ECE 2200 and ENGRD 2220
Signals and Systems
Spring 2016
Preliminary Exam 1
Thursday February 25, 2016
11:40AM–12:55PM
Phillips Hall Room 101
No calculator!
Only the provided formula sheet!
Work alone!

1. (20 = 10 + 10 pts.) Expressions involving Dirac δ functions are defined by what they do under an integral sign. We have the expression

$$\delta(at) = \frac{1}{|a|}\delta(t). \tag{18}$$

   (a) Consider the function $g(t) = \delta(t^2 - a^2)$ where a is a non-zero real number and $-\infty < t < +\infty$. How many times does $g(t)$ "blow up" and what are the values of t where the "blow ups" occur?

   (b) Consider the follwing integral where a is a non-zero real number and $f(t)$ is a smooth function:

$$\int_{-\infty}^{+\infty} \delta(t^2 - a^2) f(t) \mathrm{d}t$$

$$= \int_{-\infty}^{0} \delta(t^2 - a^2) f(t) \mathrm{d}t + \int_{0}^{+\infty} \delta(t^2 - a^2) f(t) \mathrm{d}t \tag{19}$$

$$t' = -t$$

$$= \int_{0}^{\infty} \delta(t'^2 - a^2) f(-t') \mathrm{d}t' + \int_{0}^{\infty} \delta(t^2 - a^2) f(t) \mathrm{d}t \tag{20}$$

$$= \int_{0}^{\infty} \delta(t^2 - a^2) \left[f(-t) + f(t)\right] \mathrm{d}t \tag{21}$$

$$\lambda = t^2 - a^2 \Longrightarrow t = \pm\sqrt{\lambda + a^2}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = 2t \Longrightarrow \mathrm{d}t = \frac{\mathrm{d}\lambda}{2t} = \frac{\mathrm{d}\lambda}{\pm 2\sqrt{\lambda + a^2}}$$

Only the "+" sign causes $t \in [0, \infty)$

$$= \int_{-a^2}^{\infty} \delta(\lambda) \left[f(-\sqrt{\lambda + a^2}) + f(\sqrt{\lambda + a^2})\right] \frac{\mathrm{d}\lambda}{2\sqrt{\lambda + a^2}} \tag{22}$$

$$= \left[f(-\sqrt{a^2}) + f(\sqrt{a^2})\right] \frac{1}{2\sqrt{a^2}} \tag{23}$$

$$= \frac{1}{2|a|}\left[f(-|a|) + f(|a|)\right]. \tag{24}$$

Please give a formula analogous to Eq. 18 where the right hand side of the formula is $\delta(t^2 - a^2)$ and the left hand side has δ functions whose arguments are not more complicated than $t - t_0$ for some time t_0 where t_0 might be positive or negative.

2. (40 = 4 + 8 × 4 + 4 pts.) Please consider Figures 1–9. Figure 1 shows the cosine signal $x(t)$ and Figures 2–9 show the signal $x(t)$ and the times and values of samples of $x(t)$ at various different sampling intervals T_s.

   (a) What is the period of $x(t)$?

   (b) We hope that the samples (times and values) will be a description of a unique signal. In other words, no two signals share the same samples (times and values). "Aliasing" means, however, that there is a second signal that has the same samples (times and values). It is always possible to "connect the dots" with a second signal but we are only interested in cases where you can "connect the dots" with a second sinusoid at a lower frequency. When possible, please draw such a second signal on Figures 2–9. It may be easiest to first try to draw a second signal on Figure 9.

   (c) Hopefully you were successful sometimes and unsuccessful other times in drawing a second signal in Problem 2b. What does your success and failure tell you about the range of sampling intervals T_s which do not allow aliasing? You have basically discovered the Nyquist Sampling Theorem!

