12 Nov. 2014 – Due WEDNESDAY, 19 Nov. 2014 in Dropbox
20% grade reduction per late day submission
There are 6 problems to this homework

1. Read the book, Chapter 7.

2. *Problem 7.1.* The times between successive customer arrivals at a facility are independent and identically distributed random variables with the following PMF:

$$p(k) = \begin{cases} 0.2, & k = 1, \\ 0.3, & k = 3, \\ 0.5, & k = 4, \\ 0, & \text{otherwise.} \end{cases}$$

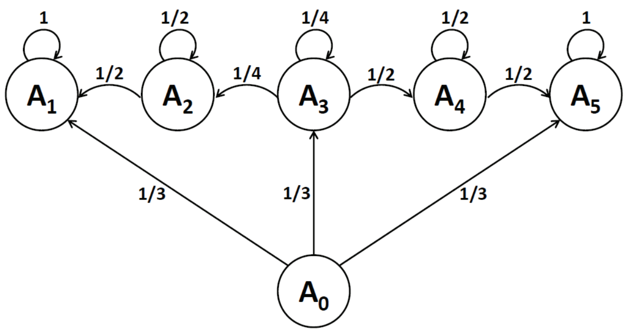
Construct a four-state Markov chain model that describes the arrival process. In this model, one of the states should correspond to the times when an arrival occurs.

3. *Problem 7.2.* A mouse moves along a tiled corridor with $2m$ tiles, where $m > 1$. From each tile $i \neq 1$, $2m$, it moves to either tile $i - l$ or $i + 1$ with equal probability. From tile 1 or tile $2m$, it moves to tile 2 or $2m$ - 1, respectively, with probability 1. Each time the mouse moves to a tile $i \leq m$ or $i > m$, an electronic device outputs a signal L or R, respectively. Can the generated sequence of signals L and R be described as a Markov chain with states L and R? Explain.

4. *Problem 7.4.* A spider and a fly move along a straight line in unit increments. The spider always moves towards the fly by one unit. The fly moves towards the spider by one unit with probability 0.3, moves away from the spider by one unit with probability 0.3, and stays in place with probability 0.4. The initial distance between the spider and the fly is integer. When the spider and the fly land in the same position, the spider captures the fly.

(a) Construct (and draw it) a Markov chain that describes the relative location of the spider and fly.
(b) Identify the transient and recurrent states.

5. Consider the Markov below chain. Assume that the process begins in state A_0. Calculate:



(a) The probability that the process enters the state A_2 for the first time on trial k.
(b) The probability that the process never enters the state A_4.
(c) The probability that the process enters state A_2 and then leaves A_2 on the next trial.
(d) The probability that the process enters A_1 for the first time on the 3^{rd} trial.
(e) The probability that the process is in state A_3 immediately after the nth trial.

6. *Problem 7.11: For this problem, it is easier to use a transition probability matrix, and that is explained on pages 341-342 and also in Example 7.3.*
A professor gives tests that are hard, medium, or easy. If she gives a hard test, her next test will be either medium or easy, with equal probability. However, if she gives a medium or easy test, there is a 0.5 probability that her next test will be of the same difficulty, and a 0.25 probability for each of the other two levels of difficulty. Construct an appropriate Markov chain and find the steady-state probabilities.