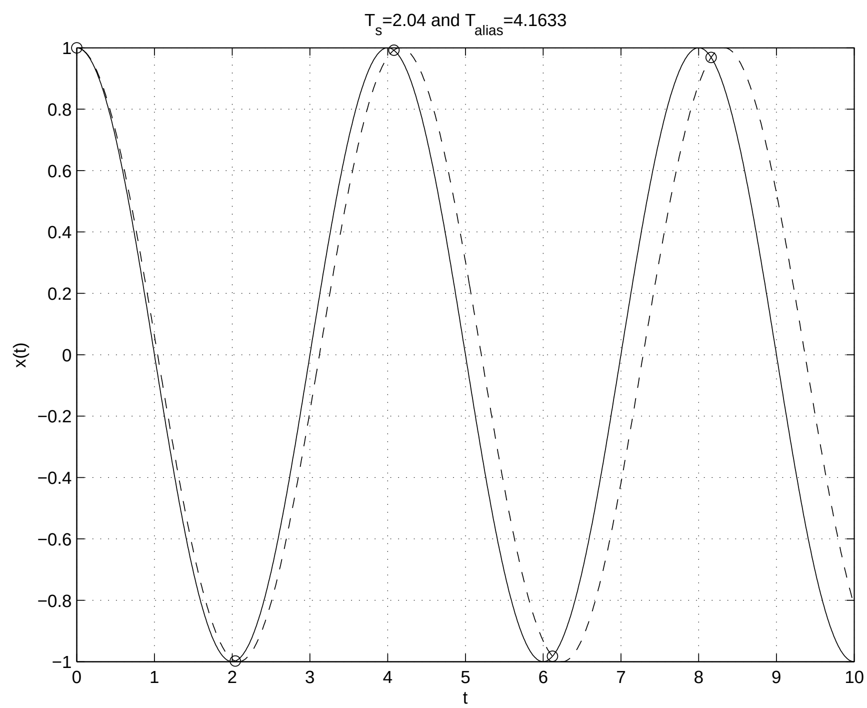

Figure 10: $T_s = 0.51T$

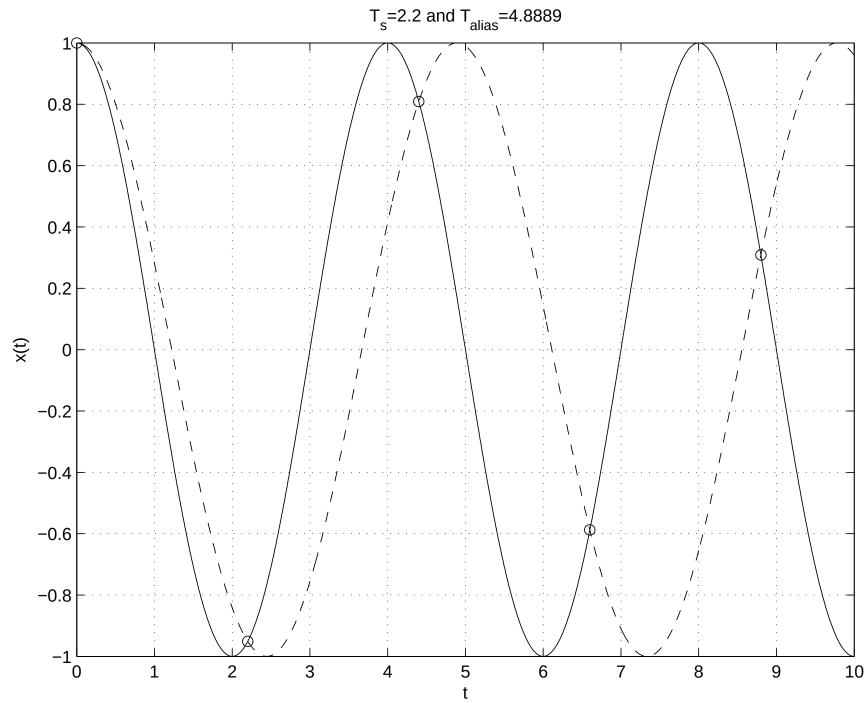

Figure 11: $T_s = 0.55T$

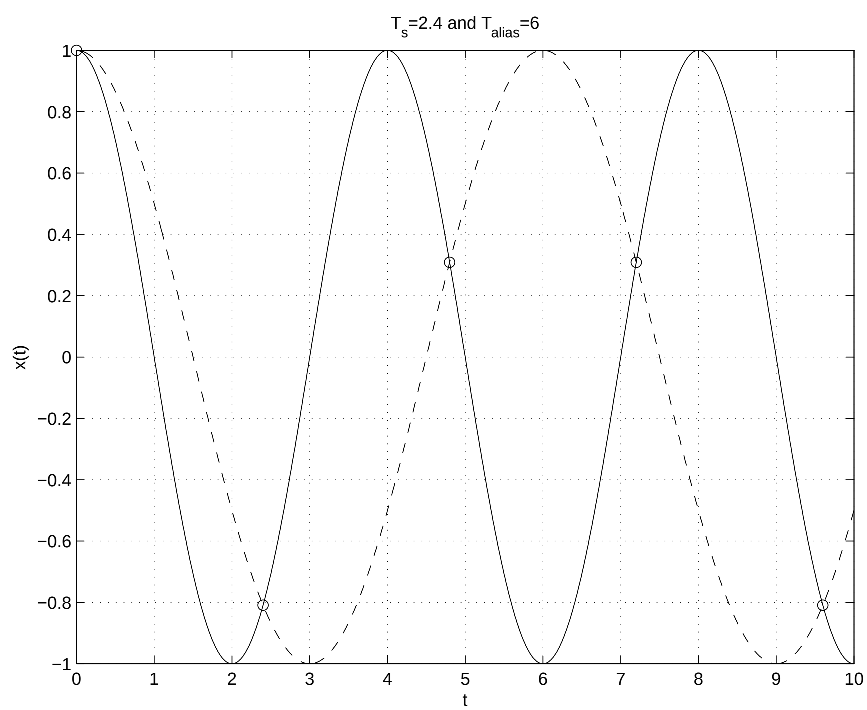

Figure 12: $T_s = 0.6T$

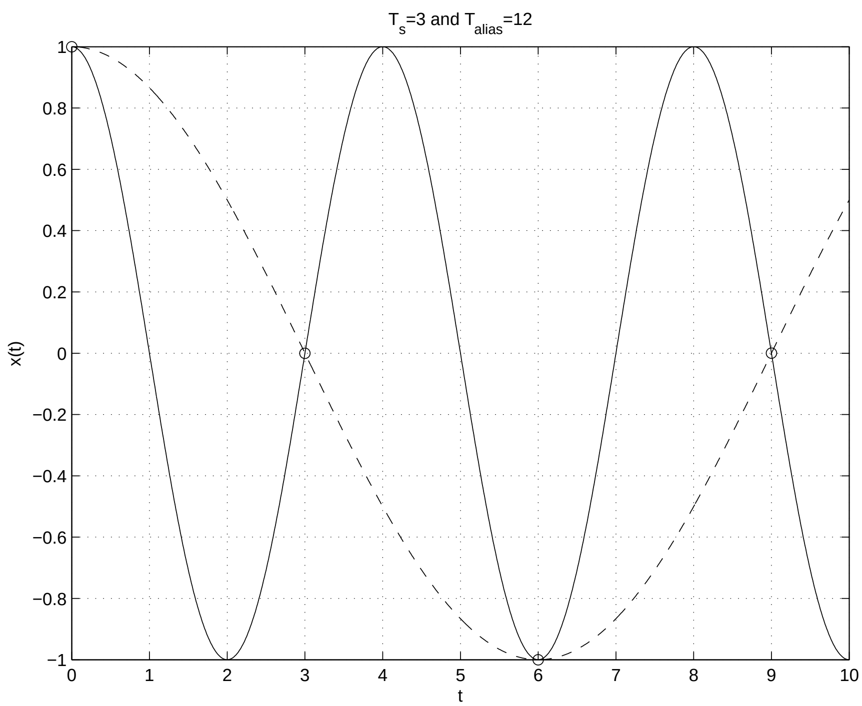

Figure 13: $T_s = 0.75T$

3. (a) i. Please write $x_2(t)$ as a function of $x_1(t)$. $x_2(t) = 2x_1(t)$.
   ii. Is it correct to write that $x_3(t) = x_1(t/2)$? No, the pulse is stretched by a factor of 2 but the period remains the same.
   iii. Please write $x_4(t)$ as a function of $x_1(t)$. $x_4(t) = x_1(t - \tau/2)$.
   iv. Please write $x_5(t)$ as a function of $x_3(t)$ and $x_1(t)$. $x_5(t) = x_3(t) + x_1(t - \tau/2)$.
   v. Please write $x_5(t)$ as a function of $x_1(t)$. $x_5(t) = x_1(t + \tau/2) + 2x_1(t - \tau/2)$ or $x_5(t) = x_1(t - \tau/2) + x_1(t/2)$.