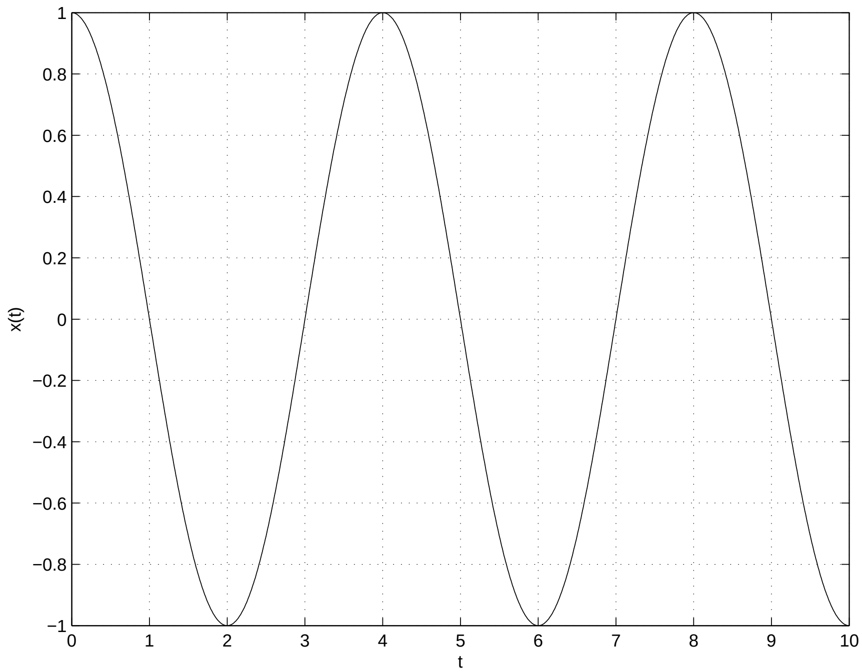


Figure 1: Original signal.

