27 Oct. 2014 – Due MONDAY, 03 Nov. 2014 in Dropbox
20% grade reduction per late day submission
There are 9 problems to this homework

1. Read the book, Chapter 6.

2. *Problem 6.1.* Each of n packages is loaded independently onto either a red truck (with probability p) or onto a green truck (with probability $1 - p$). Let R be the total number of items selected for the red truck and let G be the total number of items selected for the green truck.

(a) Determine the PMF, expected value, and variance of the random variable R.
(b) Evaluate the probability that the first item to be loaded ends up being the only one on its truck.
(c) Evaluate the probability that at least one truck ends up with a total of exactly one package.
(d) Evaluate the expected value and the variance of the difference $R - G$.
(e) Assume that $n. \geq 2$. Given that both of the first two packages to be loaded go onto the red truck, find the conditional expectation, variance, and PMF of the random variable R.

3. *Problem 6.2.* Dave fails quizzes with probability 1/4, independent of other quizzes.

(a) What is the probability that Dave fails exactly two of the next six quizzes?
(b) What is the expected number of quizzes that Dave will pass before he has failed three times?
(c) What is the probability that the second and third time Dave fails a quiz will occur when he takes his eighth and ninth quizzes, respectively?
(d) What is the probability that Dave fails two quizzes in a row before he passes two quizzes in a row?

4. *Problem 6.3.* A computer system carries out tasks submitted by two users. Time is divided into slots. A slot can be idle, with probability $p_I = 1/6$, and busy with probability $p_B = 5/6$. During a busy slot, there is probability $p_{1|B} = 2/5$ (respectively, $p_{2|B} = 3/5$) that a task from user 1 (respectively, 2) is executed. We assume that events related to different slots are independent.

(a) Find the probability that a task from user 1 is executed for the first time during the 4th slot.
(b) Given that exactly 5 out of the first 10 slots were idle, find the probability that the 6th idle slot is slot 12.
(c) Find the expected number of slots up to and including the 5th task from user 1.
(d) Find the expected number of busy slots up to and including the 5th task from user 1.
(e) Find the PMF, mean, and variance of the number of tasks from user 2 until the time of the 5th task from user 1.

5. *Problem 6.8.* During rush hour, from 8 a.m. to 9 a.m., traffic accidents occur according to a Poisson process with a rate of 5 accidents per hour. Between 9 a.m. and 11 a.m., they occur as an independent Poisson process with a rate of 3 accidents per hour. What is the PMF of the total number of accidents between 8 a.m. and 11 a.m.?

6. *Problem 6.9.* An athletic facility has 5 tennis courts. Pairs of players arrive at the courts and use a court for an exponentially distributed time with mean 40 minutes. Suppose a pair of players arrives and finds all courts busy and k other pairs waiting in queue. What is the expected waiting time to get a court?

7. *Problem 6.12.* A pizza parlor serves n different types of pizza, and is visited by a number K of customers in a given period of time, where K is a Poisson random variable with mean λ. Each customer orders a single pizza, with all types of pizza being equally likely, independent of the number of other customers and the types of pizza they order. Find the expected number of different types of pizzas ordered.

8. *Problem 6.13* Transmitters A and B independently send messages to a single receiver in a Poisson manner, with rates of λ_A and λ_B, respectively. All messages are so brief that we may assume that they occupy single points in time. The number of words in a message, regardless of the source that is transmitting it, is a random variable with PMF

$$p_W(w) = \begin{cases} 2/6, & \text{if } w = 1, \\ 3/6, & \text{if } w = 2, \\ 1/6, & \text{if } w = 3, \\ 0, & \text{otherwise,} \end{cases}$$

and is independent of everything else.

(a) What is the probability that during an interval of duration t, a total of exactly nine messages will be received?

(b) Let N be the total number of words received during an interval of duration t. Determine the expected value of N.

(c) Determine the PDF of the time from $t = 0$ until the receiver has received exactly eight three-word messages from transmitter A.

(d) What is the probability that exactly eight of the next twelve messages received will be from transmitter A?

9. A desktop computer is used until it fails, at which point it is sent out to be repaired. The time between failures (this is also the length of time that the computer is functional until it needs to be repaired) is a random variable T. The times between failures (i.e., $T_1, T_2, ..., T_n$) are independent random variables that are identically distributed. For $t > 0$, let $N(t)$ be the number of computers that have failed.

(a) What type of process is $N(t)$ if the time between failures of each computer has an exponential PDF?

(b) Say that you just bought 10 new computers, and that each as a 90 day warranty. If the mean time between failures (MTBF) is 250 days, what is the probability that at least one computer will fail before the end of the warranty period?