(b) Please consider the following plots of Fourier series coefficients. Which plot corresponds to which signal $x_1(t)$, $x_2(t)$, $\ldots$, $x_5(t)$?

$$c_n^{(1)} = \frac{1}{T}\int_T x_1(t)\exp(-j(2\pi/T)nt)\mathrm{d}t \tag{26}$$

$$= \frac{1}{T}\int_{-T/2}^{+T/2} x_1(t)\exp(-j(2\pi/T)nt)\mathrm{d}t \tag{27}$$

$$= \frac{1}{T}\int_{-\tau/2}^{+\tau/2} A\exp(-j(2\pi/T)nt)\mathrm{d}t \tag{28}$$

$$= \begin{cases} \frac{1}{T}\int_{-\tau/2}^{+\tau/2} A\exp(-j(2\pi/T)nt)\mathrm{d}t & n \neq 0 \\ \frac{1}{T}\int_{-\tau/2}^{+\tau/2} A\mathrm{d}t & n = 0 \end{cases} \tag{29}$$

$$= \begin{cases} \frac{A}{T}\frac{1}{-j(2\pi/T)n}\exp(-j(2\pi/T)nt)\Big|_{-\tau/2}^{+\tau/2} & n \neq 0 \\ \frac{A\tau}{T} & n = 0 \end{cases} \tag{30}$$

$$= \begin{cases} \frac{A}{T}\frac{1}{-j(2\pi/T)n}\left[\exp(-j(2\pi/T)n\tau/2) - \exp(j(2\pi/T)n\tau/2)\right] & n \neq 0 \\ \frac{A\tau}{T} & n = 0 \end{cases} \tag{31}$$

$$= \begin{cases} \frac{A}{T}\frac{1}{-j(2\pi/T)n}\left[-2j\sin\left((2\pi/T)n\tau/2\right)\right] & n \neq 0 \\ \frac{A\tau}{T} & n = 0 \end{cases} \tag{32}$$

$$= \begin{cases} \frac{A}{T}\frac{1}{(\pi/T)n}\left[\sin\left((\pi/T)n\tau\right)\right] & n \neq 0 \\ \frac{A\tau}{T} & n = 0 \end{cases} \tag{33}$$

$$= \begin{cases} \frac{A\tau}{T}\frac{1}{\pi(\tau/T)n}\left[\sin\left(\pi(\tau/T)n\right)\right] & n \neq 0 \\ \frac{A\tau}{T} & n = 0 \end{cases} \tag{34}$$

$$\mathrm{sinc}(z) = \sin(\pi z)/(\pi z)$$

$$= \begin{cases} \frac{A\tau}{T}\,\mathrm{sinc}((\tau/T)n) & n \neq 0 \\ \frac{A\tau}{T} & n = 0 \end{cases} \tag{35}$$

$$\mathrm{sinc}(0) = 1$$

$$= \frac{A\tau}{T}\,\mathrm{sinc}((\tau/T)n) \tag{36}$$

$$c_n^{(2)} = 2c_n^{(1)} \tag{37}$$

$$c_n^{(3)} = c_n^{(1)}\Big|_{\tau\to 2\tau} \tag{38}$$

$$= \frac{2A\tau}{T}\,\mathrm{sinc}((2\tau/T)n) \tag{39}$$

$$c_n^{(4)} = \exp(-j(2\pi/T)n\xi)\big|_{\xi=\tau/2}\, c_n^{(1)} \tag{40}$$

$$c_n^{(5)} = \exp(-j(2\pi/T)n\xi)\big|_{\xi=-\tau/2}\, c_n^{(1)} + 2\exp(-j(2\pi/T)n\xi)\big|_{\xi=\tau/2}\, c_n^{(1)}. \tag{41}$$

The key part of the software is

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n=[-10:10];

A=0.5;
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T=1;
tau=0.25;

c1=(A*tau/T).*sinc( (tau/T).*n );
c2=2*c1;
c3=(2*A*tau/T).*sinc( (2*tau/T).*n );
c4=exp(-i*(2*pi/T)*(tau/2).*n).*c1;
c5=exp( j*(2*pi/T)*(tau/2).*n).*c1 + 2*exp(-j*(2*pi/T)*(tau/2).*n).*c1;
```

The answers are

$x_1(t)$	A
$x_2(t)$	C
$x_3(t)$	B
$x_4(t)$	E
$x_5(t)$	D