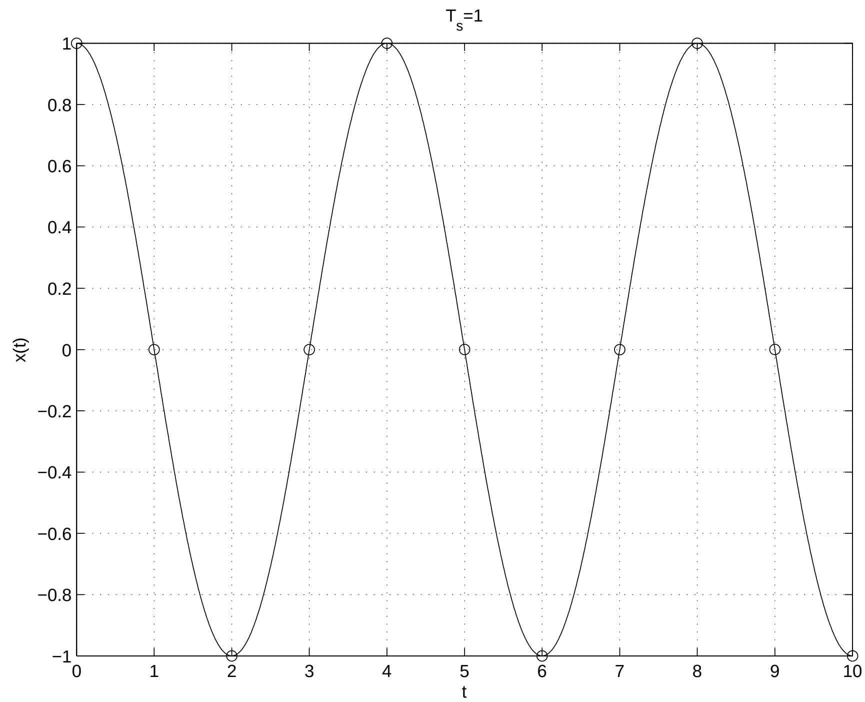


Figure 2: $T_s = 0.25T$

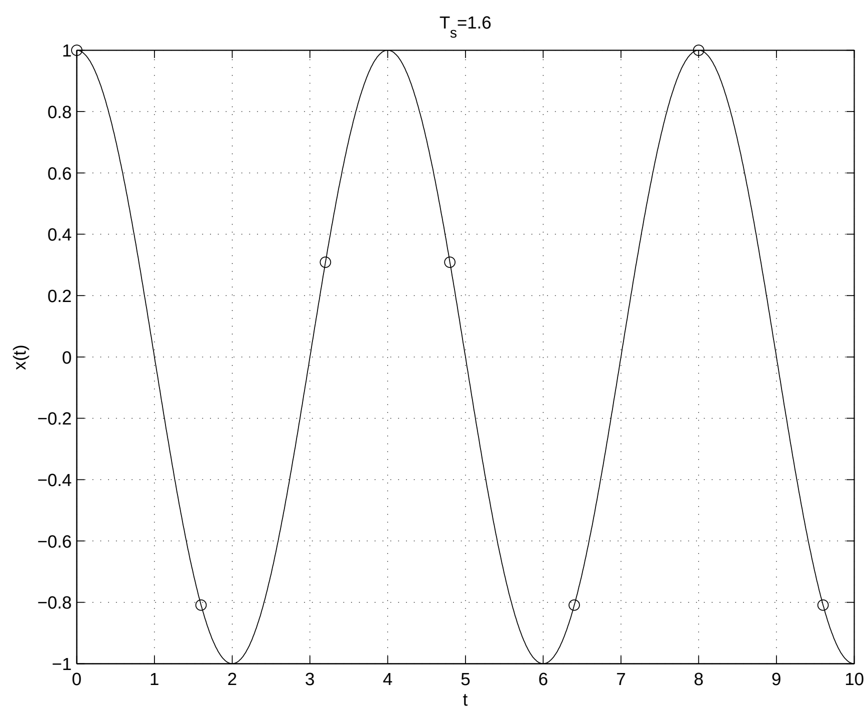


Figure 3: $T_s = 0.4T$

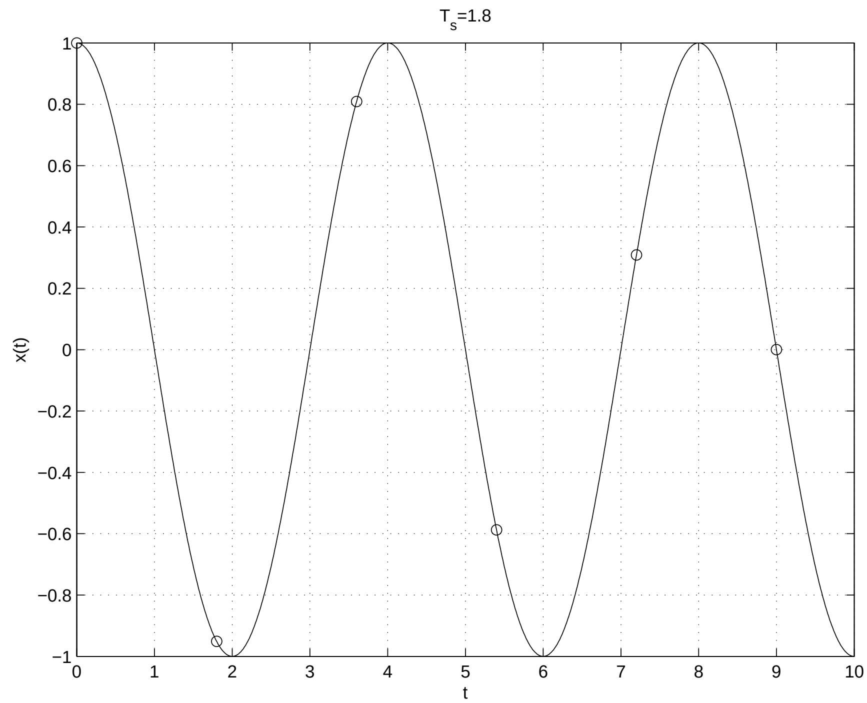


Figure 4: $T_s = 0.45T$

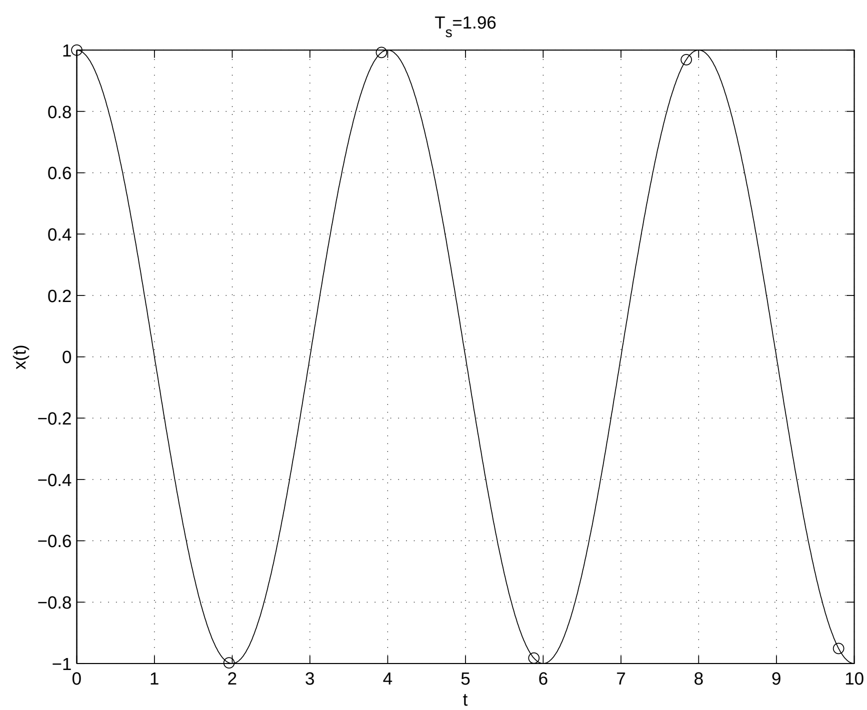


Figure 5: $T_s = 0.49T$

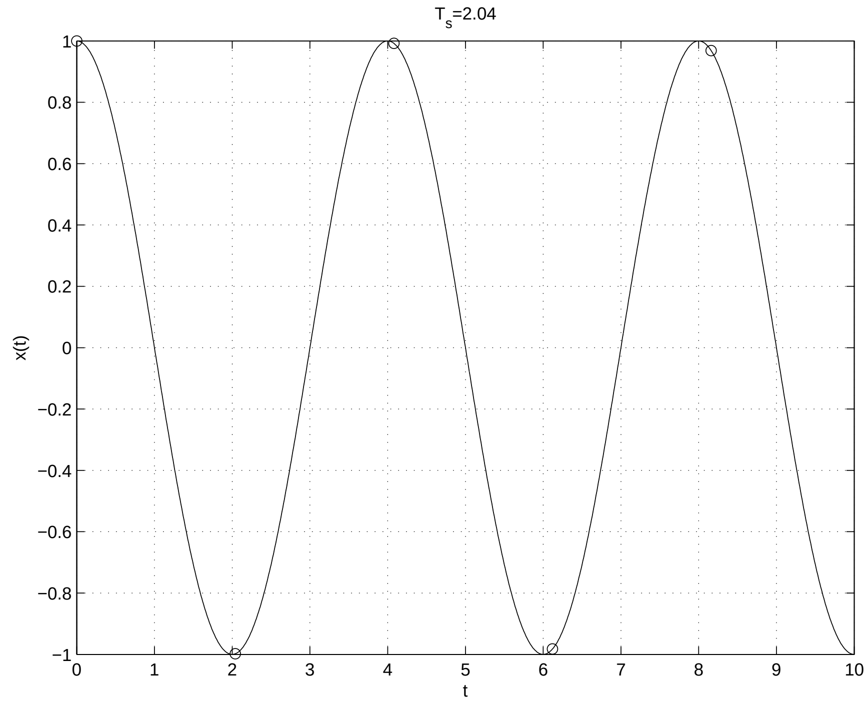


Figure 6: $T_s = 0.51T$

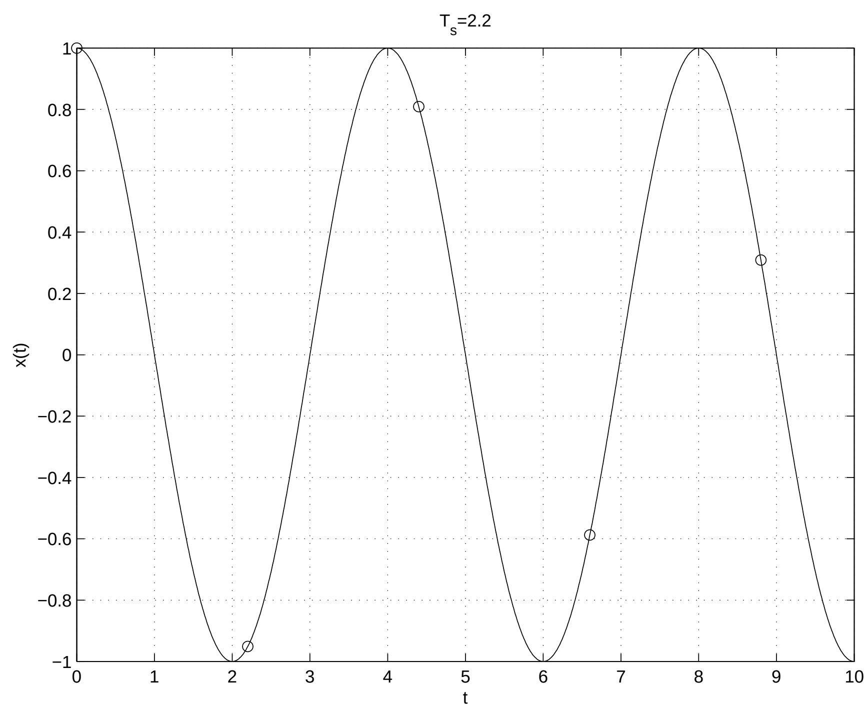


Figure 7: $T_s = 0.55T$

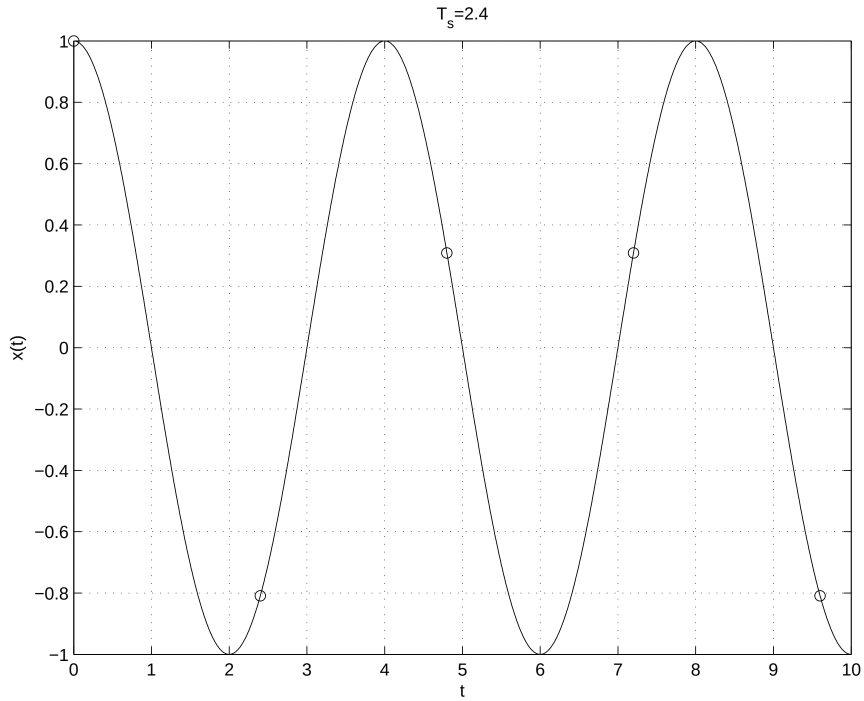


Figure 8: $T_s = 0.6T$

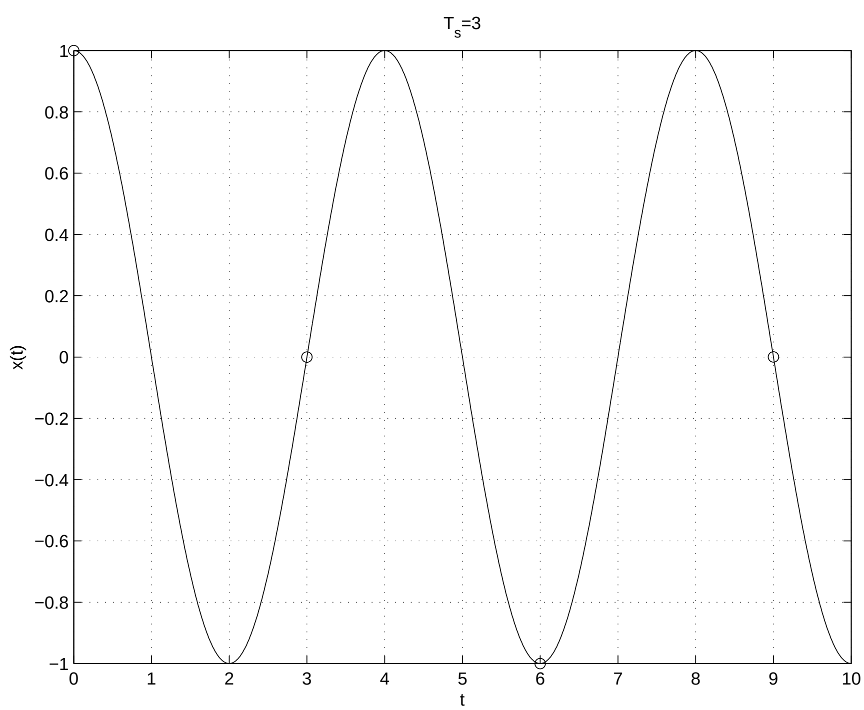
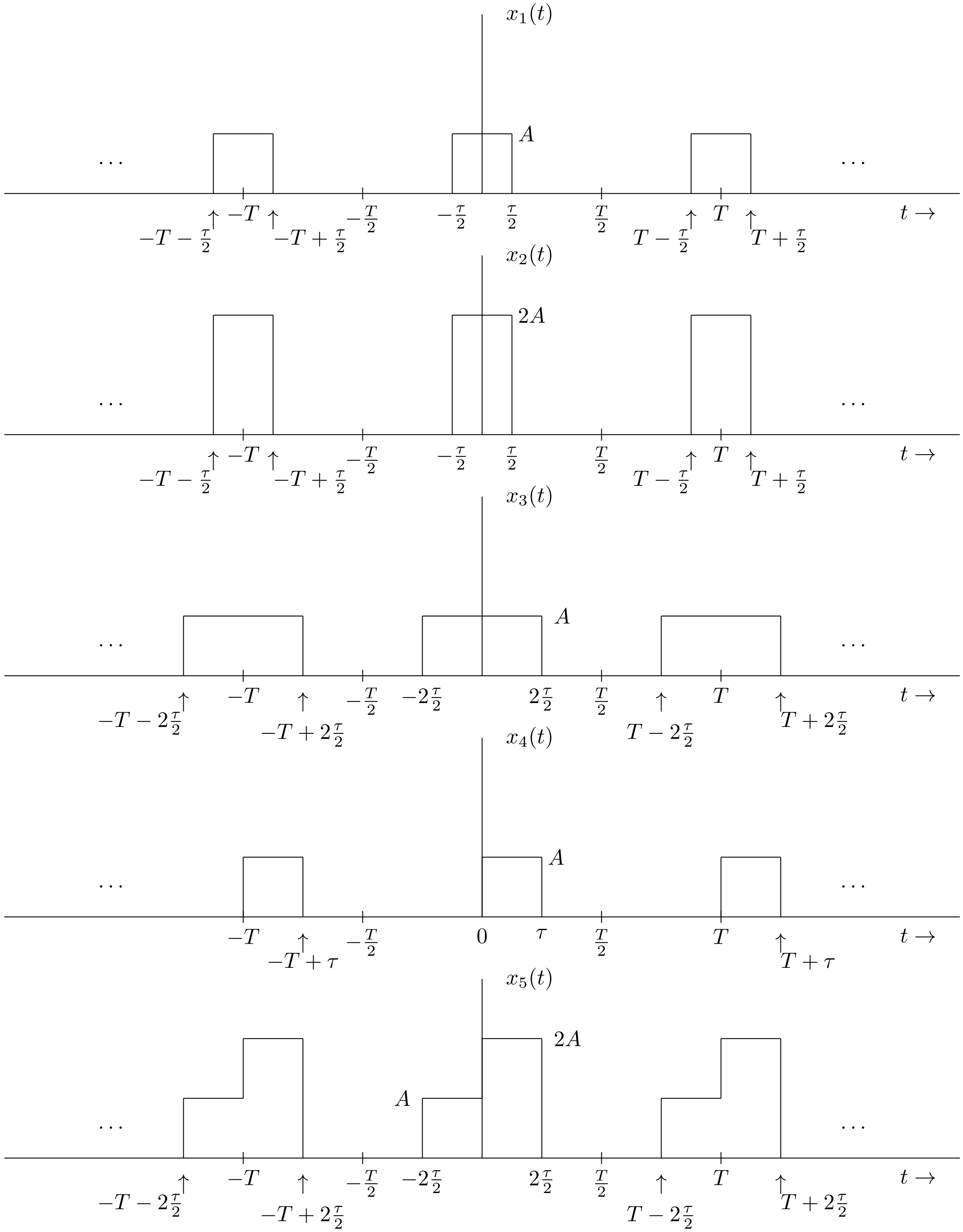


Figure 9: $T_s = 0.75T$

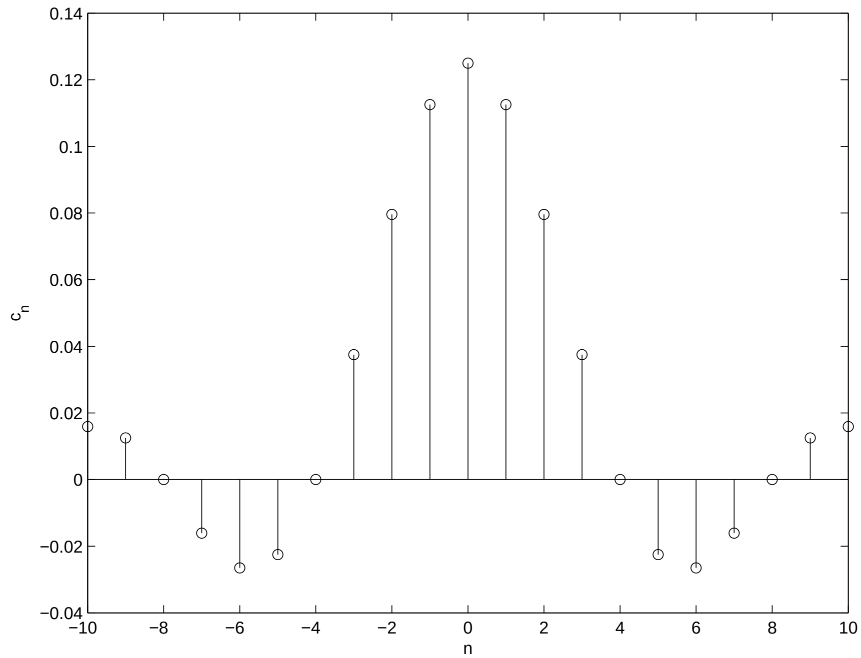
3. ($40 = 5 \times 2 + 5 \times 6$ pts.) Please consider the following periodic time signals (fundamental period is T).



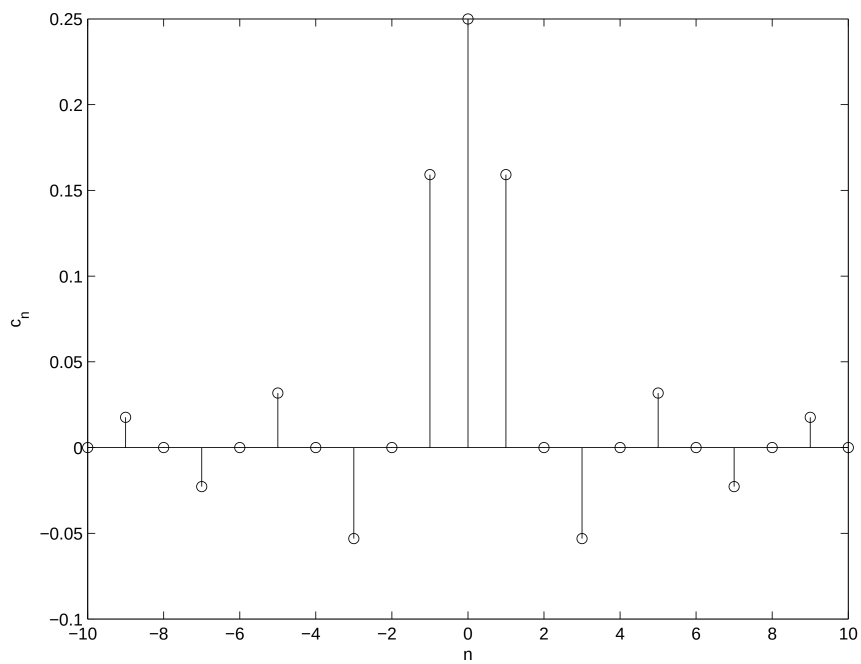
(a)
   i. Please write $x_2(t)$ as a function of $x_1(t)$.
   ii. Is it correct to write that $x_3(t) = x_1(t/2)$?
   iii. Please write $x_4(t)$ as a function of $x_1(t)$.
   iv. Please write $x_5(t)$ as a function of $x_3(t)$ and $x_1(t)$.
   v. Please write $x_5(t)$ as a function of $x_1(t)$.

(b) Please consider the following plots of Fourier series coefficients. Which plot corresponds to which signal $x_1(t)$, $x_2(t)$, ..., $x_5(t)$?

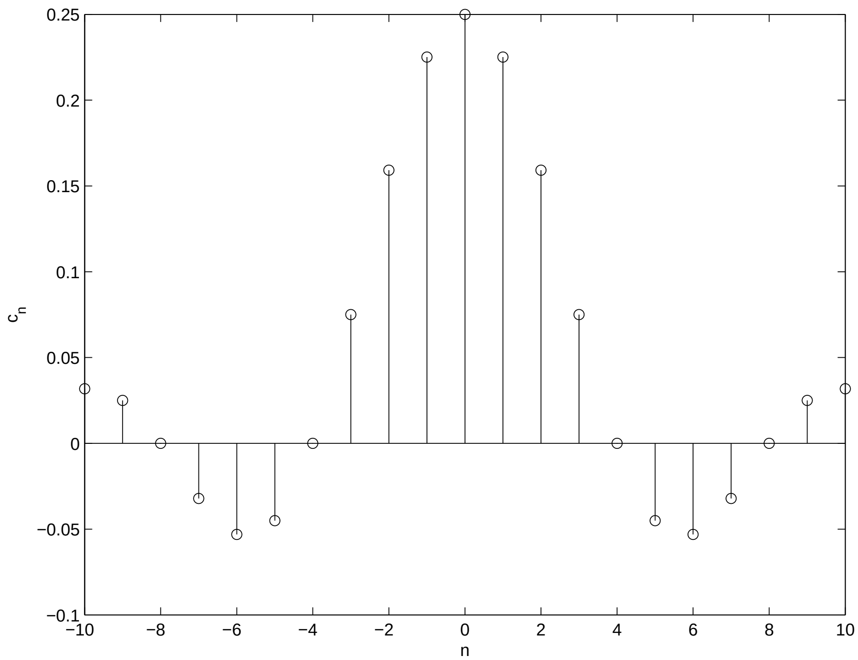
Option A



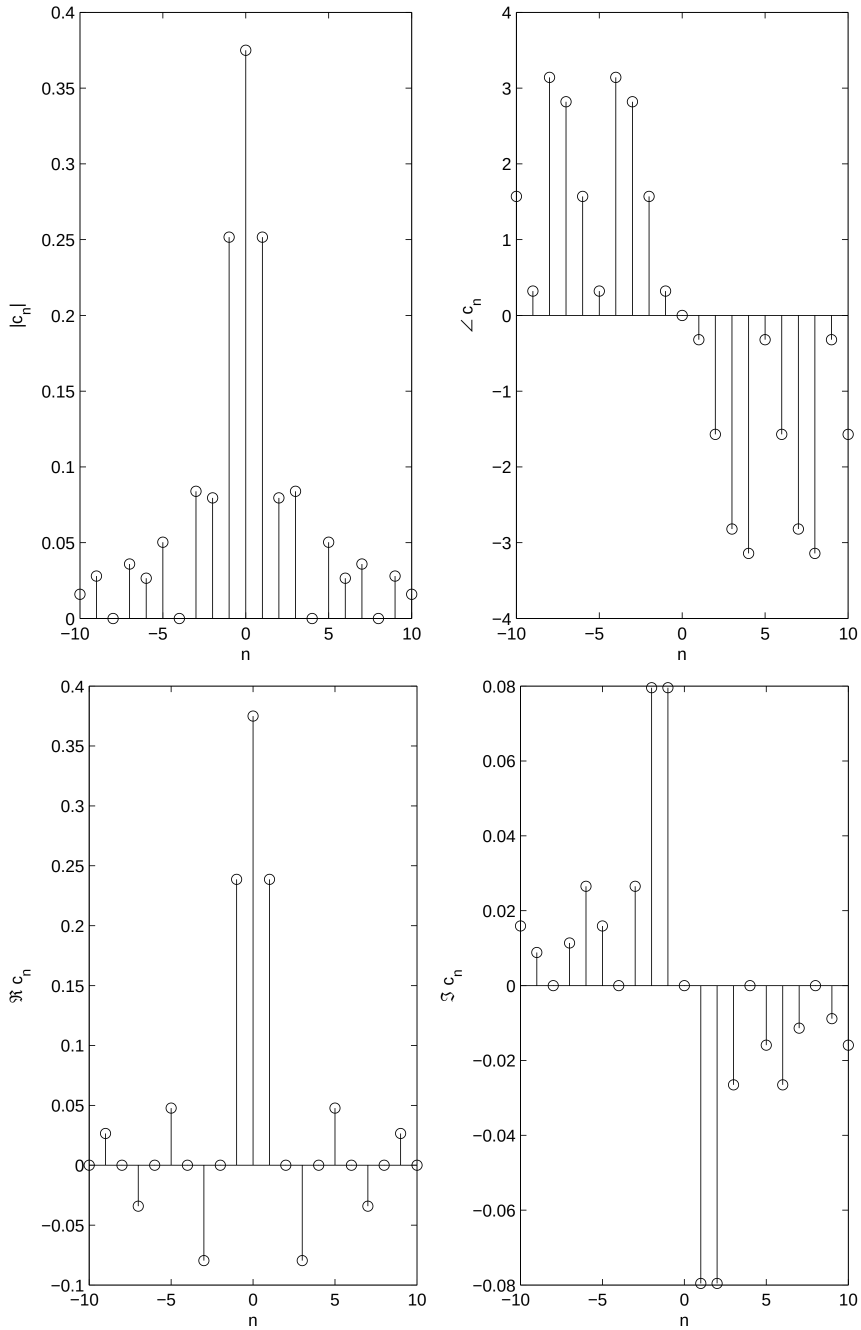
Option B



Option C



Option D The Fourier series coefficients are complex and both the magnitude and phase and the real and imaginary parts are shown.



Option E The Fourier series coefficients are complex and both the magnitude and phase and the real and imaginary parts are shown.